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A STUDY ON FUZZY EQUITABLE EDGE COLORING OF WHEEL RELATED GRAPHS

K. SIVARAMAN[1] AND R. VIKRAMA PRASAD

ABSTRACT. Equitable edge coloring is a kind of graph labeling with the following restrictions. No two adjacent edges receive same label (color). and number of edges in any two color classes differ by at most one. In this work we are going to present the Fuzzy equitable edge coloring of some wheel related graphs.

1. INTRODUCTION

Here we are going to connect fuzzy graphs with Equitable edge coloring by using the sequence methodology. All the preliminaries are in [2–8]. Before that we have stated the preliminaries requisite for our work below.

1.1. PRELIMINARY AND FUNDAMENTAL DEFINITIONS.

Definition 1.1. *[4] **Proper edge coloring:** The Proper edge coloring of a graph means a kind of graph labeling with the following conditions.*
(i) *no two incident edges at any vertex receive same color;*
(ii) *number of edges in any two color classes differ by at most one.*

Definition 1.2. *[4] Proper edge coloring of a graph G is called an equitable edge coloring if* $|M(s) - M(t)| \leq 1$ *for all* $s, t \in \{1, 2, 3, \ldots, \Delta\}$*, where ,* Δ *is the*

[1]*corresponding author*

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maximum degree of the graph $G, M(s)$ and $M(t)$ represents the number of edges in the color classes s and t respectively.

Definition 1.3. *[1] **Path:** A Path P_n is defined as a walk such that there is no repetition of vertices and edges.*

Definition 1.4. *[1] **Cycle:** A Cycle $C_n,\ n \geq 3$ is defined as a simple regular graph of degree 2, i.e., all of the vertices in the cycle C_n have same degree. The length of a cycle C_n is the number of edges in that cycle.*

Definition 1.5. *[10] **Fuzzy set:** A Fuzzy Set is a pair $(A,\ m)$ where A is a non-empty set and $m:\ A \to [0,1]$, a membership function. The Reference set A is called the Universe of Discourse and for each $x \epsilon A$, the value $m(x)$ is called the grade or membership of x in (A, m). The function $f = m = \mu_U$ is called the membership function of the fuzzy set $U = (A, m)$.*

Definition 1.6. *[9] **Fuzzy Graph:** A fuzzy graph $\xi = (V, \sigma, \mu)$ is a triple consists of a non-empty set V together with a pair of functions, $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that for all $x, y \in V,\ \mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ and μ is a symmetric fuzzy relation on σ.*

Lemma 1.1. *[4] Every simple undirected graph G may be Edge colored using a number of colors that is at most one greater than the maximum degree Δ of the graph G.*

2. MAIN RESULTS

Definition 2.1. *Let (a_n) be an infinite increasing sequence of positive integers. A mapping $\mu : E(G) \to [0,1]$ is called fuzzy equitable edge coloring if it is induced by the function $\sigma : V(G) \to [0,1]$ defined as $\sigma : (v_i) = \frac{a_i}{a_i+1}$ such that*

- (i) *$\mu(uv) \leq \sigma(u) \wedge \sigma(v)$;*
- (ii) *$\mu : E(G) \to \{b, b^2, b^3, \dots b^{\Delta}\}$ defines a proper edge coloring to the edges of the graph G; and*
- (iii) *number of edges in any two color classes differ by at most one, i.e., $|l_f(s) - l_f(t)| \leq 1$ for all $i, j \in \{b, b^2, b^3, \dots.\}$ where (i) $b \epsilon [0,1]$ is calculated from the vertex labels, (ii) $l_f(s)$ and $l_f(t)$ denote the number of edges in the color classes s and t respectively. The minimum number of colors needed for*

the b equitable edge coloring of G is called fuzzy equitable edge chromatic number and it is denoted by χ'_{fe}.

2.1. **CONSTRUCTIVE ALGORITHM.** [Prism-Wheel Graph]

Step 1: Draw two Cycles C_n such that one is dawn inside the other. Let the vertices of these two Cycles be $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$

Step 2: Introduce a central vertex say v_0.

Step 3: Join the vertex v_0 with the vertices $u_1, u_2, \ldots, u_n$ and join the vertex u_i with the vertex v_i, $i = 1, 2, 3, \ldots n$. Thus we get a graph called Prism-Wheel graph $(PW(n))$.

Step 4: It is observe that this graph has $2n + 1$ vertices, $4n$ edges. Maximum Degree $\Delta = n.$ Degree of the vertex v_0 is n,degree of the vertices $u_1, u_2, \ldots, u_n$ are equal to 4 and Degree of the vertices $v_1, v_2, \ldots, v_n$ are equal to 3. Sum of the degree of all vertices of this graph $= 8n = 2 * 4n =$ Twice the number of edges of this graph.

Example 1.

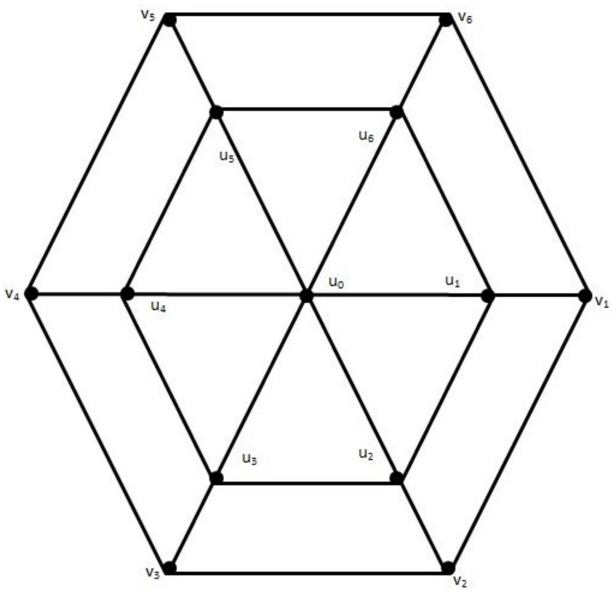

FIGURE 1. Prism-Wheel Graph PW(n)

Theorem 2.1. *The Prism-Wheel Graph admits the fuzzy equitable edge coloring and its fuzzy equitable edge Chromatic number is* $\chi'_{fe}(PW(n)) = n$.

Proof. Let G be a Prism-wheel graph. To prove that the graph G admits fuzzy equitable edge coloring, we first define a function $\sigma : V(G) \to [0.1]$ by

$$\sigma(v) = \begin{cases} \frac{a_i}{a_i+1} & \text{for} \quad v = u_i \ , i = 1, 2, 3, \ldots, n \\ \frac{a_{n+i}}{a_{n+i}+1} & \text{for} \quad v = v_i, i = 1, 2, 3, \ldots, n \\ \frac{a_{2n+1}}{a_{2n+1}+1} & \text{for} \quad v = v_0 \end{cases} .$$

Now let $b = \prod_{v \epsilon V(G)} \sigma(v)$, By Lemma 1.1.1, we need $\Delta = n$ colors for proper edge coloring of this graph G.

$$\mu(u\ v) = \begin{cases} b^i & \text{for} \quad u = v_0 \text{ and } v = u_i, i = 1, 2, 3, \ldots, n \\ b^{i+1} & \text{for} \quad u = u_i \text{ and } v = v_i, i = 1, 2, 3, \ldots, n-1 \\ b & \text{for} \quad u = u_n \text{ and } v = v_n \\ b^{i+3} & \text{for} \quad u = x_i \text{ and } v = x_{i+1}, x = u, \\ & \quad\quad v \text{ and } i = 1, 2, 3, \ldots, n-3 \\ b^{i+3 \bmod n} & \text{for} \quad u = x_i \text{ and } v = x_{i+1}, i = n-2, n-1, n. \end{cases}$$

Therefore from this mapping we find that $|l_{fe}(i) - l_{fe}(j)| \leq 1$ for all $i, j \in \{b, b^2, b^3, \ldots b^n\}$ and hence the graph G admits the fuzzy equitable edge coloring. Also the graph G is transferred into a Fuzzy graph with the above edge coloring, the color assigned to each edge is its membership value and so $\chi'_{fe}(G) = n$. □

2.2. CONSTRUCTIVE ALGORITHM. [Star wheel graph $SW(n)$]

Step 1: Draw a cycle C_n with n vertices say $v_1, v_2, \ldots, v_n$.

Step 2: Introduce a central vertex say v_0.

Step 3: Join the Central vertex v_0 with the vertices $, v_2, \ldots, v_n$.

Step 4: Introduce a vertex between the adjacent vertices of the cycle $C_n$and let them be $u_1, u_2, \ldots, u_n$.

Step 5: Introduce a pair of vertices $t_i, w_i, \ i = 1, 2, 3, \ldots, n$ outside the wheel graph. Join the vertex v_i with the vertices $t_i, \ w_i$ and join t_i with $w_i, , \ i = 1, 2, 3, \ldots, n$. Thus we get a graph called Star-Wheel Graph $(SW(n))$.

Step 6: It is observed that this graph has $4n + 1$ vertices, $6n$ edges, Maximum degree of the this graph. Degree of the vertex v_0 is n, degree of the vertices v_i

are equal to 3, degree of the vertices u_i are equal to 4 , degree of the vertices $t_i,\ w_i$ are equal to 2, where $i = 1, 2, 3, \ldots, n$. Sum of the degree of all vertices of this graph is $12n = 2*$Number edges of this graph

Example 2.

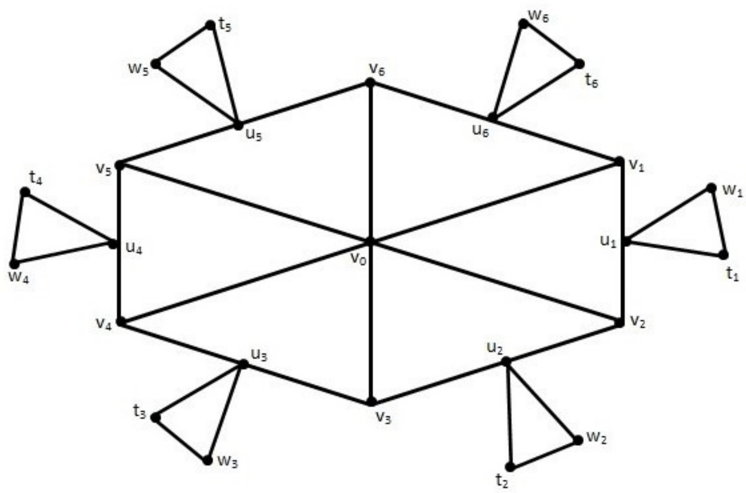

FIGURE 2. Double Wheel Graph DW(n)

Theorem 2.2. *The Star-Wheel Graph admits the fuzzy equitable edge coloring and its fuzzy equitable edge Chromatic number is* $\chi'_{fe}\ (DW(n)) = n$.

Proof. Let G be a Prism-wheel graph. To prove that the graph G admits fuzzy equitable edge coloring, we first define a function $\sigma : V(G) \to [0.1]$ by

$$\sigma(v) = \begin{cases} \frac{a_i}{a_i+1} & \text{for } v = u_i\ , i = 1, 2, 3, \ldots, n \\ \frac{a_{n+i}}{a_{n+i}+1} & \text{for } v = v_i, i = 1, 2, 3, \ldots, n \\ \frac{a_{2n+1}}{a_{2n+2}} & \text{for } v = v_0 \\ \frac{a_{2n+1+i}}{a_{2n+1+i}+1} & \text{for } v =\ w_i, i = 1, 2, 3, \ldots, n \\ \frac{a_{3n+1+i}}{a_{3n+1+i}+1} & \text{for } v =\ t_i, i = 1, 2, 3, \ldots, n \end{cases}$$

Now let $b = \prod_{v\epsilon V(G)} \sigma(v)$. By Lemma 1.1.1, we need Δ=n colors for proper edge coloring of this graph G. So to color the edges of G properly, let us define a function $\mu : E\left(G\right) \to \{\ b, b, b^3, \ldots .b^n\}$ by

$$\mu\left(u\ v\right) = \begin{cases} b^i & \text{for} \quad u = v_0 \text{ and } v = v_i, i = 1,2,3,\ldots,n \\ b^{i+1} & \text{for} \quad u = u_i \text{ and } v = v_i, i = 1,2,3,\ldots,n-1 \\ b & \text{for} \quad u = u_n \text{ and } v = v_n \\ b^i & \text{for} \quad u = v_{i+1} \text{ and } v = u_i, i = 1,2,3,\ldots,n \\ b^{i+1} & \text{for} \quad u = w_i \text{ and } v = t_i, i = 1,2,3,\ldots,n-1 \\ b & \text{for} \quad u = w_n \text{ and } v = t_n \\ b^{i+3} & \text{for} \quad u = w_i \text{ and } v = u_i, i = 1,2,3,\ldots,n-3 \\ b^{i+3 \bmod n} & \text{for} \quad u = t_i \text{ and } v = u_i, i = n-2, n-1, n \\ b^{i+2} & \text{for} \quad u = t_i \text{ and } v = u_i, i = 1,2,3,\ldots,n-2 \\ b^{i+2 \bmod n} & \text{for} \quad u = t_i \text{ and } v = u_i, i = n-1, n. \end{cases}$$

Therefore from this mapping we find that $|l_{fe}(i) - l_{fe}(j)| \leq 1$ for all $i, j \in \{b, b^2, b^3, \ldots, b^n\}$ and hence the graph G admits the fuzzy equitable edge coloring. Also the above graph G is transformed into a Fuzzy graph with the above edge coloring, the color assigned to each edge is its membership value and so $\chi'_{fe}(G) = n$. □

CONCLUSION

Hence we discussed about the Fuzzy equitable edge coloring of some wheel related graphs.

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RESEARCH SCHOLAR, DEPARTMENT OF MATHEMATICS
PERIYAR UNIVERSITY
SALEM - 636011 TAMIL NADU, INDIA.
Email address: sivaraman1729@gmail.com

DEPARTMENT OF MATHEMATICS, GOVERNMENT ARTS COLLEGE (AUTONOMOUS)
SALEM - 636007, TAMIL NADU, INDIA.
Email address: vikramprasad20@gmail.com