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WEAKLY (1,2)*-$\widetilde{G}$-CLOSED SETS IN BIOTOPOLOGICAL SPACES

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ABSTRACT. The concept of bitopological spaces was first introduced by J.C. Kelly [2] in 1963. Regular open sets have been introduced and investigated by Stone [5]. In this paper, we introduce a new class of generalized closed sets called weakly (1,2)*-$\widetilde{g}$-closed sets which contains the above mentioned class. Also, we investigate the relationships among the related generalized closed sets.

1. INTRODUCTION

The concept of bitopological spaces was first introduced by J.C. Kelly [2] in 1963. Regular open sets have been introduced and investigated by Stone [5]. in this paper, we introduce a new class of generalized closed sets called weakly (1,2)*-$\widetilde{g}$-closed sets which contains the above mentioned class. Also, we investigate the relationships among the related generalized closed sets.

2. PRELIMINARIES

Definition 2.1. *Let is be a subset of X. Then is s said to be $\tau_{1,2}$-open [3] $fis = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets of X is denoted by (1,2)*-O(X) (resp. (1,2)*-C(X)).*

Definition 2.2 (3). *Let is be a subset of a bitopological space X. Then*

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(1) *the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined by $\cup\{V : V \subseteq S$ and V is $\tau_{1,2} -$ open$\}$;*
(2) *the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined by $\cap\{V : is \subseteq V$ and $V\ s\ \tau_{1,2} -$ closed$\}$.*

Remark 2.1 (3). *Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.*

Definition 2.3 (3). . *Let is be a subset of a bitopological space X. Then A is called*
(1) *(1,2)*-semi-open set $fis \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(S)); The complement (1,2)*-semi-open set (1,2)*-semi -closed.*
(2) *regular (1,2)*-open set if is = $\tau_{1,2}$-int($\tau_{1,2}$-cl(S)). The complement of regular (1,2)*-open set is regular (1,2)*-closed.*
(3) *(1,2)*-π-open if the finite union of regular (1,2)*-open sets.*

Definition 2.4. *A subset is of a bitopological space X is called*
(i) *(1,2)*-$\hat{g}$-closed set [1] f $\tau_{1,2}$-$cl(S) \subseteq V$ whenever $S \subseteq V$ and V is (1,2)*-semi-open in X. The complement of (1,2)*-$\hat{g}$-closed set is called (1,2)*-$\hat{g}$-open set.*
(ii) *(1,2)*-αg-closed set [4] if (1,2)*-$\alpha cl(S) \subseteq V$ whenever $S \subseteq V$ and V is $\tau_{1,2}$-open in X. The complement of (1,2)*-αg-closed set is called (1,2)*-αg-open set.*

3. WEAKLY (1,2)*-$\widetilde{g}$-CLOSED SETS

We introduce the definition of weakly (1,2)*-$\widetilde{g}$-closed sets in bitopological spaces and study the relationships of such sets.

Definition 3.1. *A subset is of a bitopological space X is called a weakly (1,2)*-$\widetilde{g}$-closed (briefly, (1,2)*-w$\widetilde{g}$-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(S)) $\subseteq V$ whenever $S \subseteq V$ and V is (1,2)*-$\hat{g}$-open in X.*

Definition 3.2. *A subset is of a bitopological space X is called*
(i) *weakly (1,2)*-πg-closed (briefly, (1,2)*-wπg-closed) set if $\tau_{1,2}$-cl($\tau_{i,j}$-int(S)) $\subseteq$ V whenever $S \subseteq V$ and V is (1,2)*-π-open in X.*
(ii) *regular weakly (1,2)*-generalized closed (briefly, (1,2)*-rwg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(S)) $\subseteq V$ whenever $S \subseteq V$ and V is regular (1,2)*-open in X.*

Theorem 3.1. *Every (1,2)*-$\hat{g}$-closed set is (1,2)*-w$\widetilde{g}$-closed but not conversely.*

Proof. Let is be any (1,2)*-$\hat{g}$-closed set and V be any (1,2)*-semi-open set containing A. Every (1,2)*-semi-open is (1,2)*-$\hat{g}$-open set. We have $\tau_{1,2}$-cl($\tau_{i,j}$-int(S)) $\subseteq V$. Thus, is is (1,2)*-w$\widetilde{g}$-closed. □

Example 1. *Let $X = \{a_1, b_1, c_1\}$, $\tau_1 = \{\emptyset, X, \{a_1, b_1\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then $\{\emptyset, X, \{a_1, b_1\}\}$ are called $\tau_{1,2}$-open and the $\{\emptyset, X, \{c_1\}\}$ are called $\tau_{1,2}$-closed. Then $\{b_1\}$ is (1,2)*-w$\widetilde{g}$-closed set but it is not a (1,2)*-$\hat{g}$-closed in X.*

Theorem 3.2. *Every (1,2)*-w$\widetilde{g}$-closed set is (1,2)*-wπg-closed but not conversely.*

Proof. Let is be any (1,2)*-w$\widetilde{g}$-closed set and V be any (1,2)*-π-open set containing S. Then V is a (1,2)*-$\hat{g}$-open set containing S. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(S)) $\subseteq V$. Thus, is is (1,2)*-wπg-closed. □

Example 2. *Let $X = \{a_1, b_1, c_1\}$, $\tau_1 = \{\emptyset, X, \{a_1\}\}$ and $\tau_2 = \{\emptyset, X, \{a_1, b_1\}\}$. Then $\{\emptyset, X, \{a_1\}, \{a_1, b_1\}\}$ are called $\tau_{1,2}$-open and the $\{\emptyset, X, \{c_1\}, \{b_1, c_1\}\}$ are called $\tau_{1,2}$-closed. Then $\{a_1, b_1\}$ is (1,2)*-wπg-closed but it is not a (1,2)*-w$\widetilde{g}$-closed.*

Theorem 3.3. *Every (1,2)*-w$\widetilde{g}$-closed set is (1,2)*-rwg-closed but not conversely.*

Proof. Let is be any (1,2)*-w$\widetilde{g}$-closed set and V be any regular (1,2)*-open set containing S. Then V is a (1,2)*-$\hat{g}$-open set containing S. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(S)) $\subseteq V$. Thus, is is (1,2)*-rwg-closed. □

Example 3. *Let $X = \{a_1, b_1, c_1\}$, $\tau_1 = \{\emptyset, X, \{a_1\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then $\{\emptyset, X, \{a_1\}\}$ are called $\tau_{1,2}$-open and the $\{\emptyset, X, \{b_1, c_1\}\}$ are called $\tau_{1,2}$-closed. Then $\{a_1\}$ is (1,2)*-rwg-closed but it is not a (1,2)*-w$\widetilde{g}$-closed.*

Theorem 3.4. *If a subset is of a bitopological space X is both $\tau_{1,2}$-closed and (1,2)*-α g-closed, then it is (1,2)*-w$\widetilde{g}$-closed in X.*

Proof. Let is be an (1,2)*-αg-closed set in X and V be any $\tau_{1,2}$-open set containing S. Then $V \supseteq$ (1,2)*-$\alpha cl(S) = is \cup \tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(S))). Since is is $\tau_{1,2}$-closed, $V \supseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(S)) and hence is is (1,2)*-w$\widetilde{g}$-closed in X. □

Theorem 3.5. *If a subset is of a bitopological space X is both $\tau_{1,2}$-open and (1,2)*-w$\widetilde{g}$-closed, then it is $\tau_{1,2}$-closed.*

Proof. Since is is both $\tau_{1,2}$-open and (1,2)*-w$\widetilde{g}$-closed, $S \supseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(S)) = $\tau_{1,2}$-cl(S) and hence is is $\tau_{1,2}$-closed in X. □

Remark 3.1. Diagram

$$\begin{array}{ccc} \textit{(1,2)}^*\text{-}\hat{g}\text{-closed} & \longrightarrow & \textit{(1,2)}^*\text{-}w\widetilde{g}\text{-closed} \\ \swarrow & & \searrow \\ \textit{(1,2)}^*\text{-}w\pi\ g\text{-closed} & & \textit{(1,2)}^*\text{-rwg-closed} \end{array}$$

None of the above implications is reversible as shown in the above examples and in the related paper.

4. Conclusion

General topology plays vital role in many fields of applied sciences as well as in all branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc.

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