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ON NANO $\mathcal{M}_r$-SET AND NANO $\mathcal{M}_{r^\star}$-SET IN NANOTOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce the notions of nano $\mathcal{M}_r$-sets and nano $\mathcal{M}_{r^\star}$-sets in nanotopological spaces and their properties are discussed with suitable examples.

1. INTRODUCTION

In 2017, Rajasekaran et *al*, introduced the concept of nano πg-closed sets, nano $\pi g\alpha$-closed sets, nano πgp-closed sets and M. Lellis Thivagar et al [3] introduced the concept of nano topological spaces which was defined in terms of approximations and boundary region of a subset of a universe U using an equivalence relation on it.

In this paper, we introduce the notions of nano $\mathcal{M}_r$-sets and nano $\mathcal{M}_{r^\star}$-sets in nanotopological spaces and their properties are discussed with suitable examples.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ (or X) represent nanotopological spaces on which no separation axioms are assumed unless otherwise mentioned.

We recall the following definitions which are useful in the sequel.

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Definition 2.1. *[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(i) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*
(ii) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.*
(iii) *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Proposition 2.1. *[3] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

(i) $L_R(X) \subseteq X \subseteq U_R(X)$;
(ii) $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
(vi) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
(vii) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
(viii) $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;
(ix) $U_RU_R(X) = L_RU_R(X) = U_R(X)$;
(x) $L_RL_R(X) = U_RL_R(X) = L_R(X)$.

Definition 2.2. *[3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.1, $\tau_R(X)$ satisfies the following axioms:*

(i) *U and* $\phi \in \tau_R(X)$,
(ii) *The union of the elements of any sub collection of* $\tau_R(X)$ *is in* $\tau_R(X)$,
(iii) *The intersection of the elements of any finite subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $I_n(A)$ and $C_n(A)$, respectively.

Remark 2.1. *[3] If $[\tau_R(X)]$ is the nanotopology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.3. *[3] If $(U, \tau_R(X))$ is a nanotopological space with respect to X and if $A \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of A and it is denoted by $I_n(A)$.*

That is, $I_n(A)$ is the largest nano open subset of A. The nano closure of H is defined as the intersection of all nano closed sets containing A and it is denoted by $C_n(A)$.

That is, $C_n(A)$ is the smallest nano closed set containing A.

Definition 2.4. *[3]* A subset A of $(U,\ \mathcal{N})$ is called a

(i) *$n\alpha$-open if $A \subseteq I_n(C_n(I_n(A)))$,*
(ii) *np-open if $A \subseteq I_n(C_n(A))$,*
(iii) *nr-open if $A = I_n(C_n(A))$.*

The complements of the above mentioned open sets are called their respective closed sets.

The nano preinterior, $pI_n(A)$ (resp. nano α-interior, $\alpha I_n(A)$) of A is the union of all nano preopen sets (resp. $n\alpha$-open sets) contained in A. The $n\alpha$-closure $\alpha C_n(A)$ of A is the intersection of all $n\alpha$-closed sets containing A.

Definition 2.5. A subset A of $(U,\ \mathcal{N})$ is called a

(i) *$n\pi$-open [1] if the finite union of nr-open sets,*
(ii) *$n\pi g$-open [8] if $G \subseteq I_n(A)$ whenever $G \subseteq A$ and G is $n\pi$-closed,*
(iii) *nano πgp-open [6] if $G \subseteq pI_n(A)$ whenever $G \subseteq A$ and G is $n\pi$-closed,*
(iv) *nano $\pi g\alpha$-open [7] if $G \subseteq \alpha I_n(A)$ whenever $G \subseteq A$ and G is $n\pi$-closed,*
(v) *nt-set [2] if $I_n(A) = I_n(C_n(A))$,*
(vi) *$n\alpha^\star$-set [5] if $I_n(A) = I_n(C_n(I_n(A)))$.*

The complements of the above mentioned open sets are called their respective closed sets.

Lemma 2.1. *[6] If A is a subset of* U*, then* $pI_n(A) = A \cap I_n(C_n(A))$*,*

Lemma 2.2. *[7] If A is a subset of* U*, then* $\alpha I_n(A) = A \cap I_n(C_n(I_n(A)))$ *and* $\alpha C_n(A) = A \cup C_n(I_n(C_n(A)))$.

Remark 2.2. *The following hold in a nano space.*

(i) *Every* $n\pi g$*-open set is nano* πgp*-open but not conversely. [6]*
(ii) *Every* $n\pi g$*-open set is nano* $\pi g\alpha$*-open but not conversely. [7]*

3. On nano $\mathcal{M}_r$-sets and nano $\mathcal{M}_{r^\star}$-sets

Definition 3.1. *A subset A of a nano space* $(U, \mathcal{N})$ *is called a*

(i) *nano* $\mathcal{M}_r$*-set if* $A = H \cap V$*, where* H *is* $n\pi g$*-open and* V *is a* nt*-set.*
(ii) *nano* $\mathcal{M}_{r^\star}$*-set if* $A = H \cap V$*, where* H *is* $n\pi g$*-open and* V *is a* $n\alpha^\star$*-set.*

Proposition 3.1. *For a subset of a nano space, the following hold: every nano* $\pi g\alpha$*-open set is nano* πgp*-open.*

Proof. It follows from the Definitions. □

Remark 3.1. *The converse of Proposition 3.1 is not true, in general.*

$n\pi g$-open

$\swarrow$

nano $\pi g\alpha$-open $\downarrow$

$\searrow$

nano πgp-open

Example 1. *Let* $U = \{1, 2, 3, 4\}$*,* $U/R = \{\{2\}, \{4\}, \{1, 3\}\}$ *and* $X = \{1, 4\}$*. Clear a nano topology* $\mathcal{N} = \{\phi, \{4\}, \{1, 3\}, \{1, 3, 4\}, U\}$ *and* $\mathcal{N}^c$ = {ϕ, {2}, {2, 4}, {1, 2, 3}, U}.

Remark 3.2. *By Proposition 3.1 and Remark 2.2, we have the following diagram. In this diagram, there is no implication which is reversible as shown by Examples below*

Example 2. *In Example 1, then the set* $\{1,2,4\}$ *is nano* πgp*-open set but not nano* $\pi g\alpha$*-open.*

Proposition 3.2. *For a subset of a nano space, the following hold:*

(i) nt*-set is nano* $\mathcal{M}_r$*-set.*
(ii) *every* $n\alpha^\star$*-set is nano* $\mathcal{M}_{r^\star}$*-set.*
(iii) *every nano* $\mathcal{M}_r$*-set is nano* $\mathcal{M}_{r^\star}$*-set.*
(iv) *every* $n\pi g$*-open set is nano* $\mathcal{M}_r$*-set.*

From Proposition 3.2, We have the following diagram.

Remark 3.3. *The converses of implications in Diagram need not be true as the following Examples show.*

$$\begin{array}{ccccc} n\pi g\text{-open set} & \longrightarrow & \text{nano } \mathcal{M}_r\text{-set} & \longleftarrow & nt\text{-set} \\ & & \downarrow & & \downarrow \\ & & \text{nano } \mathcal{M}_{r^\star}\text{-set} & \longleftarrow & n\alpha^\star\text{-set} \end{array}$$

Example 3. *In Example 1, then the sets*

(i) $\{2\},\{2,4\},\{1,2,3\}$ *are nano* $\mathcal{M}_r$*-set but not* $n\pi g$*-open.*
(ii) $\{1\},\{3\},\{1,4\},\{3,4\},\{1,3,4\}$ *are nano* $\mathcal{M}_r$*-set but not* nt*-set.*
(iii) $\{1,2\},\{2,3\},\{1,2,4\},\{2,3,4\}$ *are nano* $\mathcal{M}_{r^\star}$*-set but not nano* $\mathcal{M}_r$*-set.*
(iv) $\{1\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\}$ *are* $n\alpha^\star$*-set but not* nt*-set.*
(v) $\{1,3,4\}$ *is nano* $\mathcal{M}_{r^\star}$*-set but not* $n\alpha^\star$*-set.*

Proposition 3.3. *A subset* A *of* U *is* $n\pi g$*-open* $\iff$ *it is both nano* πgp*-open and nano* $\mathcal{M}_r$*-set.*

Proof. Necessity is trivial.

We prove the sufficiency. Assume that A is nano πgp-open and nano $\mathcal{M}_r$-set in U. Let $F \subseteq A$ and F is $n\pi$-closed in U. Since A is a nano $\mathcal{M}_r$-set in U, $A = H \cap V$, where H is $n\pi g$-open and V is a nt-set. Since A is nano πgp-open, $F \subseteq pI_n(A) = A \cap I_n(C_n(A)) = (H\cap V)\cap I_n(C_n(H\cap V)) \subseteq (H\cap V)\cap I_n(C_n(H)\cap C_n(V)) = (H\cap V)\cap I_n(C_n(H))\cap I_n(C_n(V))$. This implies $F \subseteq I_n(C_n(V)) = I_n(V)$, since V is a nt-set. Since F is $n\pi$-closed, H is $n\pi g$-open and $F \subseteq H$, we have $F \subseteq I_n(H)$. Therefore, $F \subseteq I_n(H)\cap I_n(V) = I_n(H\cap V) = I_n(A)$. Hence A is $n\pi g$-open in U. □

Corollary 3.1. *A subset A of U is $n\pi g$-open $\iff$ it is both nano $\pi g\alpha$-open and a nano $\mathcal{M}_r$-set in U.*

Proof. This is an immediate consequence of Proposition 3.3. □

Proposition 3.4. *A subset A of U is $n\pi g$-open $\iff$ it is both nano $\pi g\alpha$-open and a nano $\mathcal{M}_{r^\star}$-set in U.*

Proof. Necessity is trivial.

We prove the sufficiency. Assume that A is nano $\pi g\alpha$-open and a nano $\mathcal{M}_{r^\star}$-set in U. Let $F \subseteq A$ and F is $n\pi$-closed in U. Since A is a nano $\mathcal{M}_{r^\star}$-set in U, $A = H \cap V$, where H is $n\pi g$-open and V is a $n\alpha^\star$-set. Now since F is $n\pi$-closed, $F \subseteq H$ and H is $n\pi g$-open, $F \subseteq I_n(H)$. Since A is nano $\pi g\alpha$-open, $F \subseteq \alpha I_n(A) = A \cap I_n(C_n(I_n(A))) = (H \cap V) \cap I_n(C_n(I_n(H \cap V))) = (H \cap V) \cap I_n(C_n(I_n(H) \cap I_n(V))) \subseteq (H \cap V) \cap I_n(C_n(I_n(H)) \cap C_n(I_n(V))) = (H \cap V) \cap I_n(C_n(I_n(H))) \cap I_n(C_n(I_n(V))) = (H \cap V) \cap I_n(C_n(I_n(H))) \cap I_n(V)$, since V is a $n\alpha^\star$-set. This implies $F \subseteq I_n(V)$. Therefore, $F \subseteq I_n(H) \cap I_n(V) = I_n(H \cap V) = I_n(A)$. Hence A is $n\pi g$-open in U. □

Remark 3.4. *In a nano space.*

(i) *The concepts of nano πgp-open sets and nano $\mathcal{M}_r$-sets are independent.*
(ii) *The concepts of nano $\pi g\alpha$-open sets and nano $\mathcal{M}_r$-sets are independent.*

Example 4. *In Example 1, then the set $\{1, 2, 4\}$ is nano πgp-open but not nano $\mathcal{M}_r$-set and $\{1, 2, 3\}$ is nano $\mathcal{M}_r$-set but not nano πgp-open.*

Example 5. *In Example 1, then the set $\{2\}$ is nano $\mathcal{M}_r$-set but not nano $\pi g\alpha$-open set. Also $\{1, 3, 4\}$ is nano $\pi g\alpha$-open set but not nano $\mathcal{M}_r$-set.*

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