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IMPROVED HUNGARIAN METHOD TO SOLVE FUZZY ASSIGNMENT PROBLEM AND FUZZY TRAVELING SALESMAN PROBLEM

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ABSTRACT. In this paper an improved Hungarian method is introduced for solving fuzzy assignment problem. When we apply Hungarian method to solve fuzzy assignment problem, if the minimum number of lines crossing the fuzzy zeros are not equal to the order of the fuzzy cost matrix, this method can be used to get the optimal solution with less computational work. This method reduces the computational work of getting the optimal solution. Further this method can also be applied for finding the Hamiltonian circuit with minimum fuzzy cost in the fuzzy traveling salesman problem. Some numerical examples are furnished to understand the algorithm.

1. INTRODUCTION

Assignment problem deducted from a transportation problem by imposing some additional constraints. It is to find one to one mapping from machines to jobs with the objective that the assignment cost is least. To obtain the optimum assignment, several approaches have been proposed. Hungarian method is the most acclaimed method for solving assignment problem. Kuhn [1] proposed this method and named it as Hungarian method since it is developed from the works of the two Hungarian mathematicians D Konig and Egerváry. Further Munkers [2] gave structure for algorithm. Travelling Salesman problem is to

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deduct a Hamiltonian circuit with minimum weight. In real life problems, the decision parameters are not certain. To overcome this ambiguity fuzzy notions introduced Zadeh [3,4] may be applied. This leads in to a new field called fuzzy optimization. Many researchers worked on Fuzzy assignment problem, such as Balinski and Gomory [5], Chanas et al [6], Chi-Jen Lin and Ue-pyng Wen [7], Dubois and Fortemps [9]. Chen [8] developed a fuzzy assignment problem and proposed basic theorems. Wang [12] presented a fuzzy assignment problem where the cost varies with the job's quality. Mukherjee and Basu [13] proposed a novel algorithm for fuzzy assignment problems. Amit kumar and Anil gupta [15] furnished algorithm to solve fuzzy assignment problems and fuzzy Travelling salesman problem with LR fuzzy parameters. Dubois and Fortemps [9] developed an algorithm which amalgamation of fuzzy theory, multiple criteria decision making and constrain-directed methodology. Long Sheng Huang and Guang-hui Xu [16] proposed method to solve assignment problems with constraint on restriction of qualification. Dhanasekar et al., [11,17] applied element wise subtraction in Hungarian algorithm for solving fuzzy assignment and fuzzy travelling salesamn problem. Dhanasekar et al., [18] applied fuzzy diagonal optimal algorithm to solve fuzzy assignment problem. Dhanasekar et al., [14, 20] solved fuzzy assignment problem and fuzzy travelling salesman problem using Haar Hungarian algorithm. In recent years,many algorithms are proposed to obtain the optimal solutions of fuzzy TSP. Hansen [21] solved fuzzy tsp using tabu search algorithm. Jaszkiewicz [22] solved fuzzy TSP by appling genetic local search algorithm. Yan et al., [23] developed the fuzzified version of evolutionary algorithm for fuzzy TSP. Sepideh Fereidouni [24] applied multi objective linear programming to solve fuzzy TSP. Angel et al., [26] presented dynamic search algorithm to obtain optimal route of fuzzy TSP. Paraquete et al., [25] proposed the pareto local search algorithm for fuzzy TSP.

The choice of a ranking technique to order the fuzzy numbers induces the fuzzy optimization process. In this paper, Yager's ranking technique [10] is used to order the fuzzy numbers. This ranking technique satisfies all the necessary properties of ranking. In this method a slight modification is done on the Hungarian method to reduce the computational complexity. Illustrations are given to validate the algorithm.

In this paper, Section 2 deals with fuzzy preliminaries followed by Section 3 in which the improved Hungarian algorithm for fuzzy assignment problem is given in detail with numerical example. In Section 4, the improved Hungarian algorithm for fuzzy travelling salesman problem is furnished with numerical example. Section 5 deliberates the conclusion.

2. Preliminaries

2.1. Basics. A *fuzzy set* is an assignment of its elements to the $[0, 1]$ by its membership function. A *fuzzy number* is a fuzzy subset $\tilde{A}$ with membership function $\mu_{\tilde{A}}(x)$ which is piece wise continuous, convex and normal.

2.2. Trapezoidal Fuzzy number. A fuzzy number $\tilde{A} = (n_1, n_2, n_3, n_4)$ with membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-n_1}{n_2-n_1} & n_1 \leq x \leq n_2 \\ 1 & n_2 \leq x \leq n_3 \\ \frac{n_4-x}{n_4-n_3} & n_3 \leq x \leq n_4 \\ 0 & otherwise \end{cases}$$

is called a *trapezoidal fuzzy number* [19]. In this if $n_2 = n_3$ then it is called *triangular fuzzy number* [19].

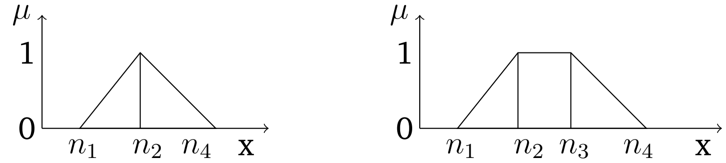

Fig 1. a) Triangular Fuzzy Number b) Trapezoidal Fuzzy Number

2.3. Fuzzy Operations. The *Fuzzy Operations* [19] are given as
Fuzzy Addition:

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$
$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

Fuzzy Subtraction:

$$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$
$$(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$$

2.4. **Yager's centroid Index ranking.** According to Yager [10] the ranking of a fuzzy number $\tilde{A}$ is given by

$$\mathsf{R}(\tilde{A}) = \int_0^1 (0.5)(A_U^\alpha + A_L^\alpha) d\alpha,$$

where A_L^α = Lower α- level cut and A_U^α = Upper α- level cut. If $\mathsf{R}(\tilde{A}) \leq \mathsf{R}(\tilde{B})$ then $\tilde{A} \leq \tilde{B}$. Let the two fuzzy numbers $\tilde{A}$ and $\tilde{B}$

- If it is element wise equal $\Longleftrightarrow \tilde{A} = \tilde{B}$.
- If $\mathsf{R}(\tilde{A}) = \mathsf{R}(\tilde{B}) \Longleftrightarrow \tilde{A} \Leftrightarrow \tilde{B}$.
- $\tilde{A}$ is negative $\Longleftrightarrow (\mathsf{R}(\tilde{A}))$ is negative.
- $\tilde{A}$ is zero fuzzy number $\Longleftrightarrow (\mathsf{R}(\tilde{A}))$ is zero.

2.5. **Fuzzy Assignment Problem.** The fuzzy assignment problem can be defined in the form of an $n \times n$ cost matrix as follows:

$$\begin{array}{c} \\ Job\ 1 \\ Job\ 2 \\ Job\ 3 \\ \cdots \\ Job\ n \end{array} \begin{array}{c} \begin{array}{ccccc} Instrument\ 1 & Instrument\ 2 & Instrument\ 3 & \cdots & Instrument\ n \end{array} \\ \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & \cdots & \tilde{C}_{1n} \\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & \cdots & \tilde{C}_{2n} \\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & \cdots & \tilde{C}_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{C}_{n1} & \tilde{C}_{n2} & \tilde{C}_{n3} & \cdots & \tilde{C}_{nn} \end{pmatrix} \end{array}$$

Mathematical formulation is

$$\min \tilde{Z} \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij} x_{ij}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \sum_{j=1}^{n} x_{ij} = 1,$$

$$x_{ij} = \begin{cases} 1, \text{if } i^{th}\text{job is assigned to the } j^{th}\text{instrument} \\ 0, \text{otherwise} \end{cases}.$$

2.6. **Fuzzy Travelling Salesman Problem.** The fuzzy travelling salesman problem can be defined in the form of an $n \times n$ cost matrix as follows:

$$\begin{array}{c c} & \begin{array}{ccccc} Place\ 1 & Place\ 2 & Place\ 3 & \cdots & Place\ n \end{array} \\ \begin{array}{c} Place\ 1 \\ Place\ 2 \\ Place\ 3 \\ \cdots \\ Place\ n \end{array} & \begin{pmatrix} \infty & \tilde{C}_{12} & \tilde{C}_{13} & \cdots & \tilde{C}_{1n} \\ \tilde{C}_{21} & \infty & \tilde{C}_{23} & \cdots & \tilde{C}_{2n} \\ \tilde{C}_{31} & \tilde{C}_{32} & \infty & \cdots & \tilde{C}_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{C}_{n1} & \tilde{C}_{n2} & \tilde{C}_{n3} & \cdots & \infty \end{pmatrix} \end{array}$$

Mathematically, it can be stated as

$$\min \tilde{Z} \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij} x_{ij}$$

subject to

(2.1) $$\textstyle\sum_{i=1}^{n} x_{ij} = 1, \quad \sum_{j=1}^{n} x_{ij} = 1,$$

(2.2) $$x_{ij} + x_{ji} \leq 1, \quad 1 \leq i \neq j \leq n$$

(2.3) $$x_{ij} + x_{jk} + x_{ki} \leq 2, \quad 1 \leq i \neq j \neq k \leq n$$

(2.4) $$x_{ip_1} + x_{p_1 p_2} + \cdots + x_{p_{n-2} i} \leq n-2, \quad 1 \leq i \neq p_1 \neq p_2 \neq \cdots \leq n.$$

Constraints (2.1) ensure that each city is visited only once. Constraint (2.2) eliminates all 2-city sub tours. Constraint (2.3) eliminates all 3-city sub tours. Constraints (2.4) eliminate all $(n-2)$ city sub tours.

3. Improved Hungarian algorithm for Fuzzy Assignment Problem

Consider the fuzzy cost matrix. If the matrix is square matrix go to Step 1, otherwise make it a square matrix by adding fuzzy zero element rows or fuzzy zero element columns.

Step 1: Select the fuzzy minimum cost in each and every row and subtract it from other fuzzy cost in the corresponding row. Do it for all the rows.

Step 2: Repeat this procedure for the columns which do not have atleast one fuzzy zero element.

Step 3: Like in Hungarian method cover the fuzzy zero elements by using minimum number(N) of horizontal and vertical lines. If N is equal to the order of the fuzzy cost matrix then do the fuzzy assignment according to Hungarian method. Otherwise proceed to next step.

Step 4: If $N \neq n$ and their difference is one then make assignments according to Hungarian method. This will leads to an assignment of $(n-1)$. There will be one row and one column in which there is no assignment. Choose the element which is in the intersection of that row and that column. Check whether it is minimum out of all uncovered fuzzy elements. If it is minimum then assign that element. This will reduce the procedure.

Step 5: If it is not the fuzzy minimum then follow the procedure of Hungarian method to obtain optimal solution

3.1. **Numeric Examples.**

3.1.1. *Example.* Consider the Fuzzy Assignment problem

$$\begin{array}{c c} & \begin{array}{ccccc} Instrument\ 1 & Instrument\ 2 & Instrument\ 3 & Instrument\ 4 & Instrument\ 5 \end{array} \\ \begin{array}{c} Job\ 1 \\ Job\ 2 \\ Job\ 3 \\ Job\ 4 \\ Job\ 5 \end{array} & \begin{pmatrix} (29,30,31) & (24,25,26) & (32,33,34) & (34,35,36) & (35,36,37) \\ (22,23,24) & (28,29,30) & (37,38,39) & (22,23,24) & (25,26,27) \\ (29,30,31) & (26,27,28) & (21,22,23) & (21,22,23) & (21,22,23) \\ (24,25,26) & (30,31,32) & (28,29,30) & (26,27,28) & (31,32,33) \\ (26,27,28) & (28,29,30) & (29,30,31) & (23,24,25) & (31,32,33) \end{pmatrix} \end{array}$$

Applying the algorithm,

$$\begin{pmatrix} (3,5,7) & (-2,0,2) & (6,8,10) & (8,10,12) & (9,11,13) \\ (-2,0,2) & (4,6,8) & (13,15,17) & (-2,0,2) & (1,3,5) \\ (6,8,10) & (3,5,7) & (-2,0,2) & (-2,0,2) & (-2,0,2) \\ (-2,0,2) & (4,6,8) & (2,4,6) & (0,2,4) & (5,7,9) \\ (1,3,5) & (3,5,7) & (4,6,8) & (-2,0,2) & (6,8,10) \end{pmatrix}$$

The minimum number of lines covers the fuzzy zeros are $N = 4$. But the $n = 5$. Therefore $N = n - 1$. By applying the last step of improved Hungarian algorithm, Assignments can be done for this cost matrix. Out of all the rows and columns, second row and fifth column have no assignment. Examining the element at the intersecting position $(2,5)$, *i.e.,)* $(1,3,5)$ is the minimum fuzzy element out of all the uncovered element. Therefore assign that position.

The optimum assignment is

$\Rightarrow Job\,1 \to Instrument\,2, Job\,2 \to Instrument\,5, Job\,3 \to Instrument\,3, Job\,4 \to Instrument\,1, Job\,5 \to Instrument\,4$.

The assignment cost is = $(24,25,26) + (25,26,27) + (21,22,23) + (24,25,26) + (23,24,25) = (117,122,127)$.

Remark 3.1. *When we apply Hungarian algorithm only* 37 *arithmetic operations are used and* 9 *lines are drawn to obtain optimal solution. But in the case of improved Hungarian method only* 25 *arithmetic operations and* 4 *lines are drawn for the optimal solution. This will reduce the complexity of the algorithm. It also reduces the computational time of the algorithm. If the order of the cost matrix becomes larger this will help a lot.*

4. IMPROVED HUNGARIAN ALGORITHM FOR FUZZY TRAVELLING SALESMAN PROBLEM

Apply the improved Hungarian algorithm to the fuzzy travelling salesman problem. Scrutinizing the obtained solution to ensure whether the route conditions are satisfied. If it satisfies then that is the solution of fuzzy TSP. If not, making adjustments in assignments to satisfy the condition with minimum increase in total cost i.e.) next best solution may be considered.

4.1. **Numerical Example.**

4.1.1. *Example.* Consider the Fuzzy Travelling salesman problem

$$\begin{array}{c} \\ Place\ 1 \\ Place\ 2 \\ Place\ 3 \\ Place\ 4 \\ Place\ 5 \end{array}
\begin{array}{c} \begin{array}{ccccc} Place\ 1 & Place\ 2 & Place\ 3 & Place\ 4 & Place\ 5 \end{array} \\
\begin{pmatrix} \infty & (15,16,17) & (17,18,19) & (12,13,14) & (19,20,21) \\ (20,21,22) & \infty & (15,16,17) & (26,27,28) & (13,14,15) \\ (11,12,13) & (13,14,15) & \infty & (14,15,16) & (20,21,22) \\ (10,11,12) & (17,18,19) & (18,19,20) & \infty & (20,21,22) \\ (15,16,17) & (13,14,15) & (16,17,18) & (11,12,13) & \infty \end{pmatrix} \end{array}$$

Applying the algorithm,

$$\begin{pmatrix} \infty & (-3,1,5) & (-1,3,7) & (-2,0,2) & (5,7,9) \\ (5,7,9) & \infty & (-4,0,4) & (11,13,15) & (-2,0,2) \\ (-2,0,2) & (-4,0,4) & \infty & (1,3,5) & (7,9,11) \\ (-2,0,2) & (1,5,9) & (2,6,10) & \infty & (8,10,12) \\ (2,4,6) & (-4,0,4) & (-1,3,7) & (-2,0,2) & \infty \end{pmatrix}$$

The minimum number of lines covers the fuzzy zeros are $N = 4$. But the $n = 5$. Therefore $N = n-1$. By applying the last step of improved Hungarian algorithm, Assignments can be done for this cost matrix. Out of all the rows and columns, fifth row and third column have no assignment. Examining the element at the intersecting position $(5,3)$, *i.e.,)*$(-1,3,7)$ is the minimum fuzzy element out of all the uncovered element. Therefore assign that position.

The optimum assignment is:
$\Rightarrow\ Place\ 1 \rightarrow Place\ 4, Place\ 2 \rightarrow Place\ 5, Place\ 3 \rightarrow Place\ 2, Place\ 4 \rightarrow Place\ 1, Place\ 5 \rightarrow Place\ 3$.

$Place\ 1 \to Place\ 4 \to Place\ 1, Place\ 2 \to Place\ 5 \to Place\ 3 \to Place\ 2$. This is not Hamiltonian circuit. $\therefore$ The next best solution is

$$Place\ 1 \to Place\ 4 \to Place\ 2 \to Place\ 5 \to Place\ 3 \to Place\ 1.$$

The optimum solution is

$$(12, 13, 14) + (13, 14, 15) + (11, 12, 13) + (17, 18, 19) + (16, 17, 18) \approx (69, 74, 79).$$

Remark 4.1. *When we apply Hungarian algorithm only* 40 *arithmetic operations are used and* 9 *lines are drawn to obtain optimal solution. But in the case of improved Hungarian method only* 28 *arithmetic operations and* 4 *lines are drawn for the optimal solution. This will reduce the complexity of the algorithm. It also reduces the computational time of the algorithm. If the order of the cost matrix becomes larger this will help a lot.*

5. Conclusions

In this paper, an improved Hungarian algorithm is presented and implemented to solve fuzzy assignment problem. Further it is extended for finding the optimum route of a fuzzy travelling salesman problem. It's also reduces the number of fuzzy arithmetic operations used to get the optimum solution. It is systematic and easy to understand. It can be applied for all the special kind of assignment problems.

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