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ABSORBING MAP IN COMPLETE INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT. In this paper, the ideas of fixed point hypothesis in intuitionistic fuzzy measurement space utilizing retaining capacities. In this paper utilize the six capacities. Our outcomes summed up and improves different outcomes

1. INTRODUCTION

In 1965 Zadeh [7] presented the thought of fuzzy sets. After this during the most recent couple of decades numerous creators have built up the presence of bunches of fixed point hypotheses in fuzzy setting,Presented the idea of intuitionistic fuzzy sets as a speculation of fuzzy sets and later there has been a lot of progress in the investigation of intuitionistic fuzzy sets. In 2004, Park [1] presented an idea of intuitionistic fuzzy measurement spaces with the assistance of consistent t-standards and nonstop t-conorms as a speculation of fuzzy measurement space due to Kramosil and Michalek [22] in certainty the ideas of triangular standards (t-standard) and triangular conorms (t-conorm) are initially presented by Schweizer and Sklar [3] in investigation of factual measurement spaces, Presented the idea of retaining mapping in metric space and demonstrate normal fixed point hypothesis in this space.

The point of this paper is to present the new thought of absorbing maps in intuitionistic fuzzy measurement space which is neither a subclass of perfect

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maps nor a subclass of non compatible maps it isn't essential that absorbing maps drive at their fortuitous event focuses notwithstanding on the off chance that the mapping pair fulfill the contractive kind condition, at that point savvy retaining maps drive at their incident focuses as well as it turns into a vital condition for acquiring a typical fixed purpose of mapping pair.

Definition 1.1. *Let X be any non empty set. A fuzzy set A in X is a function with domain X and values in $[0, 1]$.*

Definition 1.2. *Let a set E be fixed. An intuitionistic fuzzy set (IFS) A of E is an object having the form $A = \{hx, \mu_A(x), v_A(x) : x \in E\}$ where the function $\mu_A : E \to [0, 1]$ and $v_A : E \to [0, 1]$ define respectively, the degree of membership and degree of nonmembership of the element $x \in E$ to the set A, which is a subset of E, and for every $x \in E, 0 \leqslant \mu_A(x) + v_A(x) \leqslant 1$.*

Definition 1.3. *A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm, if $*$ is satisfying the following conditions:*

- (i) $*$ *is commutative and associative*
- (ii) $*$ *is continuous*
- (iii) $a * 1 = a$ *for all* $a \in [0, 1]$
- (iv) $a * b \leqslant c * d$ *whenever* $a \leqslant c$ *and* $b \leqslant d$*, for* $a, b, c, d \in [0, 1]$.

Definition 1.4. *A binary operation $\diamond : [0, 1]\times[0, 1] \to [0, 1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions:*

- (i) $\diamond$ *is commutative and associative*
- (ii) $\diamond$ *is continuous*
- (iii) $a \diamond 0 = a$ *for all* $a \in [0, 1]$
- (iv) $a \diamond b \leqslant c \diamond d$ *whenever* $a \leqslant c$ *and* $b \leqslant d$*, for* $a, b, c, d \in [0, 1]$

Definition 1.5. *A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionist fuzzy metry space (shortly IFM-Space) if X is an arbitrary set, $*$ is a permanent t-norm, $\diamond$ is a permanent t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: For all $x, y, z, s, t > 0$*

$(IFM - 1)$ $M(x, y, t) + N(x, y, t) \leqslant 1$

$(IFM - 2)$ $M(x, y, 0) = 0$

$(IFM - 3)$ $M(x, y, t) = 1$ *if and only if* $x = y$

$(IFM - 4)$ $M(x, y, t) = M(y, x, t)$

$(IFM-5)$ $M(x,y,t)+M(y,z,s)\leqslant M(x,z,t+s)$
$(IFM-6)$ $M(x,y,*):[0,\infty)\to[0,1]$ *is left permanent*
$(IFM-7)$ $\lim_{t\to\infty} M(x,y,t)=1$
$(IFM-8)$ $N(x,y,0)=1$
$(IFM-9)$ $N(x,y,t)=0$ *if and only if* $x=y$
$(IFM-10)$ $N(x,y,t)=N(y,x,t)$
$(IFM-11)$ $N(x,y,t)\diamond N(y,z,s)\geqslant N(x,s,t+s)$
$(IFM-12)$ $N(x,y,*):[0,\infty)\to[0,1]$ *is right permanent*
$(IFM-13)$ $\lim_{t\to\infty} N(x,y,t)=0$

Then (M,N) *is called an intuitionistic fuzzy metric on* X. *The functions* $M(x,y,t)$ *and* $N(x,y,t)$ *denote the degree of nearness and degree of non-nearness between* x *and* y *with respect to* t *respectively.*

Remark 1.1. *Every fuzzy metric space* $(X,M,*)$ *is an intuitionistic fuzzy metry space if* X *of the form* $(X,M,1-M,*,\diamond)$ *such that* t*-norm* $*$ *and* t*-conorm* $\diamond$ *are associated, that is,* $x\diamond y=1-((1-x)*(1-y))$ *for any* $x,y\in X$. *But the converse is not true.*

2. Main Results

Theorem 2.1. *Let* G *be point wise* S*-absorbing and* H *be point wise* T. *Absorbing self maps on a complete intuitionist fuzzy metric space* $(X,M,N,*,\diamond)$ *with permanent* t*-norm defined by* $a*b=\min\{a,b\}$ *and* $a\diamond b=max\{a,b\}$ *where* $a,b\in(0,1)$*, satisfying the conditions:*

(I) $G(X)\subset T(X), H(X)\subset S(X)$.
(II) *There exists* $k\in(0,1)$ *such that for every* $x,y\in X$ *and* $t>0$,

$$\begin{aligned}M(Gx,Hy,kt)\geqslant\min\{&M(Sx,Ty,t),M(Gx,Sx,t),M(Hy,Ty,t),\\&M(Gx,Ty,t),M(Gx,Hy,t),M(Sx,Hy,t)\}\end{aligned}$$

$$\begin{aligned}N(Gx,Hy,kt)\leqslant\min\{&N(Sx,Ty,t),N(Gx,Sx,t),N(Hy,Ty,t),\\&N(Gx,Ty,t),N(Gx,Hy,t),N(Sx,Hy,t)\}.\end{aligned}$$

(III) *for all* $x,y\in X$, $\lim_{t\to\infty} M(x,y,t)=1$ *and* $\lim_{t\to 0} N(x,y,t)=0$.

If the pair of maps (G,S) *is mutual permanent compatible maps then* G,H,S *and* T *have a unique common fixed point in* X.

Proof. Let x_0 be any arbitrary point in X, construct a sequence $y_n \in X$ such that

(2.1) $$y_{2n-1} = Tx_{2n-1} = Gx_{2n-2} \text{and} y_{2n} = Sx_{2n} = Hx_{2n+1}, n = 1, 2, 3, \cdots$$

This can be done by the virtue of (I). By using contractive condition we obtain,

$$M(y_{2n+1}, y_{2n+2}, kt) = M(Gx_{2n}, Hx_{2n+1}, kt)$$
$$\geqslant \min\{M(Sx_{2n}, Tx_{2n+1}, t), M(Gx_{2n}, Sx_{2n}, t), M(Hx_{2n+1}, Tx_{2n+1}, t),$$
$$M(Gx_{2n}, Tx_{2n+1}, t), M(Gx_{2n}, Hx_{2n}, t)M(x_{2n}, Tx_{2n+1}, t), \},$$
$$\geqslant \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t),$$
$$M(y_{2n+1}, y_{2n+1}, t)M(y_{2n+1}, y_{2n}, t), 1\}$$
$$\leqslant \min\{N(Sx_{2n}, Tx_{2n+1}, t), N(Gx_{2n}, Sx_{2n}, t), N(Hx_{2n+1}, Tx_{2n+1}, t),$$
$$N(Gx_{2n}, Tx_{2n+1}, t)M(Gx_{2n}, Hx_{2n}, t)M(Sx_{2n}, Tx_{2n+1}, t))\}$$
$$\leqslant \min\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+1}, t), 0,$$
$$M(y_{2n}, y_{2n+1}, t)\}$$

which implies,

$$M(y_{2n+1}, y_{2n+2}, kt) \geqslant M(y_{2n}, y_{2n+1}, t)N(y_{2n+1}, y_{2n+2}, kt) \leqslant N(y_{2n}, y_{2n+1}, t)$$

in general

$$M(y_n, y_{n+1}, kt) \geqslant M(y_{n-1}, y_n, t)$$
$$N(y_{2n}, y_{2n+1}, kt) \leqslant N(y_{2n-1}, y_{2n}, t)(1)$$

To prove $\{y_n\}$ is a Cauchy sequence, we have to show $M(y_n, y_{n+1}, t) \to 1$ and $N(y_n, y_{n+1}, t) \to 0$ (for $t > 0$ as $n \to \infty$uniformly on $p \in N$). For this from (1), we have,

$$M(y_n, y_{n+1}, t) \geqslant M(y_{n-1}, y_n, \frac{t}{k}) \geqslant M(y_{n-2}, y_{n-1}, \frac{t}{k^2})$$
$$\geqslant \cdots \geqslant M(y_0, y_1, t/k^{2n}) \to 1$$
$$N(y_n, y_{n+1}, t) \geqslant N(y_{n-1}, y_n, t/k)$$
$$\geqslant N(y - n - 2, y - n - 1, t/k^2)$$
$$\geqslant \cdots \geqslant N(y_0, y_1, t/k^n) \to 0.$$

As $n \to \infty$ for $p \in N$, by (1), we have

$$M(y_n, y_{n+p}, t) \geqslant \{M(y_n, y_{n+1}, (1-k)t) * M(y_{n+1}, y_{n+p}, kt)\}$$
$$\geqslant \{M(y_0, y_1, (1-k)t/k^n) * M(y_{n+1}, y_{n+2}, t) * M(y_{n+2}, y_{n+p}, (k-1)t)$$
$$\{M(y_0, y_1, (1-k)t/k^n) * M(y_0, y_1, 1/k^n) * M(y_{n+2}, y_{n+3}, t) *$$

$$\begin{aligned}&M(y_{n+3}, y_{n+p}, (k-2)t)\\ \geqslant &\{M(y_0, y_1, (1-k)t/k^n) * M(y_0, y_1, t/k^n) * M(y_0, y_1, (1-k)t/k^{n+2})\\ &* \cdots * M(y_0, y_1, (k-p)t/k^{n+p+1})\end{aligned}$$

and

$$\begin{aligned}&N(y_n, y_{n+p}, t) \leqslant \{N(y_n, y_{n+1}, (1-k)t) \diamond N(y_{n+1}, y_{n+p}, kt)\}\\ \leqslant &\{N(y_0, y_1, (1-k)t/k^n) \diamond N(y_{n+1}, y_{n+2}, t) \diamond N(y_{n+2}, y_{n+p}, (k-1)t)\}\\ \leqslant &\{N(y_0, y_1, (1-k)t/k^n) \diamond N(y_0, y1, t/k^n)\\ &\diamond N(y_{n+2}, y_{n+3}, t) \cdots \diamond N(y_0, y_1, (k-p)t/k^{n+p+1})\}\end{aligned}$$

Thus $M(y_n, y_{n+p}, t) \to 1$ and $N(y_n, y_{n+p}, t) \to 0$ (for all $t > 0$ as $n \to \infty$ uniformly on $p \in N$). Therefore $\{y_n\}$ is a Cauchy sequence in X.

But $(X, M, N, *, \diamond)$ is complete so there exists a point (say) $z \in X$ such that $\{y_n\} \to z$. Also, using (I) we have $\{Gx_{2n-2}\}, \{Tx_{2n-1}\}, \{Sx_{2n}\}, \{Hx_{2n+1}\} \to z$. Since the pair (G, S) is reciprocally continuous mappings, then we have,

$$\lim_{n\to\infty} GSx_{2n} = Gz$$

and

$$\lim_{n\to\infty} SGx_{2n} = Sz$$

and compatability of G and S yields,

$$\lim_{n\to\infty} M(GSx_{2n}, SGx_{2n}, t) = 1$$

and

$$\lim_{n\to\infty} N(GSx_{2n}, SGx_{2n}, t) = 0,$$

i.e, $M(Gz, Sz, t) = 1$ and $N(Gz, Sz, t) = 0$. Hence $Gz = Sz$.

Since $G(X) \subset T(X)$, then there exists a point $u \in X$ such that $Gz = Tu$. Now by contractive condition, we get,

$$\begin{aligned}M(Gx, Hy, kt) \geqslant \min\{&M(Sx, Ty, t), M(Gx, Sx, t), M(Hy, Ty, t),\\ &M(Gx, Ty, t), M(Gx, Hy, t), M(Sx, Hy, t)\}\\ N(Gx, Hy, kt) \leqslant \min\{&N(Sx, Ty, t), N(Gx, Sx, t), N(Hy, Ty, t),\\ &N(Gx, Ty, t), N(Gx, Hy, t), N(Sx, Hy, t)\},\end{aligned}$$

i.e. $Gz = Hu$. Thus $Gz = Sz = Hu = Tu$. Since G is S-absorbing then for $R > 0$, we have,

$$M(Sz, SGz, t) \geqslant MSz, Gz, t/R = 1$$

$$N(Sz, SGz, t) \leqslant N\{Sz, Gz, t/R\} = 0,$$

i.e. $Gz = SGz = Sz$.

Now by contractive condition, we have,

$$\begin{aligned}
M(Gx, Hy, kt) &\geqslant \min\{M(Sx, Ty, t), M(Gx, Sx, t), M(Hy, Ty, t),\\
&\quad M(Gx, Ty, t), M(Gx, Hy, t), M(Sx, Hy, t)\}\\
&= M(GGz, Az, t)\\
N(Gz, GGz, t) &= N(GGz, Hu, t)\\
N(Gx, Hy, kt) &\leqslant \min\{N(Sx, Ty, t), N(Gx, Sx, t), N(Hy, Ty, t),\\
N(Gx, Ty, t), N(Gx, Hy, t),\\
N(Sx, Hy, t)\} &= N(GGz, Gz, t),
\end{aligned}$$

i.e., $GGz = Gz = SGz$.

Therefore Gz is a common fixed point of G and S. Similarly, T is H-absorbing. Therefore we have,

$$M(Tu, THu, t) \geqslant M\{Tu, Hu, t/R\} = 1$$

$$N(Tu, THu, t) \leqslant N\{Tu, Hu, t/R\} = 0,$$

i.e. $Tu = THu = Hu$.

Now, by contractive condition, we have

$$\begin{aligned}
M(Gx, Hy, kt) &\geqslant \min\{M(Sx, Ty, t), M(Gx, Sx, t), M(Hy, Ty, t),\\
&\quad M(Gx, Ty, t), M(Gx, Hy, t), M(Sx, Hy, t)\}\\
&= M(GGz, Gz, t) = M(HHu, Gz, t)\\
N(Gx, Hy, kt) &\leqslant \min\{N(Sx, Ty, t), N(Gx, Sx, t), N(Hy, Ty, t),\\
&\quad N(Gx, Ty, t), N(Gx, Hy, t), N(Sx, Hy, t)\}\\
&= N(GGz, Gz, t).
\end{aligned}$$

i.e., $HHu = Hu = THu$.

Hence $Hu = Gz$ is a common fixed point of G, H, S and T.

Uniqueness of Gz can easily follows from contractive condition. The proof is similar when H and T are assumed compatible and reciprocally permanent. This completes the proof. □

Now we prove the result by assuming the range of one of the mappings G, H, S or T to be a complete subspace of X.

Corollary 2.1. *Let G be point wise S-absorbing and H be point wise T-absorbing self maps on an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ with continuous t-norm defined by $a * b = \min\{a, b\}$ and $a \diamond b = max\{a, b\}$ where $a, b \in [0, 1]$ satisfying the conditions:*

(I) $G(X) \subseteq T(X)$, $H(X) \subseteq S(X)$

(II) *there exists* $k \in (0,1)$ *such that for every* $x, y \in X$ *and* $t > 0$ $M(Gx, Hy, kt) \geqslant \min\{M(Sx, Ty, t), M(Gx, Sx, t), M(Hy, Ty, t), M(Gx, Ty, t), M(Gx, Hy, t), M(Sx, Hy, t)\}$
$N(Gx, Hy, kt) \leqslant \min\{N(Sx, Ty, t), N(Gx, Sx, t), N(Hy, Ty, t), N(Gx, Ty, t), N(Gx, Hy, t), N(Sx, Hy, t)\}$

(III) *for all* $x, y \in X$, $\lim_{n\to\infty} M(x, y, t) = 0$ *and* $\lim_{n\to\infty} N(x, y, t) = 0$.

If the range of one of the mappings G, H, S *or* T *be a complete subspace of* X. *Then* G, H, S *and* T *have a unique common fixed point in* X.

3. Conclusion

In this paper we have used the concept of complete intutitionistic fuzzy metric space at the place of a normal fuzzy metric space and introduce the notionn of absorbing map. We proved the result for absorbing map in complete intutionistic fuzzy Metric space. In view of support to the proof, we derived one corollary related to our result.

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