MBJW - FILTERS OF LATTICE WAJSBERG ALGEBRAS

T. ANITHA¹, V. AMARENDRABABU, AND G. BHANU VINOLIA

ABSTRACT. In this paper we define the $MBJ^w$ – filters of Lattice wajsberg algebras and proved the properties of $MBJ^w$ – filters. We derive some relation between fuzzy ideals, interval valued fuzzy ideals to neutrosophic ideals. Further we prove that cut sets of $MBJ^w$ – sets formed $MBJ^w$ – filter. Finally define the $MBJ^w$– lattice filters and proved every $MBJ^w$ – filter is a $MBJ^w$ – lattice filter and converse is not true.

1. INTRODUCTION


¹corresponding author

2020 Mathematics Subject Classification. 16P70, 16D25.

Key words and phrases. Lattice wajsberg algebra, MBJ-neutrosophic sets, MBJW – filters and MBJW– lattice filters.

In this paper we consider MBJ-neutrosophic sets \((M_J)_B\) defined by Y.B. Jun and introduce the concept \((M_J)_B\) filter of lattice wajsberg algebra and obtain some results on them. For further information of lattice wajsberg algebra refer the wajsberg algebra [5] by Front, Antonio and Torrens and for MBJ-neutrosophic sets refer the [10] MBJ-neutrosophic structures.

2. Preliminaries

**Definition 2.1.** [5] Let \((w, \rightarrow, ^{'}, 1_m)\) be a wajsberg algebra if it satisfies the following axioms for all \(x_m, y_m, z_m \in w\)

\[
\begin{align*}
(i) & 
1_m \rightarrow x_m = x_m \\
(ii) & (x_m \rightarrow y_m) \rightarrow ((y_m \rightarrow z_m) \rightarrow (x_m \rightarrow z_m)) = 1_m \\
(iii) & (x_m \rightarrow y_m) \rightarrow y_m = (y_m \rightarrow x_m) \rightarrow x_m \\
(iv) & (x'_m \rightarrow y'_m) \rightarrow (y_m \rightarrow x_m) = 1_m
\end{align*}
\]

**Definition 2.2.** [5] The wajsberg algebra \(W\) is called a lattice wajsberg algebra with the bounds \(0_m, 1_m\) if it satisfies the following axioms for all \(x_m, y_m \in W\):

A partial ordering \(\leq\) on \(W\), such that \(x_m \leq y_m\) if and only if \(x_m \rightarrow x_m = 1_m\), \((x_m \vee y_m) = (x_m \rightarrow y_m) \rightarrow y_m\) and \((x_m \wedge y_m) = ((x'_m \rightarrow y'_m) \rightarrow y'_m)\).

Let \(I\) denote the family of all intervals numbers of \([0, 1]\). If \(I_1 = [a_1, b_1]\), \(I_2 = [a_2, b_2]\) are two elements of \(I[0,1]\), we call \(I_1 \geq^* I_2\) if \(a_1 \geq a_2\) and \(b_1 \geq b_2\). we define the term rmax to mean the maximum of two interval as \(\text{rmax } [I_1, I_2] = [\max(a_1, a_2), \max(b_1, b_2)]\). Similarly, me can define the term rmin of any two intervals.

**Definition 2.3.** [10] A neutrosophic set \((N^*)\), if the structure \(A_m = < y_m, w^A_T(y_m), w^A_I(y_m), w^A_F(y_m) >, y_m \in x\) where \(w^A_T\) is truth membership function, \((w^A_I)\) is an indeterminate membership function and \((w^A_F)\) is false membership function, on a nonempty set \(X\).
Definition 2.4. [10] A MBJ neutrosophic set \( A_m = \langle y_m, M_i^A(y_m), B_i^A(y_m), J_i^A(y_m) \rangle \) where \( M_i^A \) is truth membership function, \( B_i^A \) is an indeterminate interval -valued membership function and \( J_i^A \) is false membership function, on a nonempty set \( X \). The \( M_i^A \)-set is simply denoted by \( A_m = (M_i^A, B_i^A, J_i^A) \). Throughout this paper \( W \) denotes the lattice wajsberg algebra and \( M_i^A \)-set denotes the MBJ-neutrosophic set.

3. \( M_i^A \)-FILTERS

Definition 3.1. A \( M_i^A \)-set \( A_m = (M_i^A, B_i^A, J_i^A) \) on \( W \) is called a \( M_i^A \)-filter if it satisfies for all \( x_m, y_m \in W \),

1. \( M_i^A(1_m) \geq M_i^A(x_m), B_i^A(1_m) \geq B_i^A(x_m) \) and \( J_i^A(1_m) \leq J_i^A(x_m) \).
2. \( M_i^A(y_m) \geq \min \{ M_i^A(x_m \rightarrow y_m), M_i^A(x_m) \} \), \( B_i^A(y_m) \geq \min \{ B_i^A(x_m \rightarrow y_m), B_i^A(x_m) \} \) and \( F_i^A(y_m) \leq \max \{ J_i^A(x_m \rightarrow y_m), J_i^A(x_m) \} \).

Example 1. Let \( W = \{0_m, x_m, y_m, 1_m\} \) with the binary operation \( \rightarrow \) as follows: The \( M_i^A \)-set \( A_m = (M_i^A, B_i^A, J_i^A) \) defined on \( W \) as follows is \( M_i^A \)-filter of \( W \).

### Table 1. W-Algebra

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Example 2. Let $W = \{0_m, x_m, y_m, z_m, v_m, 1_m\}$ with the binary operation $\rightarrow$ as follows:

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The $M_B^J$ set $A_m = (M_T^A, B_I^A, J_I^A)$ defined on $W$ as follows is $M_B^J$-filter of $W$.

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Theorem 3.1. Let $A_m = (M_T^A, B_I^A, J_I^A)$ is $M_B^J$- set of $W$. If $(M_T^A, J_I^A)$ is an intuitionistic fuzzy filter of $W$ and $B_I^{A^+}$ and $B_I^{A^-}$ are fuzzy filters of $W$ then $A_m = (M_T^A, B_I^A, J_I^A)$ is a $M_B^J$-filter of $W$.

Proof. For any $x_m, y_m \in W$, we have

$$B_I^A(1_m) = [B_I^{A^-}(1_m), B_I^{A^+}(1_m)] \geq^* [B_I^{A^-}(x_m), B_I^{A^+}(x_m)] = B_I^A(x_m)$$

and

$$B_I^A(y_m) = [B_I^{A^-}(y_m), B_I^{A^+}(y_m)] \geq^* [\min \{B_I^{A^-}(x_m \rightarrow y_m), B_I^{A^-}(x_m)\}, \min \{B_I^{A^+}(x_m \rightarrow y_m), B_I^{A^+}(x_m)\}]$$

$$= \min \{[B_I^{A^-}(x_m \rightarrow y_m), B_I^{A^+}(x_m \rightarrow y_m)], [B_I^{A^-}(x_m), B_I^{A^+}(x_m)]\}$$

$$= \min \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\}.$$
Therefore \( A_m = (M^A_I, B^A_I, J^A_I) \) is a \( M^B_W \) - filter of \( W \). If \( A_m = (M^A_I, B^A_I, J^A_I) \) is a \( M^B_W \) - filter of \( W \), then for all \( x, y_m \in W \),

\[
[B^A_I^- (y_m), B^A_I^+ (y_m)] = B^A_I(y_m) \geq \ast \ rmin \{ B^A_I(x_m \rightarrow y_m), B^A_I(x_m) \}
\]

\[
= rmin \{ [B^A_I^-(x_m \rightarrow y_m), B^A_I^+(x_m \rightarrow y_m)], [B^A_I^-(x_m), B^A_I^+(x_m)] \}
\]

\[
= min \{ B^A_I^-(x_m \rightarrow y_m), B^A_I^-(x_m) \}, \ min \{ B^A_I^+(x_m \rightarrow y_m), B^A_I^+(x_m) \}
\]

It follows that

\[
B^A_I^-(y_m) \geq min \{ B^A_I^-(x_m \rightarrow y_m), B^A_I^-(x_m) \} \quad \text{and}
\]

\[
B^A_I^+(y_m) \geq min \{ B^A_I^+(x_m \rightarrow y_m), B^A_I^+(x_m) \}.
\]

Thus \( B^A_I^- \) and \( B^A_I^+ \) are fuzzy filters of \( W \). But \( (M^A_I, J^A_I) \) is need not to be an intuitionistic fuzzy filter of \( W \).

For example the \( M^B_W \) - sets \( A_m = (M^B_I, B^A_I, J^B_I) \) and \( B_m = (M^B_I, B^B_I, J^B_I) \) in the example 3.3 are \( M^B_W \) - filters of \( W \) but \( (M^A_I, J^B_I) \) is an intuitionistic fuzzy filter of \( W \) and \( (M^B_I, J^B_I) \) is not an intuitionistic fuzzy filter of \( W \).

**Theorem 3.2.** If \( A_m = (M^A_I, B^A_I, J^A_I) \) is a \( M^B_W \) - filter of \( W \) then the sets

\[
(M^A_I, B^A_I^-, J^B_I)(M^A_I, B^A_I^+, J^B_I)
\]

are \( N^W \)- filters of \( W \).

**Proof.** Let \( A_m = (M^A_I, B^A_I, J^B_I) \) is a \( M^B_W \) - filter of \( W \). Then \( B^A_I(1_m) \geq \ast \ B(x_m) \) then clearly \( B^A_I^- (1_m) \geq B^A_I^- (x_m) \) and \( B^A_I^+ (1_m) \geq B^A_I^+ (x_m) \) for all \( x, y_m \in W \). And

\[
B^A_I^-(y_m) \geq \ast \ rmin \{ B^A_I^-(x_m \rightarrow y_m), B^A_I^-(x_m) \}
\]

that is

\[
B^A_I^-(y_m) \geq min \{ B^A_I^-(x_m \rightarrow y_m), B^A_I^-(x_m) \},
\]

\[
B^A_I^+(y_m) \geq min \{ B^A_I^+(x_m \rightarrow y_m), B^A_I^+(x_m) \}.
\]

\( B^A_I^- \) and \( B^A_I^+ \) satisfies the necessary conditions. So the sets \( (M^A_I, B^A_I^-, J^B_I) \) and \( (M^A_I, B^A_I^+, J^B_I) \) are \( N^W \)- filters of \( W \).

**Theorem 3.3.** Let \( A_m = (M^A_I, B^A_I, J^B_I) \) is \( M^B_W \) - filter of \( W \). If \( x \leq y_m \) then \( \{ M^I_I(x_m) \leq M^I_I(y_m), B^I_I(x_m) \leq B^I_I(y_m) \} \) for all \( x, y_m \in W \).

**Proof.** Since \( x \leq y_m \), then \( x \rightarrow y_m = 1 \). By \( A_m \) is \( M^B_W \) -filter of \( W \), We have

\[
M^I_I(y_m) \geq min \{ M^I_I(x_m \rightarrow y_m), M^I_I(x_m) \}
\]

\[
= min \{ M^I_I(1_m), M^I_I(x_m) \} = M^I_I(x_m),
\]

\[
B^I_I(y_m) \geq \ast \ rmin \{ B^I_I(x_m \rightarrow y_m), B^I_I(x_m) \}
\]

\[
= min \{ B^I_I(1_m), B^I_I(x_m) \} = B^I_I(x_m)
\]
and
\[ J_F^A(y_m) \leq \max \{ J_F^A(x_m \to y_m), J_F^A(x_m) \} = \max \{ J_F^A(1_m), J_F^A(x_m) \} = J_F^A(x_m). \]

\[ \square \]

**Theorem 3.4.** A $M_B^\text{w}$ set $A_m = (M_T^A, B_I^A, J_F^A)$ is $M_B^\text{w}$-filter of $W$ if and only if it holds (3.1) and for all $x_m, y_m, z_m \in W$,

\[ M_T^A(x_m \to y_m) \geq \min \{ M_T^A(y_m \to (x_m \to z_m)), M_T^A(y_m) \}, \]

\[ B_I^A(x_m \to z_m) \geq rmin \{ B_I^A(y_m \to (x_m \to z_m)), B_I^A(y_m) \} \]

and
\[ J_F^A(x_m \to z_m) \leq \max \{ J_F^A(y_m \to (x_m \to z_m)), J_F^A(y_m) \}. \]

**Proof.** Let $A_m$ be a $M_B^\text{w}$-filter of $W$ such that it holds (3.1) and (3.3). Conversely, suppose that $A_m$ is a $M_B^\text{w}$-set with (3.1) and (3.3). Taking $x_m = 1_m$ in (3.3), we get

\[ M_T^A(1_m \to z_m) \geq \min \{ M_T^A(y_m \to (1_m \to z_m)), M_T^A(y_m) \}, \]

\[ M_T^A(z_m) \geq \min \{ M_T^A(y_m \to z_m), M_T^A(y_m) \}, \]

\[ B_I^A(1_m \to z_m) \geq rmin \{ B_I^A(y_m \to (1_m \to z_m)), B_I^A(y_m) \}, \]

\[ B_I^A(z_m) \geq rmin \{ B_I^A(y_m \to z_m), B_I^A(y_m) \}, \]

\[ J_F^A(1_m \to z_m) \leq \max \{ J_F^A(y_m \to (1_m \to z_m)), J_F^A(y_m) \}, \]

\[ J_F^A(z_m) \leq \max \{ J_F^A(y_m \to z_m), J_F^A(y_m) \}. \]

Hence $A_m$ is a $M_B^\text{w}$-filter of $W$. \[ \square \]

**Theorem 3.5.** A $M_B^\text{w}$ set $A_m = (M_T^A, B_I^A, J_F^A)$ is $M_B^\text{w}$-filter of $W$ if and only if it holds (3.1) and

\[ M_T^A((x_m \to (y_m \to z_m)) \to z_m) \geq \min \{ M_T^A(x_m), M_T^A(y_m) \}, \]

\[ B_I^A((x_m \to (y_m \to z_m)) \to z_m) \geq rmin \{ B_I^A(x_m), B_I^A(y_m) \} \]

and
\[ J_F^A((x_m \to (y_m \to z_m)) \to z_m) \leq \max \{ J_F^A(x_m), J_F^A(y_m) \}, \]

for all $x_m, y_m, z_m \in W$.

**Proof.** Suppose that $A_m$ is a $M_B^\text{w}$-filter of $W$ and $x_m, y_m, z_m \in W$. Clearly

\[ M_T^A((x_m \to (y_m \to z_m)) \to z_m) \]

\[ \geq \min \{ M_T^A((x_m \to (y_m \to z_m)) \to (y_m \to z_m)), M_T^A(y_m) \} \]

and
Theorem 3.6. Suppose
\((x_m \to (y_m \to z_m)) \to (y_m \to z_m) = (x_m(y_m \to z_m) \geq x_m)\).

So, \(M^A_I(((x_m \to (y_m \to z_m)) \to (y_m \to z_m)) \geq M^A_I(x_m)\).

From above we get,
\(M^A_I(((x_m \to (y_m \to z_m)) \to z_m) \geq \min \{M^A_I(x_m), M^A_I(y_m)\}\).

Clearly,
\(B^A_I((x_m \to (y_m \to z_m)) \to z_m) \geq \min \{B^A_I((x_m \to (y_m \to z_m)) \to (y_m \to z_m)), B^A_I(y_m)\}\)

and
\(B^A_I(((x_m \to (y_m \to z_m)) \to (y_m \to z_m)) \geq B^A_I(x_m)\).

From above we get,
\(B^A_I((x_m \to (y_m \to z_m)) \to z_m) \geq \star \min \{B^A_I(x_m), B^A_I(y_m)\}\).

Clearly,
\(J^A_F((x_m \to (y_m \to z_m)) \to z_m) \leq \min \{J^A_F((x_m \to (y_m \to z_m)) \to (y_m \to z_m)), J^A_I(y_m)\}\)

and
\(J^A_F(((x_m \to (y_m \to z_m)) \to z_m) \leq J^A_F(x_m)\).

From above we get,
\(J^A_F((x_m \to (y_m \to z_m)) \to z_m) \leq \max \{J^A_F(x_m), J^A_I(y_m)\}\).

Conversely suppose that \(A_m\) is a \(M^I_B\) -set with (3.1) and (3.4).
\(M^A_I(y_m) = M^A_I(1_m \to y_m) = M^A_I(((x_m \to y_m) \to (x_m \to y_m)) \to y_m)\)

\(\geq \min \{M^A_I(x_m \to y_m), M^A_I(x_m)\}\).

\(B^A_I(y_m) = B^A_I(1_m \to y_m) = B^A_I(((x_m \to y_m) \to (x_m \to y_m)) \to y_m)\)

\(\geq \star \min \{B^A_I(x_m \to y_m), B^A_I(x_m)\}\).

\(J^A_F(y_m) = J^A_F(1_m \to y_m) = J^A_F(((x_m \to y_m) \to (x_m \to y_m)) \to y_m)\)

\(\leq \max \{J^A_F(x_m \to y_m), J^A_I(x_m)\}\).

So, \(A_m\) is a \(M^I_B\) -filter of \(W\).

\(\square\)

**Theorem 3.6.** Every \(M^I_B\)-filter \(A_m = (M^A_I, B^A_I, J^A_F)\) fulfills the following result:
If \(x_m \to (y_m \to z_m) = 1_m\) then for all \(x_m, y_m, z_m \in W\),
\(M^A_I(z_m) \geq \min \{M^A_I(x_m), M^A_I(y_m)\}, B^A_I(z_m) \geq \star \min \{B^A_I(x_m), B^A_I(y_m)\}\)

and \(J^A_F(z_m) \leq \max \{J^A_F(x_m), J^A_I(x_m)\}\).

**Proof.** Suppose \(A_m\) is a \(M^I_B\) - filter of \(W\) and \(x_m \to (y_m \to z_m) = 1_m\) and \(x_m, y_m, z_m \in W\).

We get
\(M^A_I(z_m) \geq \min \{M^A_I(x_m \to z_m), M^A_I(y_m)\}\)
\[ \geq \min \{ \min \{ M_T^A(x_m), M_T^A(x_m \rightarrow (y_m \rightarrow z_m)) \}, M_T^A(y_m) \} \]
\[ \geq \min \{ \min \{ M_T^A(x_m), M_T^A(1_m) \}, M_T^A(y_m) \} \]
\[ \geq \min \{ M_T^A(x_m), M_T^A(y_m) \} \]

\[ B_I^A(z_m) \geq^{* \ rmin} \{ B_I^A(y_m \rightarrow z_m), B_I^A(y_m) \} \]
\[ \geq^{* \ rmin} \{ \min \{ B_I^A(x_m), B_I^A(x_m \rightarrow (y_m \rightarrow z_m)) \} \} B_I^A(y_m) \]
\[ \geq^{* \ rmin} \{ \min \{ B_I^A(x_m), B_I^A(1_m) \}, B_I^A(y_m) \} \]
\[ \geq^{* \ rmin} \{ B_I^A(x_m), B_I^A(y_m) \} \]

and
\[ J_F^A(z_m) \leq \max \{ J_F^A(y_m \rightarrow z_m), J_F^A(y_m) \} \]
\[ \leq \max \{ \max \{ J_F^A(x_m), J_F^A(x_m \rightarrow (y_m \rightarrow z_m)) \}, J_F^A(y_m) \} \]
\[ \leq \max \{ \max \{ J_F^A(x_m), J_F^A(1_m) \}, J_F^A(y_m) \} \]
\[ \leq \max \{ J_F^A(x_m), J_F^A(y_m) \}. \]

**Lemma 3.1.** Every \( M_I^B \) set \( A_m = (M_I^A, B_I^A, J_F^A) \) of \( W \) fulfills the following result for all \( x((n_w), x(1_w), y_m \in W): \)

If \( x(n_w) \rightarrow (x(n-1)_w) \rightarrow \cdots \rightarrow (x(1)_w \rightarrow y_m) = 1_m \) then
\[ M_I^A(y_m) \geq \min \{ M_I^A(x(n_w)), \cdots, M_I^A(x(1)_w) \}, \]
\[ B_I^A(y_m) \geq^{* \ rmin} \{ B_I^A(x(n_w)), \cdots, B_I^A(x(1)_w) \}. \]

And \( J_F^A(y_m) \leq \max \{ J_F^A(x(n_w)), \cdots, J_F^A(x(1)_w) \}. \)

**Theorem 3.7.** Let \( A_m \) and \( B_m \) are two \( M_I^B \)-filters of \( W \), then \( A_m \cap B_m \) is also a \( M_I^B \)-filter of \( W \).

**Proof.** Let \( x_m, y_m, z_m \in W \) such that \( x_m \leq (y_m \rightarrow z_m) \), then \( x_m \rightarrow (y_m \rightarrow z_m) = 1_m \). Since \( A_m \) and \( B_m \) are two \( M_I^B \)-filters of \( W \), we have
\[ M_I^A(z_m) \geq \min \{ M_I^A(x_m), M_I^A(y_m) \}, B_I^A(z_m) \geq^{* \ rmin} \{ B_I^A(x_m), B_I^A(y_m) \} \]

and
\[ J_F^B(z_m) \leq \max \{ J_F^B(x_m), J_F^B(y_m) \}. \]
\[ M_T^A \cap B(z_m) = \min \{ M_T^A(z_m), M_T^B(z_m) \} \]
\[ = \min \{ \min \{ M_T^A(x_m), M_T^B(y_m) \}, \min \{ M_T^B(x_m), M_T^A(y_m) \} \} \]
\[ = \min \{ \min \{ M_T^A(x_m), M_T^B(x_m) \}, \min \{ M_T^A(y_m), M_T^B(y_m) \} \} \]
\[ = \min \{ M_T^A \cap B(x_m), M_T^A \cap B(y_m) \} \]
\[ B^1_B A \cap B (z_m) = \min \left\{ B^A_B (z_m), B(z_m) \right\} \]
\[ = \min \left\{ \min \left\{ B^A_B (x_m), B^A_B (y_m) \right\}, \min \left\{ B^B_B (x_m), B^B_B (y_m) \right\} \right\} \]
\[ = \min \left\{ \min \left\{ B^A_B (x_m), B^B_B (x_m) \right\}, \min \left\{ B^A_B (y_m), B^B_B (y_m) \right\} \right\} \]
\[ = \min \left\{ B^A_B (x_m \cap B), B^A_B (y_m \cap B) \right\}. \]
\[ J^1_B A \cap B (z_m) = \max \left\{ J^A_B (z_m), J^B_B (z_m) \right\} \]
\[ = \max \left\{ \max \left\{ J^A_B (x_m), J^A_B (y_m) \right\}, \max \left\{ J^B_B (x_m), J^B_B (y_m) \right\} \right\} \]
\[ = \max \left\{ \max \left\{ J^A_B (x_m), J^B_B (x_m) \right\}, \max \left\{ J^A_B (y_m), J^B_B (y_m) \right\} \right\} \]
\[ = \max \left\{ J^A_B (x_m \cap B), J^A_B (y_m \cap B) \right\}. \]

So \( A_m \cap B_m \) is a \( B^1_B \)-filter of W.

\[ \square \]

**Theorem 3.8.** The \( \text{M}_B^1 \)-set \( A_m = (M^A_B, B^A_B, J^A_B) \) is \( \text{M}_B^1 \)-filter of W if and only if its nonempty \( \text{M}_B^1 \) cut sets \( M^A_B(A_0) \) and \( J^A_B(A_0) \) are implicative filters of W and \( B^A_B(A_0) \) is an intuitionistic fuzzy filter of W for all \( \alpha, \gamma \in [0, 1] \) and \( [\beta_1, \beta_2] \in I \).

**Proof.** Suppose \( A_m \) is \( \text{M}_B^1 \)-filter of W and \( \alpha, \gamma \in [0, 1] \) and \( [\beta_1, \beta_2] \in I \). Let \( M^A_B(A_0), B^A_B(A_0) \) and \( J^A_B(A_0) \) are nonempty. Obviously \( \forall m \in M^A_B(A_0), \forall m \in B^A_B(A_0) \) and \( \forall m \in J^A_B(A_0) \). Let \( x_1, x_2, y_1, y_2, z_1 \), and \( z_2 \in W \) such that \( (x_1 \rightarrow x_2, x_1 \in M^A_B(A_0)) \) and \( (z_1 \rightarrow z_2, z_1 \in J^A_B(A_0)) \). Then:

\[ M^A_B(x_2) \geq \min \left\{ M^A_B((x_1 \rightarrow x_2), M^A_B(x_1)) \right\} \geq \alpha \] implies \( x_2 \in M^A_B(A_0) \)

\[ B^A_B(y_2) \geq \beta \min \left\{ B^A_B(y_1 \rightarrow y_2), B^A_B(y_1)) \right\} \geq [\beta_1, \beta_2] \] implies \( y_2 \in B^A_B(A_0) \).

\[ J^A_B(z_2) \leq \max \left\{ J^A_B(z_1 \rightarrow z_2), J^A_B(z_1) \right\} \leq \gamma \] implies \( z_2 \in J^A_B(A_0) \).

So, \( M^A_B(A_0) \) and \( J^A_B(A_0) \) are implicative filters of W and \( B^A_B(A_0) \) is an intuitionistic fuzzy filter of W.

Conversely, suppose that \( M^A_B(A_0) \) and \( J^A_B(A_0) \) are implicative filters of W and \( B^A_B(A_0) \) is an intuitionistic fuzzy filter of W for all \( \alpha, \gamma \in [0, 1] \) and \( [\beta_1, \beta_2] \in I \). For any \( x_m, y_m, z_m \in W \) such that \( M^A_B(x_m) = \alpha, B^A_B(y_m) = [\beta_1, \beta_2] \) and \( J^A_B(z_m) = \gamma \). Then \( x_m \in M^A_B(A_0), y_m \in B^A_B(A_0) \) and \( z_m \in J^A_B(A_0) \), so \( M^A_B(A_0), B^A_B(A_0) \) and \( J^A_B(A_0) \) are nonempty.

For any \( x_1, x_2 \in W \) , let \( \alpha = \min \{ M^A_B(x_1 \rightarrow x_2), M^A_B(x_1) \} \), \( [\beta_1, \beta_2] = \min \{ B^A_B(x_1 \rightarrow x_2), B^A_B(x_1) \} \) and \( \gamma = \{ J^A_B(x_1 \rightarrow x_2), J^A_B(x_1) \} \).

Then clearly:

\[ M^A_B(x_2) \geq \alpha = \min \{ M^A_B(x_1 \rightarrow x_2), M^A_B(x_1) \} \]

\[ B^A_B(y_2) \geq \beta \min \{ [\beta_1, \beta_2] = \min \{ B^A_B(x_1 \rightarrow x_2), B^A_B(x_1) \} \} \]
and
\[ J^A_F(z_2) \leq \gamma = \max \{ J^A_F(x_1 \text{ Re } x_2, J^A_F(x_1) \} \].
So, \( A_m = (M^A_T, B^A_I, J^A_F) \) is a \( M^I_B \) filter of \( W \).

**Lemma 3.2.** If \( A_m \) is a \( M^I_B \) filter of \( W \) then \( M^A_T \cap B^A_I \cap J^A_F \) are implicative filters of \( W \).

**Theorem 3.9.** Any implicative filter \( A \) of \( w \) is a \( (\alpha, [\alpha, \alpha], \alpha) \) cut- \( M^I_B \) of \( W \).

**Proof.** Let \( A \) is implicative filter of \( W \) and \( \alpha \in [0, 1] \). Consider a \( M^I_B \) set:
\[
A_m = (M^A_T(y_m), [B^A_I(y_m)] B^A_I(y_m)],
\]
\[
J^A_F(y_m) = (\alpha, [\alpha, \alpha], \alpha) \text{ if } y_m \in A_m \text{ and }
\]
\[
A_m = (0_m, [0_m], 0_m) \text{ if } y_m \not\in A_m. \text{ Let } x_m, y_m \in W. \text{ If } y_m \in A \text{ then }
\]
\[
M^A_T(y_m) = \alpha \geq \min \{ M^A_T(x_m \rightarrow y_m), M^A_T(x_m) \},
\]
\[
B^A_I(y_m) = [0, \alpha] \geq \alpha \text{ min } \{ B^A_I(x_m \rightarrow y_m), B^A_I(x_m) \}
\]
and
\[
J^A_F(y_m) \leq \max \{ J^A_F(x_m \rightarrow y_m), J^A_F(x_m) \}.
\]

**Suppose \( y_m \not\in A \) then \( x_m \not\in A \) or \( x_m \rightarrow y_m \not\in A \). So**
\[
M^A_T(y_m) = 0_m = \min \{ M^A_T(x_m \rightarrow y_m), M^A_T(x_m) \}
\]
\[
B^A_I(y_m) = [0_m, 0_m] = \min \{ B^A_I(x_m \rightarrow y_m), B^A_I(x_m) \}
\]
and
\[
J^A_F(y_m) = 0_m = \max \{ J^A_F(x_m \rightarrow y_m), J^A_F(x_m) \}. \text{ So, } A_m \text{ is } M^I_B \text{ filter of } W. \]

**Theorem 3.10.** If \( A_m \) is \( M^I_B \) filter of \( W \) then the set
\[
A = \{ x_m \in W/\{ M^A_T(y_m), B^A_I(y_m, y_m), J^A_F(y_m) = (M^A_T(1_m), B^A_I[1_m, 1_m], J^A_F(1_m) \}\}
\]
is a implicative filter of \( W \).

**Proof.** Clearly
\[
A = \{ x_m \in W/\{ M^A_T(y_m), B^A_I(y_m, y_m), J^A_F(y_m) = (M^A_T(1_m), B^A_I[1_m, 1_m], J^A_F(1_m) \}\},
\]
and \( 1_m \in A. \text{ Let } x_m, y_m \in w \text{ such that } x_m, x_m \rightarrow y_m \in A. \text{ Then }
\]
\[
M^A_T(x_m \rightarrow y_m) = M^A_T(x_m) = M^A_T(1_m),
\]
\[
B^A_I(x_m \rightarrow y_m) = B^A_I(x_m) = B^A_I[1_m, 1_m]
\]
and
\[
J^A_F(x_m \rightarrow y_m) = J^A_F(x_m) = J^A_F(1_m).
\]
So,
\[
M^A_T(y_m) \geq \min \{ M^A_T(x_m \rightarrow y - m), M^A_T(x_m) \} = M^A_T(1_m),
\]
\[ B_I^A(y_m) \geq^* \text{rmin} \left\{ B_I^A(x_m \rightarrow y_m), B_I^A(x_m) \right\} = B_I^A(1_m) \]

and

\[ J_F^A(y_m) \leq \text{max} \left\{ J_F^A(x_m \rightarrow y_m), J_F^A(x_m) \right\} = J_F^A(1_m). \]

That is \( y_m \in A \). So \( A \) a implicative filter of \( W \). \( \square \)

**Definition 3.2.** A \( M_B^I \) set \( A_m = (M_I^A, B_I^A, J_F^A) \) is on \( W \) is called a \( M_B^I \) \( w \)-lattice filter if it satisfies for all \( x_m, y_m \in W \),

\[
(3.5) \quad M_I^A(x_m \land y_m) \geq \min \left\{ M_I^A(x_m), M_I^A(y_m) \right\}, \\
B_I^A(x_m \land y_m) \geq^* \text{rmin} \left\{ B_I^A(x_m), B_I^A(y_m) \right\} \\
\text{and } J_F^A(x_m \land y_m) \leq \max \left\{ J_F^A(x_m), J_F^A(y_m) \right\}
\]

**Example 3.** The \( M_B^I \) set \( A_m = (M_I^A, B_I^A, J_F^A) \) defined on \( W \) as follows is \( M_B^I \) \( \text{lattice} \) filter of \( W \).

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<td>0_m</td>
<td>.547</td>
<td>[ .557 , .6 ]</td>
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<tr>
<td>( x_m )</td>
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<td>( y_m )</td>
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<td>( 1_m )</td>
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**Theorem 3.11.** Every \( M_B^I \) \( w \)-filter \( A_m \) of \( W \) is \( M_B^I \) \( \text{lattice filter} \) of \( W \).

**Proof.** Let \( A_m \) is a \( M_B^I \) \( w \)-filter of \( W \).

\[
M_I^A(x_m \land y_m) \geq \min \left\{ M_I^A(x_m \rightarrow (x_m \land y_m)), M_I^A(x_m) \right\} \\
= \min \left\{ M_I^A(x_m \rightarrow y_m), M_I^A(x_m) \right\} \\
\geq \min \left\{ \text{min} \left\{ M_I^A(y_m \rightarrow (x_m \land y_m)), M_I^A(y_m) \right\}, M_I^A(x_m) \right\} \\
\geq \min \left\{ \text{min} \left\{ M_I^A(1_m), M_I^A(y_m) \right\}, M_I^A(x_m) \right\} \\
= \min \left\{ M_I^A(y_m), M_I^A(x_m) \right\}
\]

\[
B_I^A(x_m \land y_m) \geq^* \text{rmin} \left\{ B_I^A(x_m \rightarrow (x_m \land y_m)), B_I^A(x_m) \right\} \\
= \text{min} \left\{ B_I^A(x_m \rightarrow y_m), B_I^A(x_m) \right\} \\
\geq^* \text{min} \left\{ \text{min} \left\{ B_I^A(y_m \rightarrow (x_m \land y_m)), B_I^A(y_m) \right\}, B_I^A(x_m) \right\} \\
\geq^* \text{min} \left\{ \text{min} \left\{ B_I^A(1_m), B_I^A(y_m) \right\}, B_I^A(x_m) \right\}
\]
\[
J^A_B(x_m \land y_m) \leq \min \left\{ J^A_B(x_m \to (x_m \land y_m)), J^A_B(x_m) \right\}
\]

\[
= \min \left\{ J^A_F(x_m \to y_m), J^A_F(x_m) \right\}
\]

\[
\leq \min \left\{ \min \left\{ J^A_F(y_m \to (x_m \land y_m)), J^A_F(y_m) \right\}, J^A_F(x_m) \right\}
\]

\[
\leq \min \left\{ \min \left\{ J^A_F(1_m), J^A_F(y_m) \right\}, J^A_F(x_m) \right\}
\]

\[
= \min \left\{ J^A_F(y_m), J^A_F(x_m) \right\}.
\]

So \( A_m \) of \( W \) is \( M^A_B \)-lattice filter of \( W \).

\[ \square \]

**Remark 3.1.** The \( M^A_B \)-lattice filter of \( W \) is need not to be a \( M^A_B \)-filter of \( W \). For example the \( M^A_B \)-lattice filter of \( A_m \) of \( W \) in example 3 is not a \( M^A_B \)-filter of \( W \) because \( M^A_B(z_m) \leq \min \{ M^A_I(y_m \to z_m), M^A_I(y_m) \} \).

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DEPARTMENT OF MATHEMATICS
K.L.UNIVERSITY, A.P., INDIA
Email address: anitha.t537@gmail.com

DEPARTMENT OF MATHEMATICS
NAGARJUNA UNIVERSITY, A.P., INDIA
Email address: amarendravelisela@ymail.com

DEPARTMENT OF MATHEMATICS
APIIIT NIZVID, A.P., INDIA
Email address: bnbattu@rguktn.ac.in