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EDGE ODD GRACEFUL LABELING OF SOME FLOWER PETAL GRAPHS

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ABSTRACT. A labeling of a graph G with α vertices and β edges called an edge odd graceful labeling if there is an edge labeling with odd numbers to all edges such that each vertex is assigned a label which is the sum $\mod (2\gamma)$ of labels of edge incident on it, where $\gamma = max\{\alpha, \beta\}$ and the induced vertex labels are distinct. In this paper, we discussed about edge odd gracefulness of some special class of flower petal graphs.

For graph theoretical terminology and notation, we in general follow [1]. In this paper we assume that the graph G is simple, connected, finite and undirected. Rosa [5] introduced a labeling of G called $\beta-$ valuation, later on Soloman W.Golomb [4] called as "graceful labeling" which is an injection f from the set of vertices $V(G)$ to the set $\{0, 1, 2, \ldots, \beta\}$ such that when each edge $e = st$ is assigned the label $|f(s) - f(t)|$, the resulting edge labels are distinct. A graph which admits a graceful labeling is called a graceful graph. In 1991, Gnanajothi [3] introduced a labeling of G called odd graceful labeling which is an injection f from the set of vertices $V(G)$ to the set $\{0, 1, 2 \ldots, 2\beta - 1\}$ such that when each edge $e = st$ is assigned the label $|f(s) - f(t)|$, the resulting edge labels are $\{1, 3, \ldots, 2\beta - 1\}$. A graph which admits an odd graceful labeling is called an odd graceful graph.

In 2009, Solairaju and Chitra [6] introduced a labeling of G called edge odd graceful labeling of G, which is a bijection f from the set of edges $E(G)$ to the

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set $\{1, 3, \ldots, 2\beta - 1\}$ such that the induced map f^* from the set of vertices $V(G)$ to $\{0, 1, 2, \ldots, 2\beta - 1\}$ given by $f^*(s) = \sum_{st \in E(G)} f(st) (\mod 2\beta)$ is a bijection. A graph which admits edge odd graceful labeling is called an edge odd graceful graph.

Recently, Daoud [2] has established, for $n \geq 3$; the friendship graphs $Fr_n^{(3)}$, $Fr_n^{(4)}$, $\overline{F}r_n^{(3)}$, the wheel graph $W_n = K_1 + C_n$, helm graph H_n, web graph Wb_n, double wheel graph $W_{n,n}$, fan graph $F_n = K_1 + P_n$, gear graph G_n, half gear graph HG_n, double fan graph and polar grid $P_{m,n}$ are edge odd graceful graphs. In this paper, we proved that some special class of flower petal graph is an edge odd graceful.

1. Results

The above figure shows that $F^n_{C_m}$ graph.

Theorem 1.1. *For $m \geq 7$, $n \geq 3$, and m, n odd, then the graph $F^n_{C_m}$ is an edge odd graceful graph.*

Proof. In this graph, the number of vertices is $\alpha = (m-1)n+1$, number of edges is $\beta = mn$ and $\gamma = max\{\alpha, \beta\} = mn$.

Let the graph $F^n_{C_m}$ be as in Figure 1. The cycles C_m in it are $C^1_m, C^2_m, C^3_m, \ldots, C^n_m$ and the middle vertex is t_0. Name the vertices of C^i_m by $t_{(m-1)i-(m-2)}$, $t_{(m-1)i-(m-3)}$, $t_{(m-1)i-(m-4)}, \ldots, t_{(m-1)i}$ for $i \in \{1, 2, 3, \ldots, n\}$.

Now, label the edges of C^i_m for $i \in \{1, 2, 3, \ldots, n\}$, t_0t_1, t_0t_m, t_0t_{2m-1}, t_0t_{3m-2}, $\ldots$, $t_0t_{n(m-1)-(m-2)}$ by $1, 2m+1, 4m+1, \ldots, 2m(n-1)+1$; label the edges t_0t_{m-1}, t_0t_{2m-2}, t_0t_{3m-3}, $\ldots$ $t_0t_{n(m-1)}$ by $2m-1$, $4m-1$, $6m-1$, $\ldots$, $2mn-1$; label the edges t_1t_2, t_2t_3, t_3t_4, $\ldots$, $t_{m-2}t_{m-1}$ by 3, 5, 7, $\ldots$, $2m-3$; label the edges t_mt_{m+1}, $t_{m+1}t_{m+2}$, $t_{m+2}t_{m+3}$, $\ldots$, $t_{2m-3}t_{2m-2}$ by $2m+3$, $2m+5$, $\ldots$, $4m-3$; label the edges $t_{2m-1}t_{2m}$, $t_{2m}t_{2m+1}$, $\ldots$, $t_{3m-2}t_{3m-3}$ by $4m+3$, $4m+5$, $\ldots$, $6m-3$; label the edges $t_{3m-2}t_{3m-1}$, $t_{3m-1}t_{3m}$, $\ldots$, $t_{4m-3}t_{4m-4}$ by $6m+3$, $6m+5$, $\ldots$, $8m-3$; label the edges $t_{4m-3}t_{4m-2}$, $t_{4m-2}t_{4m-1}$, $\ldots$, $t_{5m-4}t_{5m-5}$ by $8m+3$, $8m+5$, $\ldots$, $10m-3$; in general, label the edges $t_{(n-1)m-(n-2)}t_{(n-1)m-(n-3)}$, $t_{(n-1)m-(n-3)}t_{(n-1)m-(n-4)}$, $\ldots$, $t_{(m-1)n-1}t_{(m-1)n}$ by $2(n-1)m+3$, $2(n-1)m+5$, $\ldots$, $2nm-3$, Hence, the induced labeling of vertices t_1, t_2, t_3, $\ldots$, $t_{(m-1)n-1}$, $t_{(m-1)n}$ are 4, 8, 12, $\ldots$, $4(mn-1)(\mod 2\gamma)$, and the induced vertex labeling of t_0 is $2mn^2(\mod 2\gamma) = 0$. Thus, the graph is edge odd graceful. □

Theorem 1.2. *For $m \geq 6$, $n \geq 2$, and m, n even, then the graph $F^n_{C_m}$ is an edge odd graceful graph.*

Proof. In this graph, the number of vertices is $\alpha = (m-1)n+1$, number of edges is $\beta = mn$ and $\gamma = max\{\alpha, \beta\} = mn$. The cycles C_m in it are $C^1_m, C^2_m, C^3_m, \ldots, C^n_m$ and the middle vertex is t_0. Name the vertices of C^i_m by $t_{(m-1)i-(m-2)}$, $t_{(m-1)i-(m-3)}$, $t_{(m-1)i-(m-4)}, \ldots, t_{(m-1)i}$ for $i \in \{1, 2, 3, \ldots, n\}$. Now, Label alternate edges of C^i_m for $i \in \{1, 2, 3, \ldots, n\}$, t_0t_1, t_2t_3, t_4t_5, $\ldots$, $t_{m-2}t_{m-1}$ in the first copy of cycle C^1_m; t_0t_m, $t_{m+1}t_{m+2}$, $t_{m+3}t_{m+4}$, $\ldots$, $t_{2m-3}t_{2m-2}$ in the second copy of cycle C^2_m, and so on by $1,\ 3,\ 5, \ldots\ mn-1$; label alternate edges of C^i_m for $i \in \{1, 2, 3, \ldots, n\}$, t_1t_2, t_3t_4, $t_5t_6, \ldots, t_{m-1}t_0$ in the first copy of cycle C^1_m; t_mt_{m+1}, $t_{m+2}t_{m+3}$, $t_{m+4}t_{m+5}, \ldots$, $t_{2m-2}t_0$ in the second copy of cycle C^2_m, and so on by $mn+1$, $mn+3$, $mn+5, \ldots$ $2mn-1$. The induced vertex labeling of t_1, t_2 $t_3, \ldots$ $t_{(m-1)n}$ are $mn+2$, $mn+4$, $mn+6, \ldots, 3mn-2$ and the induced vertex labeling t_0 is $2mn^2 \pmod{2\gamma} = 0$. Thus, the graph is edge odd graceful. □

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