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INTUITIONISTIC FUZZY IDEALS AND INTUITIONISTIC FUZZY FILTERS OF TERNARY SEMIGROUPS

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ABSTRACT. In this paper, we consider the intuitionistic fuzzification of ternary sub semigroups (left ideals, right ideals, lateral ideals, ideals) and intuitionistic fuzzification of left filters (right filters, lateral filters, filters) of ternary semigroups and investigate some of their basic properties.

1. INTRODUCTION

Lehmer [8] the notion introduced of ternary semi groups. Any semi group can be reduced to a ternary semi group but a ternary semi group does not necessarily reduce to a semi group. Santiago [9] studied regular ternary semi groups. Chinram and Saelee [3] discussed on fuzzy ideals and fuzzy filters of ordered ternary semigroups. Dixit and Dewan [4] studied properties of quasi-ideals and bi-ideals in ternary semi groups and proved every quasi ideal is a bi ideal in ternary but the converse is not true in general by gave several examples in different context. Kar and Maity [6] derived congruence's of ternary semi groups. Imphan [5] studied minimal and maximal lateral ideals of ternary semi groups. Following the introduction of fuzzy sets by Zadeh [10] the fuzzy set theories have found many applications in the domain of mathematics. The concept of intuitionistic fuzzy set was introduced by Atanassov [1, 2] as a generalization of

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the notion of fuzzy set and various properties are proved, which are connected to the operations and relations over sets with model and topological operations, defined over the set of intuitionistic fuzzy sets. Kuroki [7] studied on fuzzy semiprime quasi-ideals in semigroups and fuzzy semi prime. In this paper, we consider the intuitionistic fuzzification of the concept of ideals and filters in a ternary semi group and some properties are investigated.

2. PRELIMINARIES

In this section we discuss some elementary definitions that we use latter.

Definition 2.1. *A non – empty set T is called a ternary semigroup if there exists a ternary operation* $T \times T \times T \rightarrow T$ *written as* $(x_{1.}, x_2, x_3) \rightarrow x_1x_2x_3$*satisfying the following identity for any* $x_1x_2x_3x_4x_5 \in T$, $[[x_1x_2x_3]\, x_4x_5] = [x_1\, [x_2x_3x_4]\, x_5] = [x_1x_2\, [x_3x_4x_5]]$. *Any semigroup can be reduced to a ternary semigroup. However, Banach showed that a ternary semigroup does not necessarily reduce to a semi-group.*

Definition 2.2. *Let T be a ternary semigroup. For non-empty subsets A,B and C, let* $ABC = \{abc|a \in A, b \in B \ \ and \ c \in C\}$. *For a non empty subset A of T, we note* $(A] = \{t \in T|t \leq h \ \ forsome \ h \in A\}$.

Definition 2.3. *A non empty subset A of T is calleda ternary subsemigroups of T if* $TTT \subseteq A$.

Definition 2.4. *A non empty subset A of T is called a left ideal of T if* $TTA \subseteq A$,*a right ideal of T if* $ATT \subseteq A$,*and a lateral ideal of T if* $TAT \subseteq A$. *If A is a left, right and lateral ideal of T, A is called an ideal of T.*

Definition 2.5. *Let T be a ternary semigroup. A non empty set F of T is called left filter of T if (i)* $F^3 \subseteq F$ *(ii) For all* $x, y, z \in T,\ xyz \in F$ *implies* $z \in F$.

Definition 2.6. *Let T be a ternary semigroup. A non empty set F of T is called right filter of T if (i)* $F^3 \subseteq F$ *(ii) For all* $x, y, z \in T,\ xyz \in F$ *implies* $x \in F$.

Definition 2.7. *Let T be a ternary semigroup. A non empty set F of T is called lateral filter of T if (i)* $F^3 \subseteq F$ *(ii) For all* $x, y, z \in T,\ xyz \in F$ *implies* $y \in F$.

Definition 2.8. *Let T be a ternary semigroup. A non empty set F of T is called filter of T if (i)* $F^3 \subseteq F$ *(ii) For all* $x, y, z \in T,\ xyz \in F$ *implies* $x, y, z \in F$.

Definition 2.9. *Let T be a ternary semigroup. A non empty set A of T is called a prime subset of T if for all* $x, y, z \in T,\ xyz \in A$ *implies*$x \in A$*or* $y \in A$*or* $z \in A$.

Definition 2.10. *Let T be a ternary semigroup. A ternary subsemigroup A of T is called a prime ternary subsemigroup of T if A is a prime subset of T.*

Definition 2.11. *Let T be a ternary semigroup. A left ideal A of T is called a prime left ideal of T if A is a prime subset of T.*

Definition 2.12. *An Intuitionistic fuzzy set (IFS) A in a non empty set X is an object having the form* $A = \{< x, \mu_A(x), \nu_A(x) > /x \in X\}$*, where the functions* $\mu_A : X \to [0,1]$ *and* $\nu_A : X \to [0,1]$ *denote the degree of membership and the degree of non membership of each element* $x \in X$ *to the set A, respectively and* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ *for all* $x \in X$*. For the sake of simplicity, we shall use the symbol* $A = \ < \mu_A, \nu_A >$ *for the IFS* $A = \{< x, \mu_A(x), \nu_A(x) > /x \in X\}$.

Definition 2.13. *Let A be an IFS in X and let* $t \in [0,1]$*. Then the sets* $U(\mu_A; t) = \{x \in X : \mu_A(x) \geq t\}$*and*$L(\nu_A; s) = \{x \in X : \nu_A(x) \geq t\}$ *are called a* μ *level t cut and* ν *level t cut of A, respectively.*

Definition 2.14. *Let A be an IFS of a set X. For each pair* $t, s \in [0,1]$ *such that* $t+s \leq 1$*, the set* $A_{<t,s>} = \{x \in X :\mu_A(x) \geq t$ *and* $\nu_A(x) \leq s\}$ *is called an (t,s) level subset of A. The set of all* $(t,s) \in Im\,(\mu_A) \times Im(\nu_A)$ *such that* $t + s \leq 1$ *is called the image of A. Clearly* $A_{<t,s>} = U(\mu_A; t) \bigcap L(\nu_A; s)$ *where* $U(\mu_A; t)$*and* $L(\nu_A; s)$ *are upper and lower level subsets of A.*

3. Main Results

In what follows, let T denote a ternary semi group unless otherwise specified.

Theorem 3.1. *Let T be a ternary semigroup and A non-empty subset of T. the following statements are true.*

(1) *A is a ternary subsemigroups of T if and only if A is an intuitionistic fuzzy ternary subsemigroups of T.*
(2) *A is left ideal (right ideal,lateral ideal,ideal) of T if and only if A is an intuitionistic fuzzy left ideal (right ideal,lateral ideal,ideal) of T.*

Proof. Assume that A is ternary subsemigroups of T, let $x, y, z \in T$.
Case (i): $x, y, z \in A$.since A is ternary subsemigroups of T, $xyz \in A$. Therefore

$$(\mu_A(xyz) = 1 \quad \geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$$

and

$$(\nu_A(xyz) = 0 \leq \max\{\nu_A(x), \nu_A(y), \nu_A(z)\}.$$

Case (ii): $x \notin A$or $y \notin A$ or $z \notin A$. Thus $\mu_A(x) = 0$ and $\nu_A(x) = 1$ or $\mu_A(y) = 0$ and

$$\nu_A(y) = 1 \text{ or } \mu_A(z) = 0 \text{ and } \nu_A(z) = 1.$$

Hence

$$(\mu_A(xyz) \quad \geq 0 = \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$$

and

$$(\nu_A(xyz) \leq 1 = \max\{\nu_A(x), \nu_A(y), \nu_A(z)\}.$$

Conversely, assume that A is an intuitionistic fuzzy ternary semigroup of T. Let $x, y, z \in A$.so $\mu_A(x) = \mu_A(y) = \mu_A(z) = 1$ and $\nu_A(x) = \nu_A(y) = \nu_A(z) = 0$. Since A is an intuitionistic fuzzy ternary subsemigroups of T, $(\mu_A(xyz) \geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\} = 1$ and $(\nu_A(xyz) \leq \max\{\nu_A(x), \nu_A(y), \nu_A(z)\}$. Then $xyz \in A$. □

Theorem 3.2. *Let F be a non empty subset of a ternary semigroup T. Then F is a left filter (right filter, lateral filter, filter) of T if and only if* $\left(\chi_F, \bar{\chi_F}\right)$*is an intuitionistic fuzzy left filter (right filter, lateral filter, filter) of T.*

Proof. Assume that F is a left filter of T. let $x, y, z \in T$.
Case (i): $x, y, z \in F$. Then $xyz \in F$. Hece $\chi_F(xyz) = 1$and $\bar{\chi}_F(xyz) = 0$. Therefore

$$\chi_F(xyz) \geq \min\{\chi_F(x), \chi_F(y), \chi_F(z)\}$$

and

$$\bar{\chi}_F(xyz) \leq \max\{\bar{\chi}_F(x), \bar{\chi}_F(y), \bar{\chi}_F(z)\}.$$

Case (ii): $x \notin F$ or $y \notin F$or $z \notin F$. So $\chi_F(x) = 0$ and $\bar{\chi_F}(x) = 1$or $\chi_F(y) = 0$and $\bar{\chi_F}(y) = 1$ or $\chi_F(z) = 0$ and $\bar{\chi_F}(z) = 1$. This implies $\chi_F(xyz) \geq \min\{\chi_F(x), \chi_F(y), \chi_F(z)\}$ and $\bar{\chi}_F(xyz) \leq \max\{\bar{\chi}_F(x), \bar{\chi}_F(y), \bar{\chi}_F(z)\}$. Finally, let $x, y, z \in T$.

Case (iii): $xyz \in F$.since F is a left filter of T and$xyz \in F, z \in F$. so $\chi_F(z) = 1$ and $\overline{\chi_F}(z) = 0$. Therefore $\chi_F(xyz) \leq \chi_F(z)$ and $\overline{\chi}_F(xyz) \geq \overline{\chi}_F(z)$.

Case (iv) : $xyz \notin F$. Then $\chi_F(xyz) = 0$and $\overline{\chi}_F(xyz) = 1$. Therefore $\chi_F(xyz) \leq \chi_F(z)$ and $\overline{\chi}_F(xyz) \geq \overline{\chi}_F(z)$.

Conversely, assume $(\chi_F, \overline{\chi_F})$ is an intuitionistic fuzzy left filter of T. Let $x, y, z \in T$. Then $\chi_F(x) = \chi_F(y) = \chi_F(z) = 1$ and $\overline{\chi_F}(x) = \overline{\chi_F}(y) = \overline{\chi_F}(z) = 0$.

Thus $\chi_F(xyz) \geq \min\{\chi_F(x), \chi_F(y), \chi_F(z)\}$ and $\overline{\chi}_F(xyz) \leq \max\{\overline{\chi}_F(x), \overline{\chi}_F(y), \overline{\chi}_F(z)\}$. Hence $xyz \in F$. Finally, let $x, y, z \in T$. Such that $xyz \in F$. So $\chi_F(xyz) = 1$ and $\overline{\chi}_F(xyz) = 0$. Since $(\chi_F, \overline{\chi_F})$ is an intuitionistic fuzzy left filter of T, $\chi_F(xyz) \leq \chi_F(z)$ and $\overline{\chi_F}(xyz) \geq \overline{\chi_F}(z)$. This implies $\chi_F(z) = 1$ and $\overline{\chi}_F(z) = 0$. So $z \in F$. The other parts can be proved in similar way. □

Theorem 3.3. *Let T be a ternary semi group and A a nonempty subset of T. Then A is a prime subset of t if and only if A is a prime intuitionistic fuzzy subset of A.*

Proof. Let A be a prime subset of T and $x, y, z \in T$.
Case(i): $xyz \in A$. Since A is a prime subset of T, $x \in A$ or $y \in A$or $z \in A$. So $\max\{\mu_A(x), \mu_A(y), \mu_A(z)\} = 1 \geq \mu_A(xyz)$and $\min\{\nu_A(x), \nu_A(y), \nu_A(z)\} = 0 \leq \nu_A(xyz)$.
Case(ii): $xyz \notin A$. So $\mu_A(xyz) = 0 \leq \max\{\mu_A(x), \mu_A(y), \mu_A(z)\}$and $\nu_A(xyz) = 1 \geq \min\{\nu_A(x), \nu_A(y), \nu_A(z)\}$.

Conversely, Let $x, y, z \in T$ such that $xyz \in A$. Thus $\mu_A(xyz) = 1$ and $\nu_A(xyz) = 0$. Since A is prime, $\max\{\mu_A(x), \mu_A(y), \mu_A(z)\} = 1$ and $\min\{\nu_A(x), \nu_A(y), \nu_A(z)\} = 0$. Then $\mu_A(x) = 1$ and $\nu_A(x) = 0$or $\mu_A(y) = 1$and$\nu_A(y) = 0$ or $\mu_A(z) = 1$and $\nu_A(z) = 0$. Hence $x \in A$or $y \in A$or $z \in A$. □

Theorem 3.4. *Let A be an intuitionistic fuzzy subset of a ternary semigroup T. Then A is a prime intuitionistic fuzzy subset of T if and only if for all* $t, s \in [0,1]$, *if* $A_{<t,s>} \neq \phi$,*then* $A_{<t,s>}$ *is a prime subset of T.*

Proof. Assume that A is a prime intuitionistic fuzzy subset of T. Let $t, s \in [0,1]$. Suppose that $A_{<t,s>} \neq \phi$. Let $x, y, z \in T$ such that $xyz \in A_{<t,s>}$. Thus $\mu_A(xyz) \geq t$ and $\nu_A(xyz) \leq s$. Since A is prime $\mu_A(x) \geq t$ and $\nu_A(x) \leq s$ or $\mu_A(y) \geq t$ and $\nu_A(y) \leq s$or $\mu_A(z) \geq t$ and $\nu_A(z) \leq s$. Hence $x \in A_{<t,s>}$ or $y \in A_{<t,s>}$ or $z \in A_{<t,s>}$.

Conversely, let $x, y, z \in T$. Choose$t = \mu_A(xyz)$ and $s = \nu_A(xyz)$. Thus $xyz \in A_{<t,s>}$.

Since $A_{<t,s>}$ is prime , $x \in A_{<t,s>}$ or $y \in A_{<t,s>}$ or $z \in A_{<t,s>}$. Then $\mu_A(x) \geq t$ and $\nu_A(x) \leq s$ or $\mu_A(y) \geq t$ and $\nu_A(y) \leq s$ or $\mu_A(z) \geq t$ and $\nu_A(z) \leq s$. Therefore $\max\{\mu_A(x), \mu_A(y), \mu_A(z)\} \geq t = \mu_A(xyz)$ and $\min\{\nu_A(x), \nu_A(y), \nu_A(z)\} \leq s = \nu_A(xyz)$. $\square$

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