EDGE ODD GRACEFUL LABELING OF SOME CYCLE RELATED GRAPHS

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ABSTRACT. A labeling of a graph $G$ with $\alpha$ vertices and $\beta$ edges is called an edge odd graceful labeling if there is an edge labeling with odd numbers to all edges such that each vertex is assigned a label which is the sum mod $(2\gamma)$ of labels of edge incident on it, where $\gamma = \max\{\alpha, \beta\}$ and the induced vertex labels are distinct.

1. INTRODUCTION

For graph theoretical terminology and notation, we in general follow [1]. In this paper we assume that the graph $G$ is simple, connected, finite and undirected. Rosa [5] introduced a labeling of $G$ called $\beta-$ valuation, later on Solomon W. Golomb [4] called as ”graceful labeling” which is an injection $f$ from the set of vertices $V(G)$ to the set $\{0, 1, 2, \ldots, \beta\}$ such that when each edge $e = st$ is assigned the label $|f(s) - f(t)|$, the resulting edge labels are distinct. A graph which admits a graceful labeling is called a graceful graph. In 1991, Gnanajothi [3] introduced a labeling of $G$ called odd graceful labeling which is an injection $f$ from the set of vertices $V(G)$ to the set $\{0, 1, 2, \ldots, 2\beta - 1\}$ such that when each edge $e = st$ is assigned the label $|f(s) - f(t)|$, the resulting edge labels are $\{1, 3, \ldots, 2\beta - 1\}$. A graph which admits an odd graceful labeling is called an odd graceful graph.

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In 2009, Solairaju and Chitra [6] introduced a labeling of $G$ called edge odd graceful labeling of $G$, which is a bijection $f$ from the set of edges $E(G)$ to the set $\{1, 3, \ldots, 2\beta - 1\}$ such that the induced map $f^*$ from the set of vertices $V(G)$ to $\{0, 1, 2, \ldots, 2\beta - 1\}$ given by $f^*(s) = \sum_{st \in E(G)} f(st)( \mod 2\beta)$ is a bijection. A graph which admits edge odd graceful labeling is called an edge odd graceful graph.

Recently, Daoud [2] has established that for $n \geq 3$ the friendship graphs $F_{r_n}^{(3)}$, $F_{r_n}^{(4)}$, $F_{r_n}^{(3)}$, the wheel graph $W_n = K_1 + C_n$, helm graph $H_n$, web graph $W_{bn}$, double wheel graph $W_{n,n}$, fan graph $F_n = K_1 + P_n$, gear graph $G_n$, half gear graph $HG_n$, double fan graph and polar grid $P_{m,n}$ are edge odd graceful graphs. In this paper, we proved that flower petals graph is an edge odd graceful.

2. Results

The flower petals graph $F_{p_n}$ with $3n + 1$ vertices and $5n$ edges is constructed by joining $n$ copies of the $K_4 - e$ with a common vertex.
Theorem 2.1. For \( n \geq 2 \), \( n \not\equiv 3 \pmod{10} \) and \( n \not\equiv 5 \pmod{10} \), the flower petals graph \( \text{F}_p^4 \) is an edge odd graceful graph.

Proof. In this graph, the number of vertices is \( \alpha = 3n + 1 \), number of edges is \( \beta = 5n \) and \( \gamma = \max\{\alpha, \beta\} = 5n \).

Case 1. \( n \) is even.
Label the outer edges \( t_1s_1, t_1s_2, t_2s_3, t_2s_4, \ldots, t_ns_{2n-1}, t_n^s_{2n} \) by \( 1, 3, 5, \ldots, 4n - 3, 4n - 1 \) and label the inner edges \( t_0s_1, t_0s_2, t_0s_3, \ldots, t_0s_{2n} \), by \( 4n + 1, 4n + 3, \ldots, 8n - 1 \), then label the middle edges \( t_0t_1, t_0t_2, t_0t_3, \ldots, t_0t_n \), by \( 8n + 1(\mod 2\gamma), 8n + 3(\mod 2\gamma), 8n + 5(\mod 2\gamma), \ldots, (10n - 1)(\mod 2\gamma) \). Hence, the induced labeling of vertices \( t_1, t_2, t_3, \ldots, t_n \) are \( (8n + 5)(\mod 2\gamma), (8n + 15)(\mod 2\gamma), \ldots, 18n - 5(\mod 2\gamma) \), the induced labeling of vertices \( s_1, s_2, s_3, \ldots, s_{2n} \) are \( (4n + 2)(\mod 2\gamma), (4n + 6)(\mod 2\gamma), \ldots, (12n - 2)(\mod 2\gamma) \) and induced vertex labeling of \( t_0 \) is \( (21n^2)(\mod 2\gamma) \). Figure 1 shows the labeling for \( n \) even.

Case 2. \( n \) is odd.
Label the middle edges \( t_0t_1, t_0t_2, t_0t_3, \ldots, t_0t_n \) by \( 1, 3, 5, \ldots, 2n - 1 \) and label the inner edges \( t_0s_1, t_0s_2, t_0s_3, \ldots, t_0s_{2n} \), by \( 2n + 1, 2n + 3, \ldots, 6n - 1 \), then label the
outer edges $t_1s_1, t_1s_2, t_2s_3, t_2s_4, \ldots t_ns_{2n-1}, t_ns_{2n}$, by $(6n + 1)(\mod 2\gamma), (6n + 3)(\mod 2\gamma), (6n + 5)(\mod 2\gamma), (6n + 7)(\mod 2\gamma), \ldots$, $(10n - 1)(\mod 2\gamma)$. Hence, the induced labeling of vertices $t_1, t_2, t_3, \ldots t_n$ are $2n + 5, 2n + 15 \ldots 2n - 5$, the induced labeling of vertices $s_1, s_2, s_3, \ldots s_{2n}$ are $8n + 2, 8n + 6 \ldots (16n - 2)(\mod 2k)$ and induced vertex labeling of $t_0$ is $9n^2(\mod 2\gamma)$. Figure 2 shows the labeling for $n$ odd. Thus, the graph is edge odd graceful.

The following figure shows the illustration for $F^7_{C_5}$.

**Theorem 2.2.** For $n \geq 3$, and $n$ odd, then the graph $F^m_{C_5}$ is an edge odd graceful graph.

**Proof.** In this graph, the number of vertices is $\alpha = 4n + 1$, number of edges is $\beta = 5n$ and $\gamma = \max\{\alpha, \beta\} = 5n$.

Let the graph $F^m_{C_5}$ be as in Figure 3. The cycles $C_5$ in it are $C^1_5, C^2_5, C^3_5, \ldots, C^m_5$ and the middle vertex is $t_0$. Name the vertices of $C^i_5$ by $t_{4i-3}, t_{4i-2}, t_{4i-1}, t_{4i}$ for
Now, label the edges of $C_i$ for $i \in \{1, 2, 3, \ldots, n\}$ by $t_0 t_1$, $t_0 t_5$, $t_0 t_9$, ..., $t_0 t_{4n-3}$ by 1, 11, 21 ..., $10n - 9$, $t_0 t_4$, $t_0 t_8$, $t_0 t_{12}$, ..., $t_0 t_{4n}$ by 9, 19, 29, ..., $10n - 1$; label the edges $t_1 t_2$, $t_5 t_6$, ..., $t_{4n-3} t_{4n-2}$ by 3, 13, 23, ..., $10n - 7$; label the edges $t_4 t_3$, $t_8 t_7$, ..., $t_{4n-1} t_{4n-1}$ by 7, 17, 27, ..., $10n - 3$; label the edges $t_2 t_3$, $t_6 t_7$, $t_{10} t_{11}$, ..., $t_{4n-2} t_{4n-1}$ by 5, 15, 25, ..., $10n - 5$. Hence, the induced labeling of vertices $t_1, t_2, t_3, \ldots, t_{4n-1}, t_{4n}$ are $4, 8, 12, \ldots, (20n-4) \pmod{2\gamma}$, and the induced vertex labeling of $t_0$ is $10n^2 \pmod{2\gamma} = 0$. Figure 3 shows the labeling for this case. Thus, the graph is edge odd graceful. □

REFERENCES


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