ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9605–9610
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.11.65 Spec. Iss. on ICRTAMS-2020

EDGE ODD GRACEFUL LABELING OF SOME CYCLE RELATED GRAPHS

B. AMBIKA[1] AND G. BALASUBRAMANIAN

ABSTRACT. A labeling of a graph G with α vertices and β edges is called an edge odd graceful labeling if there is an edge labeling with odd numbers to all edges such that each vertex is assigned a label which is the sum $\mod (2\gamma)$ of labels of edge incident on it, where $\gamma = max\{\alpha, \beta\}$ and the induced vertex labels are distinct.

1. INTRODUCTION

For graph theoretical terminology and notation, we in general follow [1]. In this paper we assume that the graph G is simple, connected, finite and undirected. Rosa [5] introduced a labeling of G called $\beta-$ valuation, later on Soloman W. Golomb [4] called as "graceful labeling" which is an injection f from the set of vertices $V(G)$ to the set $\{0, 1, 2, \dots, \beta\}$ such that when each edge $e = st$ is assigned the label $|f(s) - f(t)|$, the resulting edge labels are distinct. A graph which admits a graceful labeling is called a graceful graph. In 1991, Gnanajothi [3] introduced a labeling of G called odd graceful labeling which is an injection f from the set of vertices $V(G)$ to the set $\{0, 1, 2 \dots, 2\beta - 1\}$ such that when each edge $e = st$ is assigned the label $|f(s) - f(t)|$, the resulting edge labels are $\{1, 3, \dots, 2\beta - 1\}$. A graph which admits an odd graceful labeling is called an odd graceful graph.

[1] *corresponding author*

2020 *Mathematics Subject Classification.* 05C15, 05C22.

Key words and phrases. graceful labeling, edge odd graceful labeling.

In 2009, Solairaju and Chitra [6] introduced a labeling of G called edge odd graceful labeling of $G,$ which is a bijection f from the set of edges $E(G)$ to the set $\{1,3,\ldots,2\beta-1\}$ such that the induced map f^* from the set of vertices $V(G)$ to $\{0,1,2,\ldots,2\beta-1\}$ given by $f^*(s)=\sum_{st\in E(G)} f(st)(\mod 2\beta)$ is a bijection. A graph which admits edge odd graceful labeling is called an edge odd graceful graph.

Recently, Daoud [2] has established that for $n\geq 3$ the friendship graphs $Fr_n^{(3)}$, $Fr_n^{(4)}$, $\overline{F}r_n^{(3)}$, the wheel graph $W_n=K_1+C_n$, helm graph H_n, web graph Wb_n, double wheel graph $W_{n,n}$, fan graph $F_n=K_1+P_n$, gear graph G_n, half gear graph HG_n, double fan graph and polar grid $P_{m,n}$ are edge odd graceful graphs. In this paper, we proved that flower petals graph is an edge odd graceful.

2. RESULTS

The flower petals graph Fp_n^4 with $3n+1$ vertices and $5n$ edges is constructed by joining n copies of the K_4-e with a common vertex.

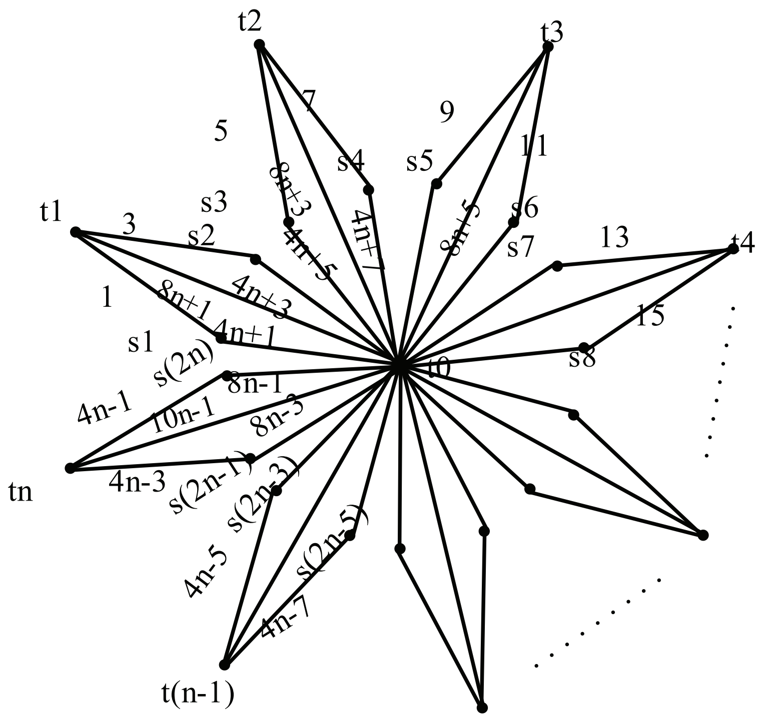

FIGURE 1

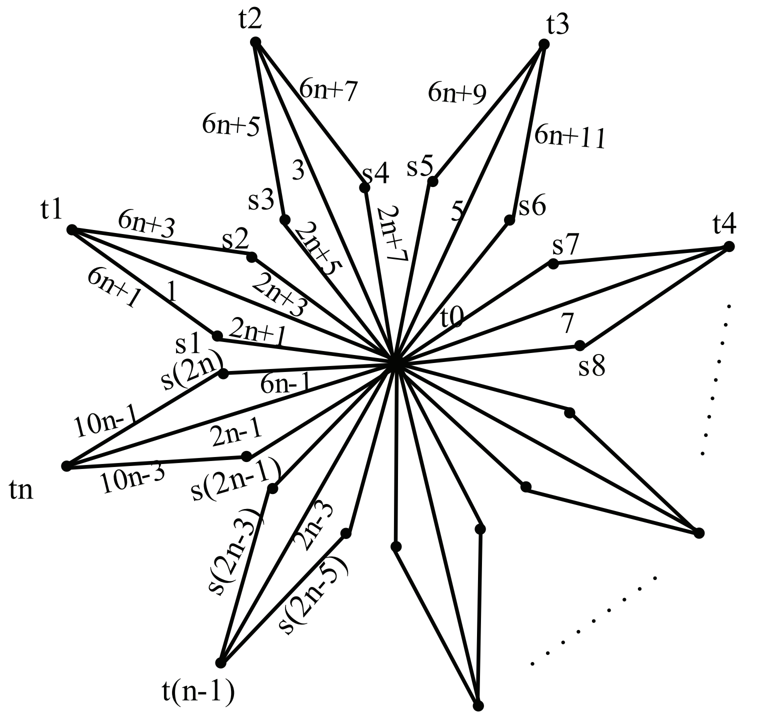

FIGURE 2

Theorem 2.1. *For $n \geq 2$, $n \not\equiv 3(\mod 10)$ and $n \not\equiv 5(\mod 10)$, the flower petals graph Fp_n^4 is an edge odd graceful graph.*

Proof. In this graph, the number of vertices is $\alpha = 3n + 1$, number of edges is $\beta = 5n$ and $\gamma = max\{\alpha, \beta\} = 5n$.

Case 1. n is even.

Label the outer edges t_1s_1, t_1s_2, t_2s_3, $t_2s_4 \dots$ t_ns_{2n-1}, t_ns_{2n} by $1, 3, 5, \dots, 4n - 3, 4n-1$ and label the inner edges t_0s_1,t_0s_2,t_0s_3,$\dots$ t_0s_{2n}, by $4n+1, 4n+3, \dots, 8n-1$, then label the middle edges t_0t_1,t_0t_2,t_0t_3,$\dots$ t_0t_n, by $8n + 1(\mod 2\gamma), 8n + 3(\mod 2\gamma), 8n+5(\mod 2\gamma), \dots, (10n-1)(\mod 2\gamma)$. Hence, the induced labeling of vertices t_1,t_2,t_3,$\dots$ t_n are $(8n + 5)(\mod 2\gamma)$, $(8n + 15)(\mod 2\gamma) \dots 18n - 5(\mod 2\gamma)$, the induced labeling of vertices s_1,s_2,s_3,$\dots$ s_{2n} are $(4n + 2)(\mod 2\gamma)$, $(4n + 6)(\mod 2\gamma)$,$\dots$ $(12n - 2)(\mod 2\gamma)$ and induced vertex labeling of t_0 is $(21n^2)(\mod 2\gamma)$. Figure 1 shows the labeling for n even.

Case 2. n is odd.

Label the middle edges t_0t_1, t_0t_2, t_0t_3, $\dots$ t_0t_n by $1, 3, 5, \dots, 2n - 1$ and label the inner edges t_0s_1,t_0s_2,t_0s_3,$\dots$ t_0s_{2n}, by $2n + 1, 2n + 3, \dots, 6n - 1$, then label the

outer edges t_1s_1,t_1s_2,t_2s_3,t_2s_3,$\ldots t_ns_{2n-1}$,t_ns_{2n}, by $(6n+1)(\mod 2\gamma), (6n+3)(\mod 2\gamma), (6n+5)(\mod 2\gamma), (6n+7)(\mod 2\gamma), \ldots,$
$(10n-1)(\mod 2\gamma)$. Hence, the induced labeling of vertices t_1,t_2,t_3,$\ldots t_n$ are $2n+5$,$2n+15\ldots 2n-5$, the induced labeling of vertices s_1,s_2,s_3,$\ldots s_{2n}$ are $8n+2$,$8n+6$,$\ldots (16n-2)(\mod 2k)$ and induced vertex labeling of t_0 is $9n^2(\mod 2\gamma)$. Figure 2 shows the labeling for n odd. Thus, the graph is edge odd graceful. □

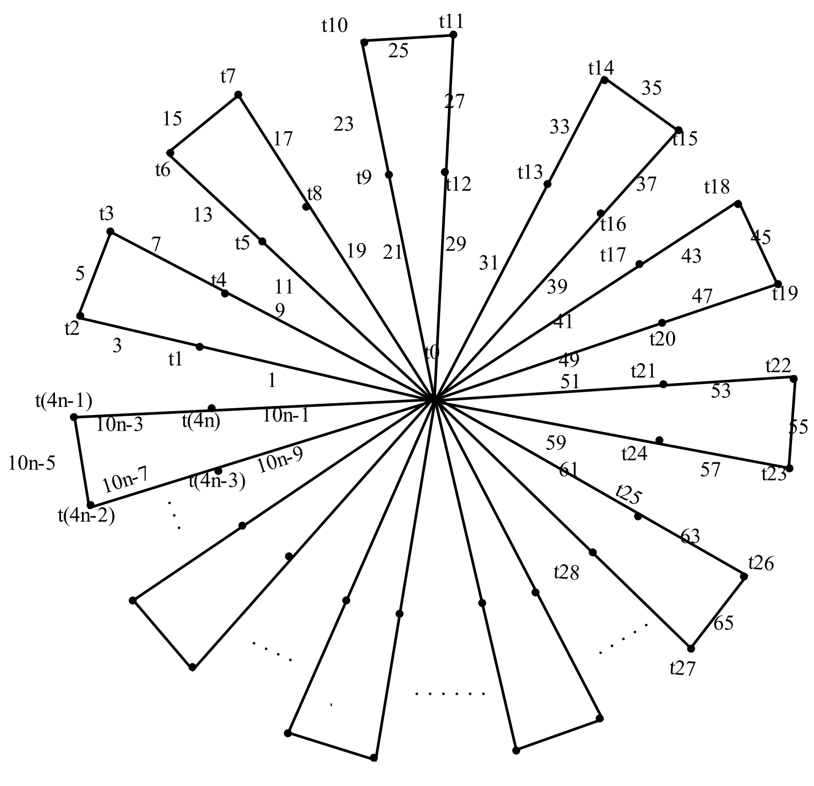

FIGURE 3

The following figure shows the illustration for $F^7_{C_5}$.

Theorem 2.2. *For $n \geq 3$, and n odd, then the graph $F^n_{C_5}$ is an edge odd graceful graph.*

Proof. In this graph, the number of vertices is $\alpha = 4n+1$, number of edges is $\beta = 5n$ and $\gamma = max\{\alpha, \beta\} = 5n$.

Let the graph $F^n_{C_5}$ be as in Figure 3. The cycles C_5 in it are $C^1_5, C^2_5, C^3_5, \ldots, C^n_5$ and the middle vertex is t_0. Name the vertices of C^i_5 by t_{4i-3},t_{4i-2},t_{4i-1},t_{4i} for

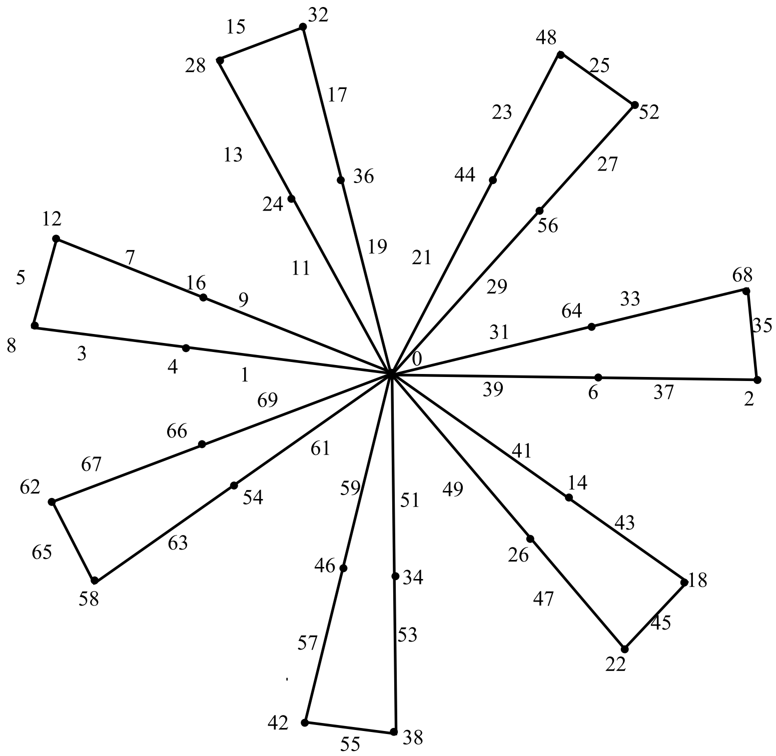

FIGURE 4

$i \in \{1, 2, 3, \ldots, n\}$. Now, label the edges of C_5^i for $i \in \{1, 2, 3, \ldots, n\}$ by t_0t_1, t_0t_5, $t_0t_9, \ldots$ t_0t_{4n-3} by 1, 11, $21 \ldots$ $10n - 9$, t_0t_4, t_0t_8, $t_0t_{12}, \ldots$ t_0t_{4n} by 9, 19, 29, $\ldots$ $10n - 1$; label the edges t_1t_2, t_5t_6,. . . ,$t_{4n-3}t_{4n-2}$ by 3, 13, 23, $\ldots$, $10n - 7$; label the edges t_4t_3, t_8t_7,. . . ,$t_{4n}t_{4n-1}$ by 7, 17, 27, $\ldots$, $10n - 3$; label the edges t_2t_3, t_6t_7, $t_{10}t_{11}$, $\ldots$,$t_{4n-2}t_{4n-1}$ by 5, 15, 25, $\ldots$, $10n - 5$. Hence, the induced labeling of vertices t_1,t_2,t_3,. . . ,t_{4n-1},t_{4n} are 4,8,12,. . . ,$(20n - 4)(\mod 2\gamma)$, and the induced vertex labeling of t_0 is $10n^2(\mod 2\gamma) = 0$. Figure 3 shows the labeling for this case. Thus, the graph is edge odd graceful. □

REFERENCES

[1] R. BALAKRISHNAN, K. RANGANATHAN: *A Textbook of Graph Theory,* Springer-Verlag, New York, 2000.

[2] S. DAOUD: *Edge odd graceful lableling of some path and cycle related graphs*, AKCE International Journal of Graphs and Combinatorics, **14**(2017), 178–203.

[3] R. B. GNANAJOTHI: *Topics in Graph Theory*, Ph D. Thesis, Madurai Kamaraj University, 1991.

[4] S. W. GOLOMB: *How to number a graph*, Graph Theory and computing, R.C.Read(Ed), Academic Press, New York, (1972), 23–37.

[5] A. ROSA: *On Certain valuations of the vertices of a graph*, Theory of graphs, Internat. Symp. Rome, July 1966, 349–355.

[6] A. SOLAIRAJU, K. CHITRA: *Edge-odd graceful graphs*, Electron. Notes Discrete Math., **33**(2009), 15–20.

DEPARTMENT OF MATHEMATICS
GOVERNMENT ARTS COLLEGE FOR WOMEN
KRISHNAGIRI-635 002,TAMIL NADU, INDIA.
Email address: ambi3900@gmail.com

PG AND RESEARCH DEPARTMENT OF MATHEMATICS
GOVERNMENT ARTS COLLEGE FOR MEN
KRISHNAGIRI-635 002,TAMIL NADU, INDIA.
Email address: gbs_geetha@yahoo.com