ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9031–9036
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.11.7

η^{**}-CLOSED SETS IN TOPOLOGICAL SPACES

E. SUBHA[1] AND D. VIDHYA

ABSTRACT. The notion of this paper is to introduce a new class of closed sets called η^{**}-closed sets and η^{**} open sets in topological spaces(TS) and we studied few of its basic properties. Also, examined the relationship of η^{**}-closed set with other sets in the TS.

1. INTRODUCTION

In 1970, the concept of gclosed sets in TS was introduced by Levine [5]. Dunham [4] introduced the concept of the closure operator cl^* and a topology τ^* and studied its few properties. Arya [2], Bhattacharyya and Lahiri [3] had introduced and investigated generalized semiclosed sets, semigeneralized closed sets respectively. In this paper, a new generalization of closed sets is obtained in the TS (X, τ). X and Y are TS(throughout this paper) where no assumptions on separation axioms are made. For a subset C of a TS X, int(C), cl(C), cl*(C), denote the interior, closure, closure* of C respectively.

2. PRELIMINARIES

Definition 2.1. *[5] In a TS X, a subset D is called generalized closed(gclosed) if* $cl(D) \subseteq P$, $D \subseteq P$ *and P is open in X.*

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 54A05.
Key words and phrases. η^{**}-closed set, η^{**}-open set.

Definition 2.2. *[3] In a TS X, a subset D is called semigeneralized closed(sgclosed) if scl(D)$\subseteq$ P, P $\subseteq$ G and P is semiopen in X.*

Definition 2.3. *[2] In a TS X, a subset D is called Generalized semiclosed(gsclosed) ifscl (D)$\subseteq$ P, D $\subseteq$ P and P is open in X.*

Definition 2.4. *[7] In a TS X, a subset D is called α-closed if cl(int(cl(D))) $\subseteq$ D.*

Definition 2.5. *[8] In a TS X, a subset D is called α generalized closed (α g-closed) if α cl(D)$\subseteq$ P , whenever D$\subseteq$ P and P is open in X.*

Definition 2.6. *[9] In a TS X, a subset D is called Generalized closed (gclosed) if spcl(D)$\subseteq$ P, D $\subseteq$ P and P is open in X.*

Definition 2.7. *[2] In a TS X, a subset D is called Generalized semipre closed (gspclosed) if scl(D) $\subseteq$ P, D $\subseteq$ P and P is open in X.*

Definition 2.8. *[11] In a TS X, a subset D is called Strongly generalized closed (strongly gclosed) if cl (D)$\subseteq$ P, D $\subseteq$ P and P is gopen in X.*

Definition 2.9. *[10] In a TS X, a subset D is called*
- *preclosed if cl(int(D)) $\subseteq$ D.*
- *semiclosed if int(cl(B))$\subseteq$ B.*
- *semipre closed(sp-closed)[1] if int(cl(int(D)))$\subseteq$ D.*

Definition 2.10. *[5] For the subset B of a TS X, the intersection of all gclosed sets containing B is defined as the generalized closure operator cl*.*

Definition 2.11. *For the subset B of a TS X,*
- *the semiclosure of B (scl(B)) [6] $= \cap\{$all semiclosed sets containing $B\}$;*
- *the semipreclosure of B (spcl(B)) [1] $= \cap\{$all semipreclosed sets containing $B\}$;*
- *the closure of B (briefly cl(B)) [7] - the intersection of all closed sets containing B.*

3. η^{**}-CLOSED SETS

Definition 3.1. *A subset D of a TS X is called η^{**}-closed set if cl* (D)$\subseteq$ H whenever D $\subseteq$ H and H is semiopen in X. The complement of η^{**}-closed set is called η^{**}-open set.*

Theorem 3.1. *Every closed set is η^{**}-closed set.*

Proof. Let F be a closed set in X such that F $\subseteq$ I, I is semiopen in X. Since F is closed, cl (F)= F. Since cl* (F)$\subseteq$ cl (F) = F . Therefore, cl* (F) $\subseteq$ I. Hence F is a η^{**}-closed set in X. □

Remark 3.1. *Example 1 proves the converse part of theorem 3.1 may not be true.*

Example 1. *Let $X = \{d, e, f\}$ with the topology $\{\emptyset, X, \{d, e\}\}$. Let $B = \{e, f\}$. Here B is η^{**}-closed set but not a closed set of (X, τ).*

Theorem 3.2. *Every gclosed set is a η^{**}-closed set.*

Proof. Let D be a gclosed set. Assume that D $\subseteq$ H, H is semiopen in TS X. Then cl (D) $\subseteq$ H. But cl* (D)$\subseteq$ cl (D). Therefore, cl* (D)$\subseteq$ H. Hence D is η^{**}-closed. □

Remark 3.2. *Example 2 explains the converse part of theorem 3.2 may not be true.*

Example 2. *Consider the TS $X = \{d, e, f\}$ with topology $\tau = \{\emptyset, X, \{d\}\}$. Then the set $\{d\}$ is η^{**}-closed but not gclosed.*

Remark 3.3. *The following example proves that η^{**}-closedness and preclosedness are independent. Let $X = \{d, e, f\}$ be the TS.*

(i) *In the topology $\tau = \{\emptyset, X, \{d\}\}$. Then the sets $\{d\}, \{d, e\}, \{d, f\}$ are η^{**}-closed set but not preclosed set.*
(ii) *In the topology $\tau = \{\emptyset, X, \{d, e\}\}$. Then the set $\{e\}$ is preclosed set but not η^{**}-closed set.*

Remark 3.4. *The following example proves that η^{**}-closedness and α-closedness are independent. Let $X = \{d, e, f\}$ be the topological space.*

(i) *In the topology $\tau = \{\emptyset, X, \{\alpha\}\}$, then $\{d\}, \{d, f\}, \{d, e\}$ are η^{**}-closed set but not α-closed.*
(ii) *In the topology $\tau = \{\emptyset, X, \{d\}, \{d, e\}\}$, then $\{e\}$ is α-closed but not η^{**}-closed set.*

Remark 3.5. *The following example proves that η^{**}-closedness and gsclosedness are independent. Let $X = \{d, e, f\}$ be the topological space.*

(i) *In the topology $\{\emptyset, X, \{d\}\}$, then $\{d\}$ is η^{**}-closed set but not gsclosed.*
(ii) *In the topology $\{\emptyset, X, \{d\}, \{d, e\}\}$, then $\{d\}$ is gsclosed but not η^{**}-closed.*

Remark 3.6. *The following example proves that η^{**}-closedness and sgclosedness are independent. Let $X = \{d, e, f\}$ be the topological space.*

(i) *In the topology $\{\emptyset, X, \{d\}, \{d, e\}\}$, then $\{e\}$ is sgclosed but not η^{**}-closed.*
(ii) *In the topology $\{\emptyset, X, \{d\}, \{d, e\}\}$, then $\{d, e\}, \{d, f\}$ are η^{**}-closed but not sgclosed.*

Remark 3.7. *The following example proves that η^{**}-closedness and semi closedness are independent. Let $X = \{d, e, f\}$ be the topological space.*

(i) *In the topology $\{\emptyset, X, \{d\}\}$, then $\{d\}, \{d, e\}, \{d, f\}$ are η^{**}-closed but not semiclosed.*
(ii) *In the topology $\{\emptyset, X, \{d\}, \{d, e\}\}$ then $\{e\}$ is semiclosed and not η^{**}-closed.*

Remark 3.8. *The following example proves that η^{**}-closedness and sg*-closedness are independent.*

(i) *In the topology $\{\emptyset, X, \{d\}, \{d, e\}\}$, then $\{d\}, \{d, e\}, \{d, f\}$ are η^{**}-closed set but not sg*-closed.*
(ii) *In the topology $\{\emptyset, X, \{d\}, \{d, e\}\}$, then $\{e\}$ is sg*closed and not η^{**}-closed.*
(iii) *In the topology $\{\emptyset, X, \{d\}\}$, then $\{d\}, \{d, e\}, \{d, f\}$ are η^{**}-closed but not sg*closed.*

Remark 3.9. *The following example proves that η^{**}-closedness and locally closedness are independent*

(i) *In the topology $\{\emptyset, X, \{d\}, \{d, e\}\}$, then $\{d\}$ is locally closed but not η^{**}-closed.*
(ii) *In the topology $\{\emptyset, X, \{d\}\}$ Then $\{d\}, \{e\}, \{d, e\}, \{d, f\}$ is η^{**}-closed set but not locallyclosed.*

Remark 3.10. *Consider the topology $\{\emptyset, X\}$. Then the sets $\{d\}, \{e\}, \{f\}, \{d, e\}$, $\{e, f\}$ and $\{d, f\}$ are η^{**}-closed but not regular closed. In the topology $\{\emptyset, X, \{d\}\}$, the set $\{d\}$ is η^{**}-closed but not gclosed.*

Theorem 3.3. *Let E be a η^{**}-closed in X. Then E is gclosed iff cl*(E)- E is a semiopen.*

Proof. Assume E be gclosed set in X. Thus, cl* (E)= E and so cl* (E) $-E = \emptyset$. which is semiopen in X.

In the converse part, suppose cl* (E) - E is semiopen in X. Also, E is η^{**}-closed, cl* (E)- E contains no nonempty semiclosed set in X. Therefore cl* (E)$-E = \emptyset$. Hence E is gclosed. □

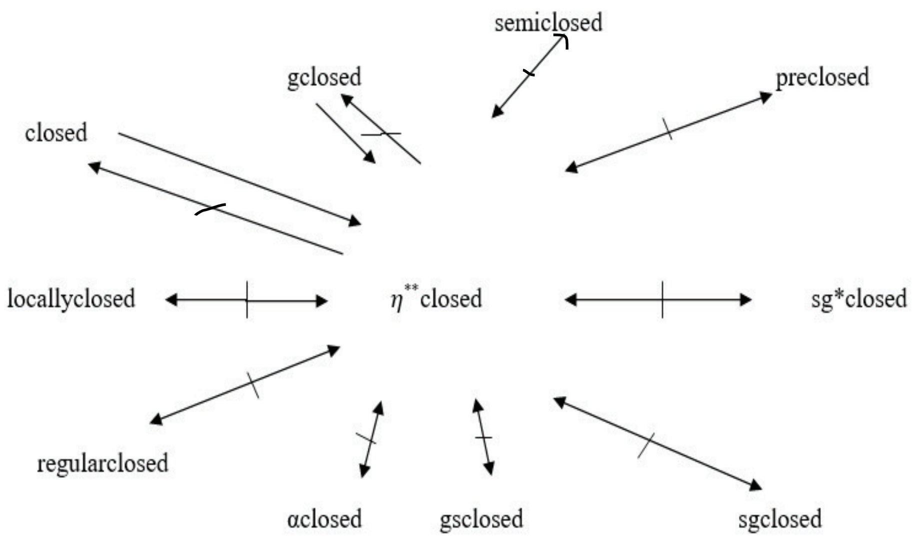
Theorem 3.4. *For $u \in X$, the set $X - \{u\}$ is η^{**}-closed set or semiopen.*

Proof. Suppose $X-\{u\}$ is not semiopen, then $X-\{u\}$ contains the only semiopen set X. Thus, cl*$(X - \{u\}) \subseteq$ X which proves that $X - \{u\}$ is a η^{**}-closed set in X. □

Theorem 3.5. *Assume D ⊆ Y⊆ X, and D is η^{**}-closed set in X, then D is η^{**}-closed relative to Y.*

Proof. It is given that D ⊆ Y⊆ X and D is η^{**}-closed set in X. To prove that D is η^{**}-closed relative to Y. Let D ⊆ Y ∩ G, where G is semiopen in X, hence Y ∩ cl*(D)⊆ Y ∩ G. Thus D is η^{**}-closed relative to Y. □

Remark 3.11. *Thus we conclude the following implications.*



REFERENCES

[1] D. ANDRIJEVIC: *Semi-preopen sets*, Mat. Vesnik, **38** (1986), 24-32.
[2] S. P. ARYA, T. NOUR: *Characterizations of s-normal spaces*, Indian J. Pure App. Math., **21** (1990), 717-719.
[3] P. BHATTACHARYYA, B.K. LAHIRI: *Semi generalized closed sets in topology*, Indian J. Math., **29** (1987), 375-382.

[4] W. DUNHAM: *A new closure operator for non -T1 topologies*, Kyungpook Math. J., **22** (1982), 55-60.

[5] N. LEVINE: *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, **19**(2) (1970), 89-96.

[6] N. LEVINE: *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, **70** (1963), 36-41.

[7] S. N. MAHESHWARI, P. C. JAIN: *Some new mappings Mathemaica*, **24**(47) (1-2) (1982), 53-55.

[8] H. MAKI, R. DEVI, K. BALACHANDRAN: *Associate topologies of generalized closed sets and generalized closed sets*, Mem. Fac. Sci. Kochi Univ. (Math.), **15** (1994), 51-63.

[9] H. MAKI, R. DEVI, K. BALACHANDRAN: *Generalized closes sets in topology*, Bull. Fukuoka Uni. Ed. Part III, **42** (1993), 13-21.

[10] A. S. MASHHOUR, M. E. ABD EI-MONSEF, N. EI-DEEB: *On precontinuous and weak precontinuous functions*, Proc. Math. Phys. Soc. Egypt, **53** (1982), 47-53.

[11] A. PUSHPALATHA, S. ESWARAN, P. RAJARUBI: *τ^*-Generalized Closed Sets in Topological Spaces*, Proceedings of the World Congress on Engineering, Vol II, 2009.

DEPARTMENT OF SCIENCE AND HUMANITIES,
KARPAGAM COLLEGE OF ENGINEERING,
TAMIL NADU, INDIA.
Email address: prajeethsubha@gmail.com

DEPARTMENT OF SCIENCE AND HUMANITIES,
SRI KRISHNA COLLEGE OF ENGINEERING AND TECHNOLOGY,
TAMIL NADU, INDIA.
Email address: vidhyanallamani@gmail.com