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GRANULATION ON FUZZY NEAR SETS

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ABSTRACT. In this paper granulation on fuzzy near sets is defined and some of its properties are established. Further, quantitative measure is defined and an example is provided.

1. INTRODUCTION

Pawlak in [2] originated a generalization of the classification of objects resulting from the solution to the problem of approximating sets of perceptual objects. This generalization lead to the inauguration of near set by James F. Peters in [3]. He considered the problem of approximation of sets of perceptual objects that have matching descriptions. In [4] Qian et al. continued Pawlak's rough set model to multi granulation rough set model which are described by using multi equivalence relations on the universe. In this paper, we have introduced the concepts of multi granulation on fuzzy near sets. Further, quantitative measure on fuzzy near sets is also developed.

2. FUZZY NEAR SET

Notation: Throughout this paper, let U be a non-empty finite universe. The definitions of fuzzy near sets, class of fuzzy near sets, lower and upper approximation and the boundary region on fuzzy near sets are given in [1].

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3. Granulation On Fuzzy Near Sets

Notation. Throughout this section, let $(U, G, \mathbb{FN}_s)$ be a fuzzy near system and $G \subseteq U$. Let $\mathbb{FN}_s(A_{\lambda 1})$ and $\mathbb{FN}_s(A_{\lambda 2})$ be two fuzzy near sets.

Definition 3.1. *The granulation on fuzzy near sets is denoted by* $(\mathbb{GFN}_s) = \mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})$. *The lower and upper approximation of the granulation are denoted by* $\underline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})}$ *and* $\overline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})}$, *respectively. They are defined as:*

$$\underline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})} = \{g \in G : [g]_{\mathbb{FN}_s(A_{\lambda 1})} \subseteq G \bigcup [g]_{\mathbb{FN}_s(A_{\lambda 2})} \subseteq G\}$$

and

$$\overline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})} = \sim (\underline{\sim \mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})}).$$

Table 1. Fuzzy near sets with decision values

U	$\mathbb{FN}_s(A_{\lambda 1})$	$\mathbb{FN}_s(A_{\lambda 2})$	$Decision$
$g_{\xi 1}$	0.5	0.7	1
$g_{\xi 2}$	0.2	0.2	1
$g_{\xi 3}$	0.6	0.3	0
$g_{\xi 4}$	0.2	0.3	0
$g_{\xi 5}$	0.5	0.7	0
$g_{\xi 6}$	0.2	0.9	1
$g_{\xi 7}$	0.3	0.4	1
$g_{\xi 8}$	0.3	0.9	1

Theorem 3.1. *For any two fuzzy near sets the following properties hold:*
(i) $\underline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})} \subseteq \underline{\mathbb{FN}_s(A_{\lambda 1}) \bigcup \mathbb{FN}_s(A_{\lambda 2})}$ *and*
(ii) $\overline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})} \supseteq \overline{\mathbb{FN}_s(A_{\lambda 1}) \bigcup \mathbb{FN}_s(A_{\lambda 2})}$.

Proof. (i) For any $g \in \underline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})}$ from Definition 3.1, it follows that $g \in [g]\mathbb{FN}_s(A_{\lambda 1})$, $g \in [g]\mathbb{FN}_s(A_{\lambda 2})$. Hence, $g \in [g]\mathbb{FN}_s(A_{\lambda 1}) \bigcap [g]\mathbb{FN}_s(A_{\lambda 2})$. But $[g]\mathbb{FN}_s(A_{\lambda 1}) \bigcap [g]\mathbb{FN}_s(A_{\lambda 2}) \subseteq [g]\mathbb{FN}_s(A_{\lambda 1}) \bigcup [g]\mathbb{FN}_s(A_{\lambda 2})$ and $\underline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})}$ $= \{g \in [g]\mathbb{FN}_s(A_{\lambda 1}) \bigcup [g]\mathbb{FN}_s(A_{\lambda 2})\}$ by Definition 3.1.

Therefore, $g \in \underline{\mathbb{FN}_s(A_{\lambda 1}) \bigcup \mathbb{FN}_s(A_{\lambda 2})}$. i.e., $\underline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})} \subseteq \underline{\mathbb{FN}_s(A_{\lambda 1}) \bigcup \mathbb{FN}_s(A_{\lambda 2})}$.

(ii) $\overline{\mathbb{FN}_s(A_{\lambda 1}) \bigcup \mathbb{FN}_s(A_{\lambda 2})} = \sim \underline{(\sim \mathbb{FN}_s(A_{\lambda 1}) \bigcup \mathbb{FN}_s(A_{\lambda 2}))}$

$\subseteq \sim \underline{(\sim \mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2}))} = \overline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})}$

i.e., $\overline{\mathbb{FN}_s(A_{\lambda 1}) + \mathbb{FN}_s(A_{\lambda 2})} \supseteq \overline{\mathbb{FN}_s(A_{\lambda 1}) \bigcup \mathbb{FN}_s(A_{\lambda 2})}$. □

Definition 3.2. *Let $FP_1, FP_2, \ldots, FP_m$ be m partitions induced by the $\mathbb{FN}_s$'s respectively. The lower and the upper approximation of G related to $FP_1, FP_2, \ldots, FP_m$ are*

$$\underline{G}_{\sum_{\lambda=1}^{m} FP_\lambda} = \{g : \bigcup FP_\lambda \subseteq G, \lambda \leq m\} \text{ and } \overline{G}^{\sum_{\lambda=1}^{m} FP_\lambda} = \sim \underline{(\sim G)}_{\sum_{\lambda=1}^{m} FP_\lambda}.$$

Theorem 3.2. *Let $FP_1, FP_2, \ldots, FP_m$ be m partitions induced by $\mathbb{FN}_s$'s respectively. Then, the following properties hold:*

(i) $\underline{(\bigcap_{\omega=1}^{n} G_\omega)}_{\sum_{\lambda=1}^{m} FP_\lambda} \subseteq \bigcap_{\omega=1}^{m} (\underline{G_\omega}_{\sum_{\lambda=1}^{m} FP_\lambda})$

(ii) $\overline{(\bigcup_{\omega=1}^{n} G_\omega)}^{\sum_{\lambda=1}^{m} FP_\lambda} \supseteq \bigcup_{\omega=1}^{m} (\overline{G_\omega}^{\sum_{\lambda=1}^{m} FP_\lambda})$

(iii) $\underline{(\bigcup_{\omega=1}^{n} G_\omega)}_{\sum_{\lambda=1}^{m} FP_\lambda} \supseteq \bigcup_{\omega=1}^{m} (\underline{G_\omega}_{\sum_{\lambda=1}^{m} FP_\lambda})$

(iv) $\overline{(\bigcap_{\omega=1}^{n} G_\omega)}^{\sum_{\lambda=1}^{m} FP_\lambda} \subseteq \bigcap_{\omega=1}^{n} (\overline{G_\omega}^{\sum_{\lambda=1}^{m} FP_\lambda})$.

Proof.

(i) $\underline{(\bigcap_{\omega=1}^{n} G_\omega)}_{\sum_{\lambda=1}^{m} FP_\lambda} = \bigcup_{\lambda=1}^{m} (\bigcap_{\omega=1}^{n} \underline{G_\omega}_{FP_\lambda})$

$= \bigcap_{\omega=1}^{n} (\bigcup_{\lambda=1}^{m} \underline{G_\omega}_{FP_\lambda}) \bigcap \ldots = \bigcap_{\omega=1}^{n} (\underline{G_\omega}_{\sum_{\lambda=1}^{m} FP_\lambda}) \bigcap \ldots \subseteq \bigcap_{\omega=1}^{n} (\underline{G_\omega}_{\sum_{\lambda=1}^{m} FP_\lambda})$.

(ii) $\overline{(\bigcup_{\omega=1}^{n} G_\omega)}^{\sum_{\lambda=1}^{m} FP_\lambda} = \bigcap_{\lambda=1}^{m} (\bigcup_{\omega=1}^{n} \overline{G_\omega}^{FP_\lambda})$

$= \bigcup_{\omega=1}^{n} (\bigcap_{\lambda=1}^{m} \overline{G_\omega}^{FP_\lambda}) \bigcup \ldots = \bigcup_{\omega=1}^{n} (\overline{G_\omega}^{\sum_{\lambda=1}^{m} FP_\lambda}) \bigcup \ldots \supseteq \bigcup_{\omega=1}^{n} (\overline{G_\omega}^{\sum_{\lambda=1}^{m} FP_\lambda})$.

(iii) It follows from $G_\omega \subseteq \bigcup\limits_{\omega=1}^{n} G_\omega$ that $\underline{G_\omega}_{\sum\limits_{\lambda=1}^{m} FP_\lambda} \subseteq \underline{\bigcup\limits_{\lambda=1}^{m}} G_\omega{}_{\sum\limits_{\lambda=1}^{m} FP_\lambda}$.

Hence, $\underline{(\bigcup\limits_{\omega=1}^{n} G_\omega)}_{\sum\limits_{\lambda=1}^{m} FP_\lambda} \supseteq \bigcup\limits_{\omega=1}^{m} (\underline{G_\omega}_{\sum\limits_{\lambda=1}^{m} FP_\lambda})$.

(iv) It follows from $(\bigcap\limits_{\omega=1}^{n} G_\omega \subseteq G_\omega(\omega \in \{1, 2, \ldots, n\}))$ that

$\overline{G_\omega}^{\sum\limits_{\lambda=1}^{m} FP_\lambda} \supseteq \overline{\bigcap\limits_{\omega=1}^{n} G_\omega}^{\sum\limits_{\lambda=1}^{m} FP_\lambda}$.Hence,$\overline{(\bigcap\limits_{\omega=1}^{n} G_\omega)}^{\sum\limits_{\lambda=1}^{m} FP_\lambda} \subseteq \bigcap\limits_{\omega=1}^{n} (\overline{G_\omega}^{\sum\limits_{\lambda=1}^{m} FP_\lambda})$.

□

4. Quantitative measure on Fuzzy Near Sets

Definition 4.1. *FP_i is lower approximation significant in $[g]_{\mathbb{FN}_s(A)}$ w.r.t. G if $\underline{G}_S \supset \underline{G}_{\mathbb{S}}$, and FP_i is not lower in $[g]_{\mathbb{FN}_s(A)}$ with respect to G if $\underline{G}_S = \underline{G}_{\mathbb{S}}$ where $|\mathbb{FN}_s(A)|$ is the cardinality of the fuzzy near set $\mathbb{FN}_s(A)$, $S = \sum\limits_{i=1}^{|\mathbb{FN}_s(A)|} FP_i$ and $\mathbb{S} = \sum\limits_{i=1}^{|\mathbb{FN}_s(A)|}\ \ \sum\limits_{[g]_{\mathbb{FN}_s(A)} \neq FP_i} FP_i$.*

Definition 4.2. *FP_i is upper approximation significant in $[g]_{\mathbb{FN}_s(A)}$ w.r.t. G if $\overline{G}^S \subset \overline{G}^{\mathbb{S}}$ and FP_i is not upper in $[g]_{\mathbb{FN}_s(A)}$ with respect to G if $\overline{G}^S = \overline{G}^{\mathbb{S}}$.*

Definition 4.3. *The measure of the lower and the upper approximation of $\mathbb{GFN}_s$ w.r.t. the decision class D in fuzzy near set is defined as*

$$\underline{M} = \bigcup \{\underline{G}_S \backslash \underline{G}_{\mathbb{S}} : G \in D\} \text{ and } \overline{M} = \bigcup \{\overline{G}^S \backslash \overline{G}^{\mathbb{S}} : G \in D\}.$$

Definition 4.4. *The Quantitative measure of the lower and upper approximation of $\mathbb{GFN}_s$ w.r.t. the decision class D of the fuzzy near set using the membership value are defined as*

$$\underline{QM} = \frac{\sum \mu_{\mathbb{FN}_s(A_{\lambda i})}(g_i)}{|U|} : \forall g_i \in \underline{M} \text{ and } \overline{QM} = \frac{\sum \mu_{\mathbb{FN}_s(A_{\lambda i})}(g_i)}{|U|} : \forall g_i \in \overline{M}.$$

Example 1. *Computing the Measures and Quantitative measures w.r.t. the decision D in Table 1, we have* $\underline{M}_{\mathbb{FN}_s(A_{\lambda 1})} = \{g_{\xi 7}, g_{\xi 8}\} \backslash \{\phi\} \bigcup \{g_{\xi 3}\} \backslash \{g_{\xi 3}\} = \{g_{\xi 7}, g_{\xi 8}\}$ *and*

$\underline{M}_{\mathbb{FN}_s(A_{\lambda 2})} = \{g_{\xi 2}, g_{\xi 6}, g_{\xi 7}, g_{\xi 8}\} \backslash \{g_{\xi 2}, g_{\xi 6}, g_{\xi 8}\} \bigcup \{g_{\xi 3}, g_{\xi 4}\} \backslash \{g_{\xi 3}, g_{\xi 4}\} = \{g_{\xi 7}\}$

$\overline{M}^{\mathbb{FN}_s(A_{\lambda 1})} = \{g_{\xi 1}, g_{\xi 2}, g_{\xi 4}, g_{\xi 5}, g_{\xi 6}, g_{\xi 7}, g_{\xi 8}\} \backslash \{g_{\xi 1}, g_{\xi 2}, g_{\xi 4}, g_{\xi 5}, g_{\xi 6}\}$

$\bigcup\{g_{\xi 1}, g_{\xi 2}, g_{\xi 3}, g_{\xi 4}, g_{\xi 5}, g_{\xi 6}\}\backslash\{g_{\xi 1}, g_{\xi 2}, g_{\xi 3}, g_{\xi 4}, g_{\xi 5}, g_{\xi 6}\} = \{g_{\xi 7}, g_{\xi 8}\}$ *and*

$\overline{M}^{\mathbb{FN}_s(A_{\lambda 2})} = \{g_{\xi 1}, g_{\xi 2}, g_{\xi 5}, g_{\xi 6}, g_{\xi 7}, g_{\xi 8}\}\backslash\{g_{\xi 1}, g_{\xi 2}, g_{\xi 5}, g_{\xi 6}, g_{\xi 8}\}$

$\bigcup\{g_{\xi 1}, g_{\xi 3}, g_{\xi 4}, g_{\xi 5}\}\backslash\{g_{\xi 1}, g_{\xi 3}, g_{\xi 4}, g_{\xi 5}\} = \{g_{\xi 7}\}$

$\underline{QM}_{\mathbb{FN}_s(A_{\lambda 1})} = \frac{0.3+0.3}{8} = 0.3375$, *and* $\underline{QM}_{\mathbb{FN}_s(A_{\lambda 2})} = \frac{0.4}{8} = 0.05.$

$\overline{QM}_{\mathbb{FN}_s(A_{\lambda 1})} = \frac{0.3+0.3}{8} = 0.3375$, *and* $\overline{QM}_{\mathbb{FN}_s(A_{\lambda 2})} = \frac{0.4}{8} = 0.05.$

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