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MULTIPLY DIVISOR CORDIAL LABELING IN CONTEXT OF RING SUM OF GRAPHS

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ABSTRACT. Multiply divisor cordial labeling of a graph G^* having set of node V^* is a bijective h from V(G^*) to {1,2,...,|V(G^*)|} such that an edge xy is assigned the label 1 if 2 divides $(h(x) \cdot h(y))$ and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph having multiply divisor cordial labeling is said to be multiply divisor cordial graph. In this paper, we have recognized seven new graph families by which the conditions of multiply divisor cordial labeling in context of ring sum of graphs are satisfied.

1. INTRODUCTION

All graphs included here are without loops and parallel edges, having no orientation, finite and connected. We follow the basic notations and terminologies of graph theory as in [2]. A graph labeling is a mapping that carries the graph components to the set of numbers, usually to the set of natural numbers. If the domain is the set of nodes the labeling is called node labeling. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both nodes and edges then the labeling is called total labeling. For a dynamic survey of various graph labeling, we refer to [1]. Other valuable references are [3–10].

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Definition 1.1. *Let $G^*(V(G^*), E(G^*))$ be a simple graph and $h : V(G^*) \to \{1, 2, \ldots, |V(G^*)|\}$ be a bijection. For each edge xy, assign the label 1 if $2 | (h(x) \cdot h(y))$ and the label 0 otherwise. The function h is called a multiply divisor cordial labeling if $|e_h(0) - e_h(1)| \leq 1$. A graph which admits multiply divisor cordial labeling is called a multiply divisor cordial graph.*

Definition 1.2. *A double-wheel graph DW_k of size k can be composed of $2C_P + K_1$, i.e. it consists of two cycles of size p, where the nodes of the two cycles are all connected to a common hub.*

Definition 1.3. *The double fan DF_p consists of two fan graph that have a common path. In other words $DF_p = P_p + \overline{K}_2$.*

Definition 1.4. *If pendent edge is attached at each node of the p-cycle of the wheel then the graph H_p is called Helm graph obtained from wheel graph W_p.*

2. Main Results

Theorem 2.1. *$C_p \oplus K_{1,p}$ is a multiply divisor cordial graph for all p.*

Proof. Let $V(C_p \oplus K_{1,p}) = V_1 \cup V_2$, where $V_1 = V(C_p) = \{x_1, x_2, \ldots, x_p\}$ and $V_2 = V(K_{1,p}) = \{y = x_1, y_1, y_2, \ldots, y_p\}$. Here $y_1, y_2, \ldots, y_p$ are pendant nodes and v is the apex node of $K_{1,p}$:

$$| V(C_p \oplus K_{1,p}) | = | E(C_p \oplus K_{1,p}) | = 2p.$$

Define labeling $h : V(C_p \oplus K_{1,p}) \to \{1, 2, 3, \ldots, |V(C_p \oplus K_{1,p})|\}$ as follows.

$h(x_i) = 2i - 1; 1 \leq i \leq p,$

$h(y_j) = 2j; 1 \leq j \leq p.$

According to this pattern the nodes are labeled such that for any edge $e = x_i x_{i+1}$ in C_p, $h(x_i) | h(x_i + 1), 1 \leq i \leq p$. Also, $h(y)$ does not divide $h(y_j), 1 \leq j \leq p$. Hence, $e_h(0) = e_h(1) = p$. Thus, $|e_h(0) - e_h(1)| \leq 1$. So, $C_p \oplus K_{1,p}$ is a multiply divisor cordial graph. □

Example 1. *Multiply divisor cordial labeling of the graph $C_6 \oplus K_{1,6}$ can be seen in Figure 1.*

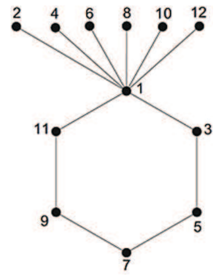

FIGURE 1

Theorem 2.2. *$G^* \oplus K_{1,p}$ is multiply divisor cordial graph for $p \geq 4$, $p \in N$, where G^* is cycle C_p with one chord and chord forms a triangle with two edges of C_p.*

Proof. Let G^* be the cycle C_p with one chord, $V(G^*) = \{x_1, x_2, \ldots, x_p\}$ and $e = x_2x_p$ be the chord of C_p. The nodes $x_1, x_2, \ldots, x_p$ forms a triangle with chord e. Let $V(K_{1,p}) = \{y, y_1, y_2, \ldots, y_p\}$, where $y = x_1$ is the apex node and $y_1, y_2, \ldots, yp$ are the pendant nodes of $K_{1,p}$,

$$|V(G^* \oplus K_{1,p})| = 2p \text{ and } |E(G^* \oplus K_{1,p})| = 2p + 1.$$

Define labeling $h : V(G^* \oplus K_{1,p}) \rightarrow \{1, 2, 3, \ldots, 2p\}$ as follows. The labeling Pattern is same as Theorem 2.1. According to this pattern the nodes are labeled such that for any edge $e = x_ix_{i+1}$ in C_p, $h(x_i)|h(x_{i+1})1 \leq i \leq p$. Also $h(y)$ does not divide $h(y_j)1 \leq j \leq p$. Hence $e_h(0) = p + 1$ and $e_h(1) = p$. Thus, $|e_h(0) - e_h(1)| \leq 1$. So, $G^* \oplus K_{1,p}$ is a multiply divisor cordial graph, where G^* is the cycle C_p with one chord. □

Example 2. *Multiply divisor cordial labeling of ring sum of C_7 with one chord and $K_{1,7}$ can be seen in Figure 2.*

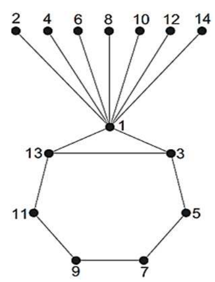

FIGURE 2

Theorem 2.3. *$C_{p,3} \oplus K_{1,p}$ is a multiply divisor cordial graph, for $p \geq 5$, $p \in N$.*

Proof. Let $V(C_{p,3}) = \{x_1, x_2, \ldots, x_p\}, e_1 = x_2x_n$ and $e_2 = x_3x_p$ be the chords of C_p. Let $V(K_{1,p}) = \{y = x_1, y_1, y_2, \ldots, y_p\}$, where y is the apex node and $y_1, y_2, \ldots, y_p$ are pendant nodes of $K_{1,p}$,

$$|V(C_{p,3} \oplus K_{1,p})| = 2p \text{ and } |E(C_{p,3} \oplus K_{1,p})| = 2p + 2.$$

Define labeling $h : V(C_{p,3} \oplus K_{1,p}) \to \{1, 2, 3, \ldots, 2p\}$ as follows. $h(x_{p-1}) = 2, h(x_p) = 2p - 1$, $h(x_i) = 2i + 1; 1 \leq i \leq p - 2$. $h(y_1) = 1$, $h(y_j) = 2j; 2 \leq j \leq p$. According to this labeling, the nodes are labeled such that $e_h(0) = e_h(1) = p + 1$. Thus $|e_h(0) - e_h(1)| \leq 1$. Hence the graph under consideration admits multiply divisor cordial labeling. Thus $C_{p,3} \oplus K_{1,p}$ is a multiply divisor cordial graph. □

Example 3. *Multiply divisor cordial labeling of $C_{6,3} \oplus K_{1,6}$ can be seen in Figure 3.*

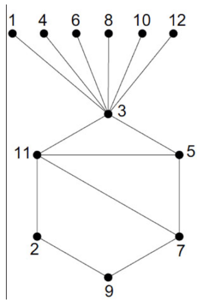

FIGURE 3

Theorem 2.4. *$C_p(1, 1, p - 5) \oplus K_{1,p}$ is a multiply divisor cordial graph, for $p \geq 6$, $p \in N$.*

Proof. Let $V(C_p(1, 1, p - 5)) = \{x_1, x_2, \ldots, x_p\}$, where $e_1 = x_1x_3, e_2 = x_3x_{p-1}$ and $e_3 = x_1x_{p-1}$ are chords of C_p which by themselves form triangle. Let $V(K_{1,p}) = \{y = x_1, y_1, y_2, \ldots, y_p\}$, where y is the apex node and $y_1, y_2, \ldots, y_p$ are the pendant nodes. $|V(C_p(1, 1, p - 5) \oplus K_{1,p})| = 2p$ and $|E(C_p(1, 1, p - 5) \oplus K_{1,p})| = 2p + 3$. Define labeling $h : V(C_p(1, 1, p - 5) \oplus K_{1,p}) \to \{1, 2, 3, \ldots, 2p\}$ as follows. $h(x_1) = 3, h(x_2) = 2$, $h(x_i) = 2i - 1; 3 \leq i \leq p$. $h(y_1) = 1$, $h(y_j) = 2j; 2 \leq j \leq p$. In view of above defined labeling pattern $e_h(0) = p + 2, e_h(1) = p + 1$. Thus $|e_h(0) - e_h(1)| \leq 1$. Hence $C_p(1, 1, p - 5) \oplus K_{1,p}$ is a multiply divisor cordial graph. □

Example 4. *Multiply divisor cordial labeling of ring sum of cycle with triangle $C_6(1,1,3)$ and $K_{1,6}$ can be seen in Figure 4.*

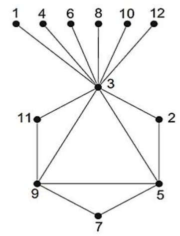

FIGURE 4

Theorem 2.5. *$DF_p \oplus K_{1,p}$ is a multiply divisor cordial graph for all p.*

Proof. Let $V(DF_p \oplus K_{1,p}) = V_1 \cup V_2, V_1 = V(DF_p) = \{x, w, x_1, x_2, \ldots, x_p\}$, where x, w are two apex nodes of $DF_p; V_2 = V(K_{1,p}) = \{y = w, y_1, y_2, \ldots, y_p\}$, where $y_1, y_2, \ldots, y_p$ are pendant nodes and y is the apex node of $K_{1,p}$. $|V(DF_p \oplus K_{1,p})| = 2p+2, |E(DF_p \oplus K_{1,p})| = 4p-1$. Define labeling $h : V(DF_p \oplus K_{1,p}) \rightarrow \{1, 2, 3, \ldots, 2p+2\}$ as follows.

$h(x) = 1, h(y = w) = 2,$

$h(x_i) = 2i + 1; 1 \leq i \leq p.$

$h(y_j) = 2j + 2; 1 \leq i \leq p.$

In view of above defined labeling pattern $e_h(0) = 2p - 1, e_h(1) = 2p$. Thus $|e_h(0) - e_h(1)| \leq 1$. Hence the graph under consideration admits multiply divisor cordial labeling. i.e. $DF_p \oplus K_{1,p}$ is a multiply divisor cordial graph. □

Example 5. *Multiply divisor cordial labeling of $DF_5 \oplus K_{1,5}$ can be seen in Figure 5.*

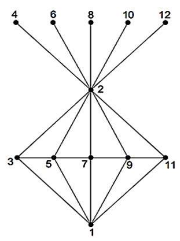

FIGURE 5

Theorem 2.6. *$DW_p \oplus K_{1,p}$ is a multiply divisor cordial graph for all p.*

Proof. Let $V(DW_p \oplus K_{1,p}) = V_1 \cup V_2, V_1 = V(DW_p) = \{x, x_1, x_2, \ldots, x_p, y_1, y_2, \ldots, y_p\}$, where x be the apex node, $x_1, x_2, \ldots, x_p$ be the nodes of inner cycle and $y_1, y_2, \ldots, y_p$ be the nodes of outer cycle of DW_p; $V_2 = V(K_{1,p}) = \{y_1 = w, w_1, w_2, \ldots, w_p\}$, where $w_1, w_2, \ldots, w_p$ are pendant nodes and w is the apex node of $K_{1,p}$. $|V(DW_p \oplus K_{1,p})| = 3p + 1$, $|E(DW_p \oplus K_{1,p})| = 5p$. Define labeling $h : V(DW_p \oplus K_{1,p}) \rightarrow \{1, 2, 3, \ldots.., 3p + 1\}$ as follows.

$h(x) = 1$,
$h(x_i) = 2i; 1 \leq i \leq p$.
$h(y_i) = 2i + 1; 1 \leq i \leq p$.
$h(w_j) = 2p + i + 1; 1 \leq j \leq p$.

Thus $|e_h(0) - e_h(1)| \leq 1$. Hence the graph under consideration admits multiply divisor cordial labeling, i.e., $DW_p \oplus K_{1,p}$ is a multiply divisor cordial graph. □

Example 6. *Multiply divisor cordial labeling of $DW_7 \oplus K_{1,7}$ can be seen in Figure 6.*

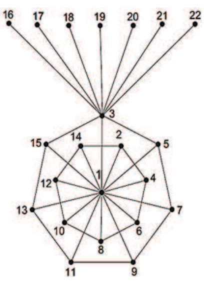

FIGURE 6

Theorem 2.7. *$H_p \oplus K_{1,p}$ is a multiply divisor cordial graph for all p.*

Proof. Let $V(DH_p \oplus K_{1,p}) = V_1 \cup V_2, V_1 = V(H_p) = \{x, x_1, x_2 \ldots, x_p, y_1, y_2, \ldots, y_p\}$, where x be the apex node, $x_1, x_2, \ldots, x_p$ be the nodes of degree 4 and $y_1, y_2, \ldots, y_p$ be the pendant nodes of H_p; $V_2 = V(K_{1,p}) = \{y_1 = w, w_1, w_2, \ldots, w_p\}$, where $w_1, w_2, \ldots, w_p$ are pendant nodes and w is the apex node of $K_{1,p}$. $|V(H_p \oplus K_{1,p})| = 3p + 1, |E(H_p \oplus K_{1,p})| = 4p$. Define labeling $h : V(H_p \oplus K_{1,p}) \rightarrow \{1, 2, 3, \ldots, 3p + 1\}$ as follows.

$h(x) = 1$,
$h(x_i) = 2i + 1; 1 \leq i \leq p$.

$h(y_i) = 2i; 1 \leq i \leq p.$
$h(w_j) = 2p + j + 1; 1 \leq j \leq p.$

In view of above defined labeling pattern $e_h(0) = e_h(1) = 2p$. Thus $|e_h(0) - eh(1)| \leq 1$. Hence $H_p \oplus K_{1,p}$ is a multiply divisor cordial graph. □

Example 7. *Multiply divisor cordial labeling of $H_5 \oplus K_{1,5}$ can be seen in Figure 7.*

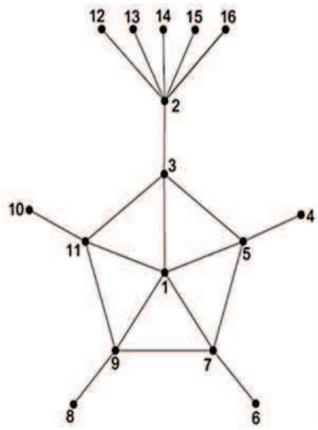

FIGURE 7

3. CONCLUSION

The multiply divisor cordial labeling is a variant of divisor cordial labeling. It is very interesting to investigate graph or graph families which are multiply divisor cordial as all the graphs do not admit multiply divisor cordial labeling. This will add new dimension to the research work in the area connecting three branches - graph labeling, number theory and networking of hyperlinks in computer engineering. Here, we have investigated seven new graph families which admit multiply divisor cordial labeling.

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