

## APPLICATION OF HEXAGONAL FUZZY NUMBER IN RECURRENT NEURAL NETWORK USING GAUSSIAN FUNCTION

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ABSTRACT. This paper deals with the application of Hexagonal fuzzy number in recurrent neural network using Gaussian activation function. An algorithm is proposed based on this concept. Further, a numerical example is given to illustrate the proposed algorithm.

### 1. INTRODUCTION

In traditional neural networks, the input of the hidden layer comes from the input layer only. In the recurrent neural network (RNN), the input of the hidden layer includes the input layer data and the output of the hidden layer neurons at the last time. Paulo [1] developed a procedure that uses Wang's RNN to solve transportation problem. Scheibel et al. [3] proposed an algorithm to connect the centers and variances of Gaussian nodes.

Pavithra et al. in [2] dealt with fuzzy transportation problem in which the values of transportation costs are represented as Hexagonal fuzzy numbers. Virgin Raj et al. in [4] proposed intuitionistic bell shape fuzzy number in recurrent feed forward neural network.

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This paper deals with the application of Hexagonal fuzzy number in RNN using Gaussian activation function. An algorithm is proposed based on this concept. Further, a numerical example is given to illustrate the proposed algorithm.

## 2. RECURRENT NEURAL NETWORK (RNN)

**Definition 2.1.** Let  $H = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ , where  $(h_1 \leq h_2 \leq h_3 \leq h_4 \leq h_5 \leq h_6)$  and the parameters  $h_1, h_2, h_3, h_4, h_5, h_6$  scale from 1 to 10, be the hexagonal fuzzy number. Its membership value is defined by  $\mu_H(x) = \{\frac{h_1}{10}, \frac{h_2}{10}, \frac{h_3}{10}, \frac{h_4}{10}, \frac{h_5}{10}, \frac{h_6}{10}\}$ .

**Definition 2.2.** Let  $H = \{H_1, H_2, \dots, H_n\}$  be the set of 'n' elements which are hexagonal fuzzy numbers. These fuzzy numbers represent the weights of the Recurrent Neural Network (RNN) denoted by  $H_1 = \{h_{11}, h_{12}, h_{13}, h_{14}, h_{15}, h_{16}\}, \dots, H_n = \{h_{n1}, h_{n2}, h_{n3}, h_{n4}, h_{n5}, h_{n6}\}$ . Let  $c_i$  be the center of the hexagonal fuzzy number defined as  $c_i = \frac{1}{n} \sum_{j=1}^n h_{ij}$   $j = 1, 2, \dots, 6$ .

**Definition 2.3.** The variance  $\sigma_i$  of RNN is defined by the following equation:  $\sigma_i = \frac{1}{n-1} \sum_{k=1}^n (x_k - c_i)$ , where  $x_k$  takes the value either 0 or 1.

**Definition 2.4.** Previous state of the recurrent neuron is denoted by  $W_{ps}$  and defined using sigmoid function.  $W_{ps} = (\frac{1}{1+\exp(-\mathbb{H}_i)})$ , where  $\mathbb{H}_i = \sum_{j=1}^n h_{ij}x_j$ ,  $j = 1, 2, \dots, 6$  are the weighted sum of RNN.

**Definition 2.5.** The activation function of RNN is a Gaussian function defined by  $\Phi(h_{ij}) = \exp(\frac{-(h_{ij}-c_i)^2}{2\sigma_i^2})$ , where  $h_{ij}$  are the weights of RNN.  $\Phi(h_{ij})$ 's are the current state  $W_{cs}$  of the recurrent neuron.

**Definition 2.6.** The output function  $y_n$  of RNN is defined as  $y_n = \tanh(W_{ps} + W_{cs})$ .

## 3. ALGORITHM

Algorithm for the Recurrent Neural Network using Gaussian Function is given below.

**Step 1:** Construct the hexagonal fuzzy number.

**Step 2:** Convert the hexagonal fuzzy number into its membership values which are weights of recurrent neural network.

**Step 3:** Assume the input values  $x_i$ 's as 0 and 1.

**Step 4:** Calculate the the weighted sum  $\mathbb{H}_i$ .

**Step 5:** Find the previous state  $W_{ps}$  of the recurrent neuron by using Definition 2.4.

**Step 6:** Calculate the activation hidden layer using Definition 2.5 which represent the current state  $W_{cs}$  of the recurrent neuron.

**Step 7:** The hidden output neuron  $y_n$  is found using Definition 2.6.

**Step 8:** Once the current state is calculated, we can determine the output state by finding the maximum output value.

#### 4. APPLICATIONS

In this section we give an example to illustrate the working of the algorithm. Let the four hexagonal fuzzy numbers  $H_1, H_2, H_3, H_4$  represent four culture media viz., Potato Dextrose Yeast Agar, Potato Dextrose Agar, Oat Meal Agar and Tapioca Agar respectively, for the vegetative growth of a species of edible mushroom. Let  $h_{1j}, j = 1, 2, \dots, 6$  represent the vegetative growth of the mushroom at the end of third, fourth, fifth, sixth, seventh and eighth day respectively in the medium  $H_1$  which is Potato Dextrose Yeast Agar. Similarly,  $h_{2j}, h_{3j}, h_{4j}$  represent the vegetative growth of the mushroom at the end of the above mentioned days in the other three mediums  $H_2, H_3$  and  $H_4$ .

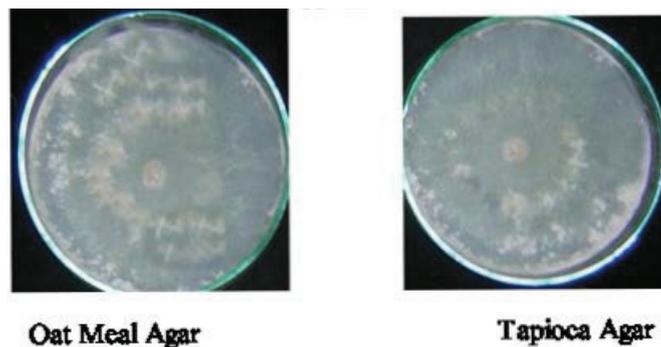
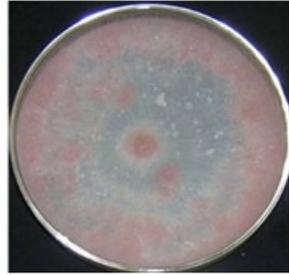


FIGURE 1. Effect of culture media on the vegetative growth of mushroom

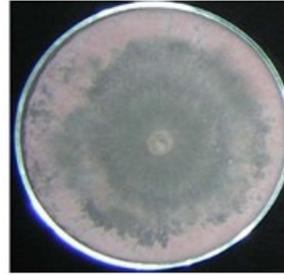
**Example 1. Step 1:** Consider the hexagonal fuzzy number

$$H_1 = (1, 2, 3, 3, 5, 6)$$

$$H_2 = (2, 2, 4, 5, 6, 8)$$



Potato dextrose Yeast Agar



Potato Dextrose Agar

$$H_3 = (1, 1, 3, 4, 6, 9)$$

$$H_4 = (0, 2, 4, 5, 7, 8)$$

**Step 2:** Convert the hexagonal fuzzy number into its membership value which represent the weights of RNN.

$$H_1 = (0.1, 0.2, 0.3, 0.3, 0.5, 0.6)$$

$$H_2 = (0.2, 0.2, 0.4, 0.5, 0.6, 0.8)$$

$$H_3 = (0.1, 0.1, 0.3, 0.4, 0.6, 0.9)$$

$$H_4 = (0.0, 0.2, 0.4, 0.5, 0.7, 0.8)$$

$$h_{11} = 0.1, h_{12} = 0.2, h_{13} = 0.3, h_{14} = 0.3, h_{15} = 0.5, h_{16} = 0.6$$

$$h_{21} = 0.2, h_{22} = 0.2, h_{23} = 0.4, h_{24} = 0.5, h_{25} = 0.6, h_{26} = 0.8$$

$$h_{31} = 0.1, h_{32} = 0.1, h_{33} = 0.3, h_{34} = 0.4, h_{35} = 0.6, h_{36} = 0.9$$

$$h_{41} = 0.0, h_{42} = 0.2, h_{43} = 0.4, h_{44} = 0.5, h_{45} = 0.7, h_{46} = 0.8$$

**Step 3:** Assume the inputs  $X = (0, 1, 0, 1, 1, 0)$  as  $x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1, x_5 = 1, x_6 = 0$ .

**Step 4:** Calculate the weighted sum

$$\mathbb{H}_1 = h_{11}x_1 + h_{12}x_2 + h_{13}x_3 + h_{14}x_4 + h_{15}x_5 + h_{16}x_6 = 1$$

$$\mathbb{H}_2 = 1.3, \mathbb{H}_3 = 1.2, \mathbb{H}_4 = 1.4.$$

**Step 5:** Determine the previous state,

$$W_{ps1} = \frac{1}{1+\exp(-\mathbb{H}_1)} = \frac{1}{1+\exp(-1)} = 0.268941,$$

$$W_{ps2} = 0.214165, W_{ps3} = 0.231475, W_{ps4} = 0.197816.$$

**Step 6:** Calculate the activation function.

$$\Phi(h_{11}) = \exp\left(-\frac{(h_{11}-c_1)^2}{2\sigma_1^2}\right) = 0.0032797.$$

$$\text{Similarly, } \Phi(h_{12}) = 0.1604136, \Phi(h_{13}) = 0.9075560,$$

$$\Phi(h_{14}) = 0.9075560, \Phi(h_{15}) = 0.0437178, \Phi(h_{16}) = 0.0003782.$$

$$W_{cs1} = 2.0229013, W_{cs2} = 0.0874356.$$

$$W_{cs3} = 0.0446027, W_{cs4} = 0.0832154.$$

**Step 7:** Determine the hidden output neuron by adding the previous state and the current state and applying the hyperbolic tangent function.

$$y_1 = \tanh(W_{ps_1} + W_{cs_1}) = 0.0400215.$$

$$y_2 = 0.0052640, y_3 = 0.0048185, y_4 = 0.0035087.$$

**Step 7:** Determine the maximum value.

$y_1 = 0.0400215$  is the maximum output value. Hence, it is found that Potato Dextrose Yeast agar medium supported the maximum vegetative growth.

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