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ON BIPOLAR FUZZY ROUGH TOPOLOGICAL SPACES

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ABSTRACT. In this paper the concepts of bipolar fuzzy rough open sets and topology are explained by an example. The boundary values of upper and lower approximations of bipolar fuzzy rough sets are defined. Using the boundary value the spread rate of five different species of Coleus plants is calculated and the maximum spread rate is determined.

1. INTRODUCTION

In [2, 3] fuzzy rough set concepts were dealt with. Bipolar fuzzy rough sets and topology, bipolar fuzzy rough open and closed sets were developed by Anita Shanthi et al., [1].

This paper illustrates bipolar fuzzy rough open sets and topology. The concept of boundary values of upper and lower approximations of bipolar fuzzy rough sets is introduced and some interesting results are established.

2. BIPOLAR FUZZY ROUGH TOPOLOGICAL SPACES

The definitions of bipolar fuzzy rough (in short BFR) topology and open sets are given in [1].

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Example 1. *The symptoms of a person infected with **niba virus** are represented by the set* $U = \{x_1, x_2, x_3\}$, *where* x_1 *=fever,* x_2 *=stomach ache and* x_3 *=hair fall. If* A *denotes the BF set representing the above symptoms for a person infected by **niba virus**, then* $A = \{x_1/(-0.1, 0.9), x_2/(-0.2, 0.8), x_3/(-0.25, 0.7)\}$, *where* $(-0.1, 0.9)$ *specifies that person is suffering from fever to the extent of 90% and at the same time this person has 10% of the counter property,* $(-0.2, 0.8)$ *represents that person is suffering from stomach ache to 80% level and at the same time this person has 20% of the counter property and so on.*

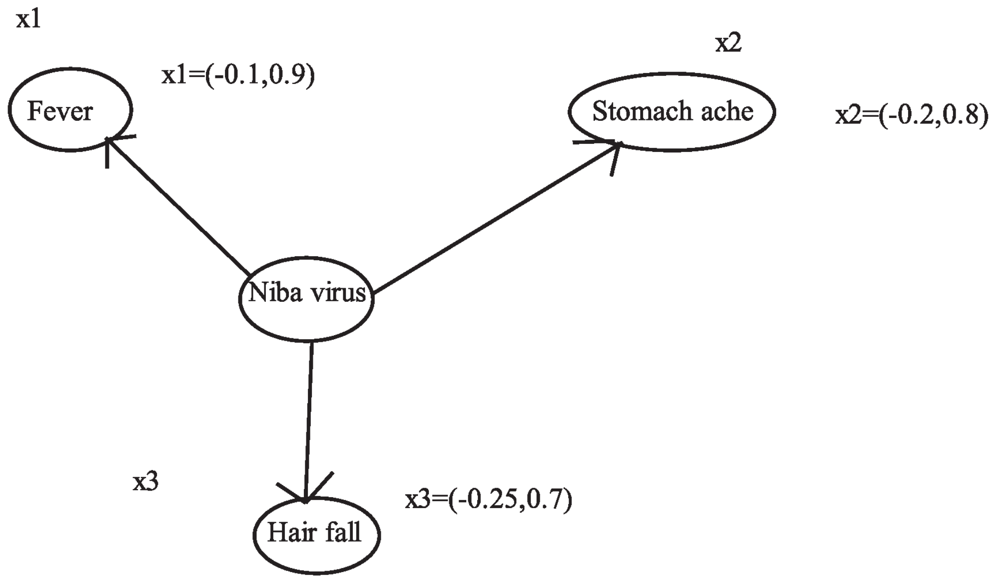


FIGURE 1. Bipolar fuzzy set A representing a person infected by niba virus

This bipolar fuzzy relation is represented in the matrix form as

$$BFR = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} (-1,1) & (-0.1,0.9) & (-0.2,0.8) \\ (-0.1,0.9) & (-1,1) & (-0.2,0.7) \\ (-0.2,0.8) & (-0.2,0.7) & (-1,1) \end{pmatrix} \end{array}$$

Using the BF set A *and bipolar fuzzy relation* BFR, *BF lower and upper approximations of the symptoms are calculated and the BFR topology constructed. Now, the elements of negative lower approximations are*

$$\mu_{BF\underline{R}^n(A)}(x_1) = -0.25,\ \mu_{BF\underline{R}^n(A)}(x_2) = -0.25,\ \mu_{BF\underline{R}^n(A)}(x_3) = -0.2.$$

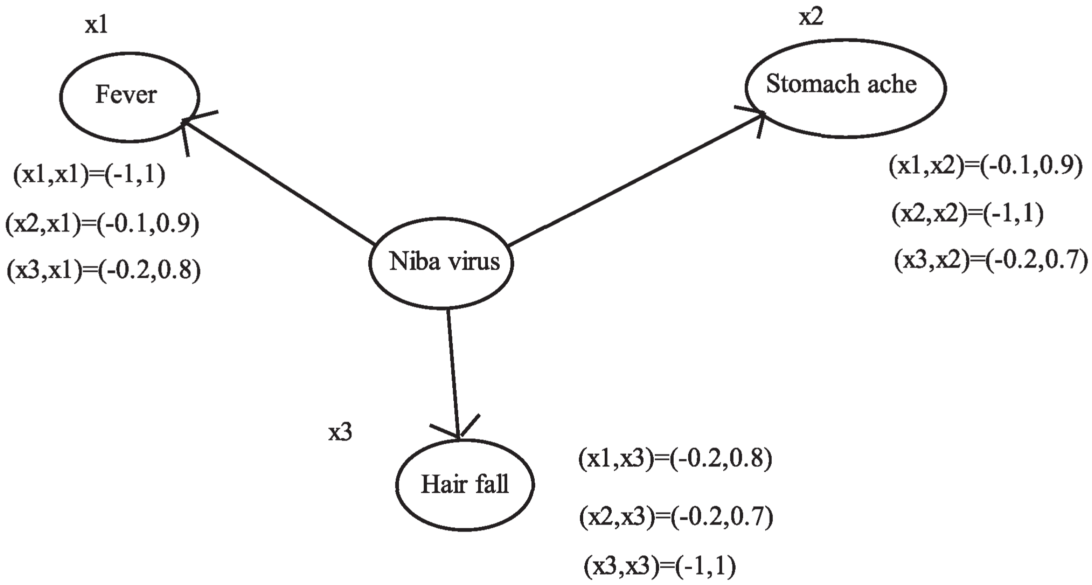


FIGURE 2. Bipolar fuzzy relation

The elements of positive lower approximations are

$$\mu_{BF\underline{R}^p(A)}(x_1) = 0.7,\ \mu_{BF\underline{R}^p(A)}(x_2) = 0.7,\ \mu_{BF\underline{R}^p(A)}(x_3) = 0.7.$$

Therefore, $BF\underline{R}(A) = \{x_1/(-0.25, 0.7), x_2/(-0.25, 0.7), x_3/(-0.2, 0.7)\}$. *The elements of negative upper approximations are*

$$\mu_{BF\overline{R}^n(A)}(x_1) = -0.2,\ \mu_{BF\overline{R}^n(A)}(x_2) = -0.1,\ \mu_{BF\overline{R}^n(A)}(x_3) = -0.1,$$

The elements of positive upper approximations are

$$\mu_{BF\overline{R}^p(A)}(x_1) = 0.9,\ \mu_{BF\overline{R}^p(A)}(x_2) = 0.9,\ \mu_{BF\overline{R}^p(A)}(x_3) = 0.8.$$

Therefore, $BF\overline{R}(A) = \{x_1/(-0.2, 0.9), x_2/(-0.1, 0.9), x_3/(-0.1, 0.8)\}$.

Hence, $\tau = \{\{x_1/(-1,1), x_2/(-0.1,0.9), x_3/(-0.2,0.8)\},\ \{x_1/(0,0), x_2/(0,0), x_3/(0,0)\},\ BF\underline{R}(A),\ BFR\overline{R}(A)$. *Therefore,* $BF\underline{R}(A), BFR\overline{R}(A)$ *are BFR open sets.*

Theorem 2.1. *If τ_1 and τ_2 are two BFR topologies on U, then $\tau_1 \cap \tau_2$ is BFR topology on U.*

Proof. Since τ_1 and τ_2 are two BFR topologies on U, $\phi, U \in \tau_1 \cap \tau_2$. Let $(BF\underline{R}^n(A_i), BF\underline{R}^p(A_i)) \in \tau_1 \cap \tau_2$ and $(BF\overline{R}^n(A_i), BF\overline{R}^p(A_i)) \in \tau_1 \cap \tau_2$, for $i = 1, 2$ with $(BF\underline{R}^n(A_1), BF\underline{R}^p(A_1)) \in \tau_1$,$(BF\overline{R}^n(A_1), BF\overline{R}^p(A_1)) \in \tau_1$ and $(BF\underline{R}^n(A_2),$

$BF\underline{R}^p(A_2)) \in \tau_2$, $(BF\overline{R}^n(A_2), BF\overline{R}^p(A_2)) \in \tau_2$. Thus, $BF\underline{R}(A_i) \cap BF\overline{R}(A_i) \in \tau_1 \cap \tau_2,\ i = 1, 2$.

Also, $\cup BFR(A_i) \in \tau_1 \cap \tau_2,\ i = 1, 2$. Thus $\tau_1 \cap \tau_2$ is a BFR topology on U. □

3. Application

In this section we define boundary with respect to $BF\underline{R}$ and $BF\overline{R}$ approximations in BFR topology.

Definition 3.1. *The boundary values of upper and lower approximations of BFR sets are $B^p = |BF\overline{R}^p - BF\underline{R}^p|$ and $B^n = |BF\overline{R}^n - (-BF\underline{R}^n)| = |BF\overline{R}^n + BF\underline{R}^n|$.*

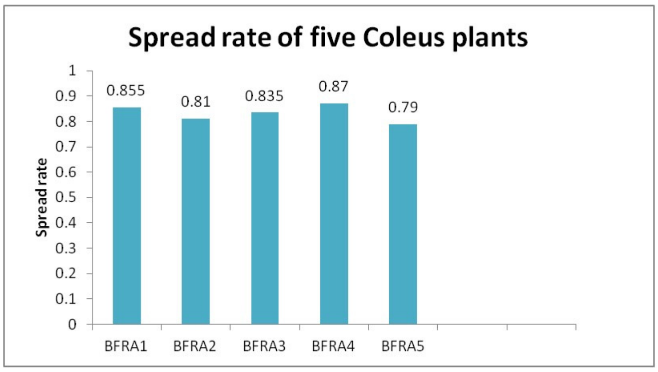
Definition 3.2. *The spread rates are defined using boundary values as $\frac{B^p}{t}, \frac{B^n}{t}$, where t denotes the time.*

FIGURE 3. Coleus blumei

Example 2. *Let $BFR(A_1), BFR(A_2), BFR(A_3), BFR(A_4), BFR(A_5)$ represent five different species of Coleus plants viz., Coleus caninus, Coleus amboinicus, Coleus barbatus, Coleus blumei and Coleus esculentus. The boundary value $B^p = |BF\overline{R}_i^p - BF\underline{R}_i^p|, B^n = |BF\overline{R}_i^n + BF\underline{R}_i^n|, i = 1$ to 5 are calculated. Taking $t = 1(week)$ the average spread rate of the plants is calculated as $\frac{B^p+B^n}{2}$. These values are tabulated as follows.*

	$BF\underline{R}$	$BF\overline{R}$	$B^n = \lvert BF\overline{R}^n + BF\underline{R}^n\rvert$	$B^p = \lvert BF\overline{R}^p - BF\underline{R}^p\rvert$	*Average [0.5ex]*
$BFRA_1$	*(-0.81,0.72)*	*(-0.79 , 0.61)*	*1.6*	*0.11*	*0.855*
$BFRA_2$	*(-0.84 , 0.61)*	*(-0.79 , 0.61)*	*1.63*	*0*	*0.81*
$BFRA_3$	*(-0.81 , 0.71)*	*(-0.76 , 0.61)*	*1.57*	*0.1*	*0.835*
$BFRA_4$	*(-0.81, 0.85)*	*(-0.69 , 0.61)*	*1.5*	*0.24*	*0.87*
$BFRA_5$	*(-0.81, 0.74)*	*(-0.64 , 0.61)*	*1.45*	*0.13*	*0.79*

The spread rate of the five Coleus plants are $BFRA_4 \succ BFRA_1 \succ BFRA_3 \succ BFRA_2 \succ BFRA_5$. *The average value of* $BFR(A_4)$ *i.e.,Coleus blumei is maximum. Hence* $BFRA_4$ *has maximum spread rate.*



4. Conclusion

In this paper we have studied BFR topology and open sets. We have calculated the boundary value using BFR lower and upper approximations and made use of this boundary value to calculate the spread rate. Further, is given an example to illustrate the spread rate of five different species of Coleus plants.

References

[1] S. A. Shanthi, M. Saranya: *On bipolar fuzzy rough connected spaces*, AIP Conference Proceedings, **2177**(1) (2019).

[2] D. Dubois, H. Prade: *Rough fuzzy set and fuzzy rough set*, International Journal of General Systems, **17**(2-3) (1990), 191–209 .

[3] S. Nanda: *Fuzzy rough sets*, Fuzzy sets and systems, **45** (1992), 157–160 .

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