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PYTHAGOREAN FUZZY MULTI SET AND ITS APPLICATIONS IN FISH FEED FOR INDIAN MAJOR CARP

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ABSTRACT. Pythagorean fuzzy set is an extension of Intutionistic fuzzy set, which is more capable of expressing and handling the uncertainty under uncertain environments, so that it was broadly applied in various fields. In this paper, we explored the concept of Pythagorean fuzzy multi set (PFMS). We describe some basic set operations of Pythagorean fuzzy multi set and also, we proposed sine exponential distance function. Finally, through an illustrative example it is shown how the proposed distance works in decision-making problem.

1. INTRODUCTION

Many fields of modern mathematics has been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. Set is a well-defined collection of distinct objects, that is, the elements of a set are pair wise different. If we relax this restriction and allow repeated occurrences of any element, then we can get a mathematical structure that is known as multi sets or bags. For example, the prime factorization of an integer $n > 0$, is a multi set whose elements are primes. The number 120, has the prime factorization $60 = 2^2 3^1 5^1$ which gives the multi set $\{2, 2, 3, 5\}$. A complete account of the development of multi set theory can be seen in [2]. As

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a generalization of multi set, Yager in [8] introduced the concept of Fuzzy Multi Set (FMS). An element of a fuzzy multi set can occur more than once with possibly the same or different membership values.

Shinoj and Sunil in [7] introduced intuitionistic fuzzy multi sets (IFMSs) from the combination of IFS and fuzzy multi set (FMS) proposed in [8] and showed its application in medical diagnosis. Obviously, IFMS is a generalized IFS or an extension of IFS, [5].

Atanassov in [1] introduced intuitionistic fuzzy sets of second type (IFSST) with the property that the sum of the square of the membership and non membership degrees is less than or equal to one. This concept generalizes IFSs in a way. The notion of IFSs provides a flexible framework to elaborate uncertainty and vagueness. The idea of IFS seems to be resourceful in modeling many real-life situations like medical diagnosis, [3, 4]. There are situations where $\mu + \nu = 1$ unlike the cases capture in IFSs. This limitation in IFS naturally led to a construct called Pythagorean fuzzy sets (PFSs). Pythagorean fuzzy set (PFS) proposed in [9, 10] is a new tool to deal with vagueness considering the membership grade, μ and non membership grade, ν satisfying the conditions $\mu + \nu = 1$ or $\mu + \nu = 1$, and also, it follows that $\mu^2 + \nu^2 + \pi^2 = 1$, where π is the Pythagorean fuzzy set index.

This paper is an attempt to combine the two concepts: Pythagorean fuzzy sets and multi sets together by introducing a new concept called Pythagorean fuzzy multi sets. The rest of the paper is organized as follows: Section 2 recalls some basic definitions of Pythagorean fuzzy sets and multi sets. Section 3, deals with the concept of Pythagorean fuzzy multi sets, operations and the proposed sine exponential distance measure. Section 4 and 5 contains the algorithm and the practical application of PFMS. Section 6 concludes the article.

2. Preliminaries

Definition 2.1. *[6], Let M be a fixed set, then a Pythagorean fuzzy set in M can be defined as follows: $P = \{(m, \lambda_p(m), \eta_p(m))/m \in M\}$, where $\lambda_p(m)$ and $\eta_p(m)$ are mappings from M to $[0,1]$ with conditions $0 \leq \lambda_p(m) \leq 1$, $0 \leq \eta_p(m) \leq 1$ and also $0 \leq \lambda_p^2(m) + \eta_p^2(m) \leq 1$, for all $m \in M$, and they denote the degree of membership and degree of non membership of element $m \in M$ to set P, respectively.*

Let $p_p(m) = \sqrt{1 - \lambda_p^2(m) - \eta_p^2(m)}$ *then it is called the Pythagorean fuzzy index of element* $m \in M$ *to set* P*, representing the degree of indeterminacy of* m *to* P*. Also* $0 \leq \pi_p(m) \leq 1$*, for every* $m \in M$*.*

Definition 2.2. *[6], Let* S *and* T *be two Pythagorean fuzzy sets. Then* S *and* T *are called similar sets if the following conditions hold:* $\lambda_S(m) = \lambda_T(m)$ *or* $\eta_S(m) = \eta_T(m)$*.*

Definition 2.3. *[6], Let* S *and* T *be two Pythagorean fuzzy sets. Then* S *and* T *are called comparable sets if the following conditions hold:* $\lambda_S(m) = \lambda_T(m)$ *and* $\eta_S(m) = \eta_T(m)$*.*

Definition 2.4. *[7], A mset* M *drawn from the set* X *is represented by a function count* M *or* C_M*, defined as* $C_M : X \to N$*, where* N *represents the set of nonnegative integers. Here,* $C_M(x)$ *is the number of occurrences of the element* x *in the mset* M *however those elements which are not included in the mset* M *have zero count.*

Example 1. *Assume that* $X = \{x, y, z, w\}$ *is a set of symbols and suppose we have a number of objects but they are not distinguishable except their labels of* x, y, z, *or* w*. For example, we have two balls with label* x *and one ball with* y*, three with label* z*, but no ball with the label* w*. Moreover we are not allowed to put additional labels to distinguish two* $x's$*. Therefore a natural representation of the situation is that we have a collection* $\{x, x, y, z, z, z\}$*.*

We can also write $\{2/x, 1/y, 3/z, 0/w\}$ *to show the number for each element of the universe* X*, or* $\{2/x, 1/y, 3/z\}$ *by ignoring zero of* w*. We will say that there are three occurrences of* x*, two occurrences of* y*, and three occurrences of* z*. We have* $Coun_M(x) = 2$*,* $Count_M(y) = 1$*,* $Count_M(z) = 3$*,* $Count_M(w) = 0$*.*

Definition 2.5. *[7], The following are basic relations and operations for multi sets.*

(i) Inclusion: $M \subseteq N \Leftrightarrow Count_M(x) \leq Count_N(x), \forall\, x \in X$

(ii) Equality : $M = N \Leftrightarrow Count_M(x) = Count_N(x), \forall x \in X$

(iii) Union :

$$Count_{M \cup N}(x) = \max\{Count_M(x), Count_N(x)\} = Count_M(x) \vee Count_N(x).$$

(iv) Intersection :

$$Count_{M \cap N}(x) = \min\{Count_M(x), Count_N(x)\} = Count_M(x) \wedge Count_N(x)$$

(v) Addition : $Count_{M\oplus N}(x) = Count_M(x) + Count_N(x)$

(vi) Subtraction : $Count_{M\ominus N}(x) = 0 \wedge (Count_M(x) - Count_N(x)$

The symbols $\bigvee$ *and* $\bigwedge$ *are the in fix notations of* max *and* min, *respectively.*

Example 2. *Consider the multi set* $M = \{2/x,\ 1/y,\ 3/z,\ 0/w\}$ *and* $N = \{1/x,\ 4/y,\ 3/w\}$. *Then,*

(i) $M \oplus N = \left\{\frac{3}{x}, \frac{5}{y}, \frac{3}{z}, \frac{3}{w}\right\}$,

(ii) $M \cup N = \{2/x,\ 4/y,\ 3/z,\ 3/w\}$,

(iii) $M \cap N = \{1/x,\ 1/y\}$

(iv) $M \ominus N = \{1/x,\ 3/z\}$.

3. Pythagorean fuzzy multi sets

Definition 3.1. *Let M be a non-empty set. An Pythagorean fuzzy multi set (PFMS) P drawn from M is defined by two functions "count membership" of P denoted as* CM_P *and "count non-membership" of P denoted as* CN_P *respectively by* $CM_P : M \to Q$ *and* $CN_P : M \to Q$ *where Q is the set of all crisp multi sets drawn from the interval* $[0,1]$. *For each* $m \in M$, *the membership and non-membership sequence may be in increasingly or decreasingly ordered sequence and it is denoted by* $(\lambda_P^1(m), \lambda_P^2(m), \ldots, \lambda_P^n(m))$, $(\eta_P^1(m), \eta_P^2(m), \ldots, \eta_P^n(m))$ *and also* $0 \le (\lambda_P^2(m))^i + (\eta_P^2(m))^i \le 1$, *for all i.*

Let $\pi_p^i(m) = \sqrt{1 - (\lambda_P^i(m))^2 - (\eta_P^i(m))^2}$ *then it is called the Pythagorean fuzzy multi index of element* $m \in M$ *to set P, representing the degree of indeterminacy of m to P. Also ,* $0 \le \pi_P^i(m) \le 1$ *for every* $m \in M$.

Example 3. *Consider a Pythagorean fuzzy multi set*

$$P = \{(x, 0.2, 0.4), (x, 0.3, 0.5), (y, 1, 0), (y, 0.5, 0.3), (z, 0.4, 0.5)\}$$

of $M = \{x,\ y,\ z,\ w\}$.

Definition 3.2. *Let* $\{P_j\}_{j\in J}$ *be an arbitrary family of PFMSs in M where* $P = \left(\lambda_{P_j}^i(m), \eta_{P_j}^i(m)\right) \in PFMS(M)$ *for each* $j \in J$, *we define*

(i) $\bigcap_{j\in J} P = (\wedge\lambda_{P_j}^i(m), \vee\eta_{P_j}^i(m))$

(ii) $\bigcup_{j\in J} P = (\vee\lambda_{P_j}^i(m), \wedge\eta_{P_j}^i(m))$

Definition 3.3. *Length of an element m is an PFMS P is defined as the cardinality of $CN_P(m)$ for which $0 \leq (\lambda_P^i(m))^2 + (\eta_P^i(m))^2 \leq 1$ and it is denoted by $L(m : P)$.*

Example 4. $P = \{x : (0.3, 0.1), (0.4, 0.3), y : (0.5, 0.5, 0.1), (0.4, 0.3, 0.2), z : (0.5, 0.4, 0.3, 0.2), (0.6, 0.4, 0.2, 0.1)\}$.

$CN_P(x) = 2$, $CN_P(y) = 3$, $CN_P(z) = 4$.

Definition 3.4. *For any two PFMSs P and Q drawn from a set M, the following operations and relations will hold. Let $P = \{m : (\lambda_P^1(m), \lambda_P^2(m), \ldots, \lambda_P^n(m)), (\eta_P^1(m), \eta_P^2(m), \ldots, \eta_P^n(m)) : m \in M\}$ and $Q = \{m : (\lambda_Q^1(m), \lambda_Q^2(m), \ldots, \lambda_Q^n(m)), (\eta_Q^1(m), \eta_Q^2(m), \ldots, \eta_Q^n(m)) : m \in M\}$ then*

(i) Inclusion : $P \subset Q \iff \lambda_P^i(m) \leq \lambda_Q^j(m)$ *and* $\eta_P^i(m) \leq \eta_Q^j(m)$, $i = 1, 2, \ldots, L(m), m \in M$

$$P = Q \iff P \subseteq Q \text{ and } Q \subseteq P$$

(ii) Complement : $P^C = \{m\,(\eta_P^i(m), \lambda_P^i(m)) : m \in M\}$

(iii) Union : $P \cup Q = \{\lambda_P^i(m) \vee \lambda_Q^i(m), \eta_P^i(m) \wedge \eta_Q^i(m) : m \in M\}$

(iv) Intersection : $P \cap Q = \{\lambda_P^i(m) \wedge \lambda_Q^i(m), \eta_P^i(m) \vee \eta_Q^i(m) : m \in M\}$

(v) Addition :
$P \oplus Q = \left(\lambda_P^i(m) + \lambda_Q^i(m) - \lambda_P^i(m)\,\lambda_Q^i(m), \eta_P^i(m)\,\eta_Q^i(m)\right) : m \in M$

(vi) Multiplication :
$P \otimes Q = \left(\lambda_P^i(m)\,\lambda_Q^i(m), \eta_P^i(m) + \eta_Q^i(m) - \eta_P^i(m)\,\eta_Q^i(m)\right) : m \in M$

(vii) Difference : $P - Q = \left\{(\lambda_P^i(m) \wedge \eta_Q^i(m), \eta_P^i(m) \vee \lambda_Q^i(m)) : m \in M\right\}$

Definition 3.5. *Let $P = \{m_i : (\lambda_P^1(m), \lambda_P^2(m), \ldots, \lambda_P^n(m)), (\eta_P^1(m), \eta_P^2(m), \ldots, \eta_P^n(m)) : m \in M\}$ be two Pythagorean fuzzy multi sets on a finite set $M = \{M_1, M_2, \ldots, M_n\}$. Then, the sine exponential distance is defined by*

$$D_{PFMS}\ (A, B) = \frac{1}{2n-1}\left[\sum_{i=1}^{q}\left[\sum_{j=1}^{n} \sin\, e^{-\left[\left|\lambda_A^i(x_j) - \lambda_B^i(x_j)\right| + \left|\eta_A^i(x_j) - \eta_B^i(x_j)\right| + \left|\pi_A^i(x_j) - \pi_B^i(x_j)\right|\right]}\right]\right]$$

Proposition 3.1.

(i) $0 \leq D_{PFMS}(A, B) \leq 1$.

(ii) $D_{PFMS}(A, B) = D_{PFMS}(B, A)$.

(iii) If $A \subseteq B \subseteq C$ then $D_{PFMS}(A,C) \geq D_{PFMS}(A,B)$ and $D_{PFMS}(A,C) = D_{PFMS}(B,C)$.

Proof.

(i) The proof is obvious.

(ii)

$$\left|\lambda_A^i(x_j) - \lambda_B^i(x_j)\right| = \left|\lambda_B^i(x_j) - \lambda_A^i(x_j)\right|$$
$$\left|\eta_A^i(x_j) - \eta_B^i(x_j)\right| = \left|\eta_B^i(x_j) - \eta_A^i(x_j)\right|$$
$$\therefore\ D_{PFMS}(A,B) = D_{PFMS}(B,A)$$

(iii) Since $A \subseteq B \subseteq C$,

$$\lambda_A^i(x_j) \leq \lambda_B^i(x_j) \leq \lambda_C^i(x_j)$$
$$\eta_A^i(x_j) \geq \eta_B^i(x_j) \geq \ \eta_C^i(x_j)$$
$$\Rightarrow \left|\lambda_A^i\left(x_j\right) - \lambda_B^i\left(x_j\right)\right| \leq \left|\lambda_A^i\left(x_j\right) - \lambda_C^i\left(x_j\right)\right|$$
$$\left|\lambda_B^i(x_j) - \lambda_C^i(x_j)\right| \leq \left|\lambda_A^i(x_j) - \lambda_C^i(x_j)\right|$$
$$\Rightarrow \left|\eta_A^i\left(x_j\right) - \eta_B^i\left(x_j\right)\right| \leq \left|\eta_A^i\left(x_j\right) - \eta_C^i\left(x_j\right)\right|$$
$$\left|\eta_B^i(x_j) - \eta_C^i(x_j)\right| \leq \left|\eta_A^i(x_j) - \eta_C^i(x_j)\right|$$

Therefore, the sine exponential distance is an increasing function and

$$D_{PFMS}(A,C) \geq D_{PFMS}(A,B) \text{ and } D_{PFMS}(A,C) = D_{PFMS}(B,C)$$

□

4. Practical application of Pythagorean fuzzy multi set in decision making

Feeding the fish is a major expenditure for fishers, when demand increases for fish and fish protein. Good fish feed management can reduce overall cost, improve fish farm environment and ensure healthy growth of fish. It includes choosing the quality fish feeds, feeding method, calculating the feeding cost and secure the effectiveness of fish farm.

Three kinds of fishes (**Catla catla, Labeo rohita, Cirrhinus mrigala)** were taken as sample and three varieties of fish feed (**Yeast, Lactobacillus, Algae)** were tried on these fishes. This was done on three months and the data were collected to prove which fish feed was best for these fishes to increase Hematological and Nutritional contents. Distance measure was used to calculate the effectiveness of the fish feeds.

Three months data of Fish feed (Yeast, Lactobacillus and Algae) and Hematological and Nutritional contents (i.e., **RBC, HB, Protein**) of the fishes were taken and it was noted in Table 1 and Table 2.

TABLE 1. The relation between Fishes and Fish feeds

Fish \ Feed	**Yeast**	**Lactobacillus**	**Algae**
F_1 (Catla catla)	(0.2,0.8) (0.8,0.1) (0.2,0.9)	(0.3,0.8) (0.2,0.7) (0.9,0.2)	(0.8,0.1) (0.3,0.6) (0.2,0.8)
F_2 (Labeo rohita)	(0.1,0.8) (0.7,0.2) (0.2,0.8)	(0.2,0.8) (0.2,0.6) (0.8,0.1)	(0.9,0.2) (0.2,0.7) (0.2,0.9)
F_3 (Cirrihinus mrigal)	(0.2,0.9) (0.8,0.3) (0.1,0.9)	(0.4,0.8) (0.3,0.9) (0.9,0.3)	(0.8,0.3) (0.2.0.7) (0.1,0.6)

TABLE 2. The relation between Fish feeds and H & N Content

Feed \ H & N Content	**RBC**	**HB**	**Protein**
Yeast	(0.4,0.6) (0.5.0.2) (0.3.0.5)	(0.5.0.2) (0.4,0.6) (0.4,0.5)	(0.5,0.7) (0.5.0.2) (0.4,0.4)
Lactobacillus	(0.5,0.6) (0.3,0.7) (0.5,0.2)	(0.6,0.3) (0.5,0.4) (0.4,0.7)	(0.5,0.4) (0.6.0.4) (0.6.0.6)
Algae	(0.8,0.5) (0.5.0.6) (0.7,0.3)	(0.9,0.3) (0.4.0.5) (0.5,0.3)	(0.8,0.2) (0.5,0.4) (0.6.0.5)

TABLE 3. Distance between Fishes and Hematological and Nutritional contents using Sine exponential distance

$\mathbf{D}_{\mathbf{P}FMS}(\mathbf{F}, \mathbf{H\&N})$	**RBC**	**HB**	**Protein**
$\mathbf{F}_1$	0.184	0.073	0.134
$\mathbf{F}_2$	0.150	0.085	0.14
$\mathbf{F}_3$	0.170	0.085	0.130

If the distance between the fishes, and a particular Hematological and Nutritional content is shortest, then the fish is likely to have the H & N content. From Table 3 the distance between the fish 1 (Catla Catla) and HB provides the shortest distance. So Fish 1 has more HB count compare to the remaining two fishes. Likewise, Fish 2 has more RBC count, and Fish 3 is rich in Protein.

5. ALGORITHM

Step 1: The feed of the fishes are given to obtain the fish - feed relation and are noted in Table 1.

Step 2: The biological knowledge related to the feed with the Hematological & Nutritional contents under consideration is given to obtain the feed - Hematological & Nutritional contents relations and are noted in Table 2.

Step 3: The computation of the relation between the fishes and Hematological & Nutritional contents is found using sine exponential distance measure and are noted in Table 3.

Step 4: Finally, the minimum value from Table 3 of each row was selected to find the possibility of the fish which is having more Hematological & Nutritional contents benefited by the fish feed.

6. CONCLUSION

This paper extends the concepts of Pythagorean fuzzy multi set, and its operations. It discovers the relationship between the feed given to the fishes and Hematological and Nutritional content of the fish. Our proffered technique is most reliable to handle decision-making problem quiet easy. This method can conquer other areas like medical diagnosis, image processing, etc.,

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