ROOT QUAD MEAN LABELING FOR SOME SPECIAL GRAPHS

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ABSTRACT. A graph $G = (p, q)$ is a Root Quad Mean graph if there is an injective function $f$ from the vertices of $G$ to $1, 2, \ldots, p^2 - 1$ such that when each edge $uv$ is labeled with $\sqrt{f(u)^4 + f(v)^4}$ then the resultant edges are distinct.

In this paper we proved Comb graph, Ladder graph, Slanting ladder graph, Gear graph, Helm graph and Heger graph are Root Quad Mean Graphs.

1. INTRODUCTION

A graph is made up of vertices which are connected by edges. In this paper a concise summary of definitions and other information is given aiming to maintain compactness. In 1967, Alexander Rosa introduced the concept of labeling. Graph labeling is the assignment of labels, normally represented by integers, to edges and vertices of a graph. A useful survey on graph labeling by J. A. Gallian (2014) can be found in [1].

Somasundaram and Ponraj [2] have introduced the notion of Mean Labeling of graphs. Sandhya, Somasundaram and Anusa have introduced Root Square Mean Labeling [3]. Gowri and Vembarasi have introduced Root cube Mean Labeling[4]. Inspired by the above works we introduced new labeling called Root Quad Mean Labeling. In this paper we probe the Root Quad Mean Labeling of

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Comb graph, Ladder graph, Slanting ladder graph, Gear graph, Helm graph and a new graph is found.

2. BASIC DEFINITIONS

**Definition 2.1.** A graph \( G = (V,E) \) is a set of all vertices and edges in which each edge is associated by a pair of two vertices.

**Definition 2.2.** The graph obtained by attaching a single pendent edge to each vertex of a path is called comb.

**Definition 2.3.** A ladder \( L_n \) is the graph obtained from two paths \( u_1, u_2, ..., u_n \) and \( v_1, v_2, ..., v_n \) by joining each \( u_i \) with \( v_i, 1 \leq i \leq n \).

**Definition 2.4.** A slanting ladder \( SL_n \) is the graph obtained from two paths \( u_1, u_2, ..., u_n \) and \( v_1, v_2, ..., v_n \) by joining each \( u_i \) with \( v_{i+1}, 1 \leq i \leq n-1 \) [5].

**Definition 2.5.** Gear graph \( G_p \) is a graph obtained from wheel by adding a vertex between each pair of adjacent vertices of rim of the cycle.

**Definition 2.6.** The Helm \( H_p \) is a graph obtained by joining pendant vertices to each rim vertex of the Wheel.

**Definition 2.7.** Heger graph \( M_p \) is graph obtained from gear graph \( G_p \) by attaching pendant edge to each vertex of the rim of \( G_p \) which is not connected to centre vertex of \( G_p \).

**Definition 2.8.** A graph \( G \) with \( p \) vertices and \( q \) edges is a Mean graph if there is an injective function \( f \) from the vertices of \( G \) to \( 0, 1, 2, \ldots, q \) such that when each edge \( uv \) is labeled with \( \frac{f(u) + f(v)}{2} \) if \( f(u) + f(v) \) is even and \( \frac{f(u) + f(v) + 1}{2} \) if \( f(u) + f(v) \) is odd then the resulting edges are distinct.

**Definition 2.9.** A graph \( G = (p, q) \) is a Root Quad Mean graph if there is an injective function \( f \) from the vertices of \( G \) to \( 1, 2, \ldots, p^2 - 1 \) such that when each edge \( uv \) is labeled with \( \frac{\sqrt{f(u)^4 + f(v)^4}}{2} \) then the resultant edges are distinct.
3. Main Results

**Theorem 3.1.** Every Comb graph is a root quad mean graph.

**Proof.** Let $G$ be a comb graph, $p$ the number of vertices on the graph $G$, $V(G)$ be the vertices of the graph such that $u_1, u_2, \ldots, u_p$ be the upper set of vertices, $v_1, v_2, \ldots, v_p$ be the lower set of vertices, and let edges be defined as

$$E(G) = \{ u_i u_{i+1} / 1 \leq i \leq \frac{p-2}{2} \} \cup \{ u_i v_i / 1 \leq i \leq \frac{p}{2} \}.$$ 

Here $V(G) = p$ and $E(G) = p - 1$.

Now define a function $f : V(G) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(u_i) = pi - 1$ for $1 \leq i \leq \frac{p}{2}$, $f(v_i) = pi - 2$ for $1 \leq i \leq \frac{p}{2}$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u_i)^4 + f(v_i)^4}}{2} \rfloor$, $e_i \in E(G)$ are all distinct. The edge sets are

$$E_1 = \{ u_i u_{i+1} / 1 \leq i \leq \frac{p-2}{2} \},$$
$$E_2 = \{ u_i v_i / 1 \leq i \leq \frac{p}{2} \}.$$

Thus the resultant edges are distinct. Hence every comb graph $G$ is a root quad mean graph. $\Box$

**Theorem 3.2.** Every Ladder graph $L_n$ is a root quad mean graph.

**Proof.** Let $L_n$ be a ladder graph. Let $V(L_n)$ be the vertices of the graph such that $u_1, u_2, \ldots, u_n$ be the upper set of vertices, $v_1, v_2, \ldots, v_n$ be the lower set of vertices. Let edges be defined as

$$E(L_n) = \{ u_i u_{i+1} / 1 \leq i \leq n-1 \} \cup \{ v_i v_{i+1} / 1 \leq i \leq n-1 \} \cup \{ u_i v_i / 1 \leq i \leq n \}.$$ 

Let $p$ the number of vertices on the graph $L_n$. Here $V(L_n) = 2n = p$, $E(L_n) = 3n - 2$.

Now define a function $f : V(L_p) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(u_i) = pi - 1$ for $1 \leq i \leq n$, $f(v_i) = pi - 2$ for $1 \leq i \leq n$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u_i)^4 + f(v_i)^4}}{2} \rfloor$, $e_i \in E(L_n)$ are all distinct. The edge sets are:

$$E_1 = \{ u_i u_{i+1} / 1 \leq i \leq n-1 \} ,$$
$$E_2 = \{ v_i v_{i+1} / 1 \leq i \leq n-1 \} ,$$
$$E_3 = \{ u_i v_i / 1 \leq i \leq n \} .$$
Thus the resultant edges are distinct. Hence every ladder graph $L_n$ is a root quad mean graph.

\section*{Theorem 3.3.} Every Slanting ladder graph $SL_n$ is a root quad mean graph.

\textbf{Proof.} Let $SL_n$ be a slanting ladder graph. Let $V(SL_n)$ be the vertices of the graph such that $u_1, u_2, \ldots, u_n$ be the upper set of vertices, $v_1, v_2, \ldots, v_n$ be the lower set of vertices. Let edges be defined as

\[ E(SL_n) = \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{v_iv_{i+1}/1 \leq i \leq n-1\} \]
\[ \cup \{u_iv_{i+1}/1 \leq i \leq n-1\}. \]

Let $p$ the number of vertices on the graph $SL_n$. Here $V(SL_n) = 2n = p$, $E(SL_n) = 3(n-1)$. Now define a function $f : V(SL_n) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(u_i) = pi - 1$ for $1 \leq i \leq n$, $f(v_i) = pi - 2$ for $1 \leq i \leq n$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \sqrt{f(u)^4 + f(v)^4} / 2 \rfloor$, $e_i \in E(SL_n)$ are all distinct. The edge sets are

\[ E_1 = \{u_iu_{i+1}/1 \leq i \leq n-1\}, \]
\[ E_2 = \{v_iv_{i+1}/1 \leq i \leq n-1\}, \]
\[ E_3 = \{u_iv_{i+1}/1 \leq i \leq n-1\}. \]

Thus the resultant edges are distinct. Hence every slanting ladder graph $SL_n$ is a root quad mean graph. \qed

\section*{Theorem 3.4.} Every Gear graph $G_p$ is a root quad mean graph.

\textbf{Proof.} Let $G_p$ be a gear graph. Let $V(G_p)$ be the vertices of the graph such that $v_0$ be the centre vertex and $v_1, v_2, \ldots, v_p$ be the vertices on the rim. Let edges be defined as

\[ E(G_p) = \{v_iv_{i+1}/1 \leq i \leq p - 1\} \cup \{vpv_1\} \cup \{v_0v_{2i-1}/1 \leq i \leq p/2\}. \]

Here $V(G_p) = p + 1$, $E(G_p) = \frac{3p^2}{2}$. Now define a function $f : V(G_p) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(v_0) = p$, $f(v_i) = pi - 1$ for $1 \leq i \leq p$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \sqrt{f(u)^4 + f(v)^4} / 2 \rfloor$, $e_i \in E(G_p)$ are
all distinct. The edge sets are

\[ E_1 = \{v_i v_{i+1} / 1 \leq i \leq p - 1\}, \]
\[ E_2 = \{v_p v_1\}, \]
\[ E_3 = \{v_0 v_{2i-1} / 1 \leq i \leq \frac{p}{2}\}. \]

Thus the resultant edges are distinct. Hence every Gear graph \( G_p \) is a root quad mean graph.

**Theorem 3.5.** Every Helm graph \( H_p \) is a root quad mean graph.

**Proof.** Let \( H_p \) be a helm graph. Let \( p \) the number of vertices on the rim. Let \( V(H_p) \) be the vertices of the graph such that \( v_0 \) be the centre vertex and \( v_1, v_2, \ldots, v_p \) be the vertices on the rim and \( u_1, u_2, \ldots, u_p \) be the pendant vertices. Let edges be defined as

\[ E(H_p) = \{v_i v_{i+1} / 1 \leq i \leq p - 1\} \cup \{v_p v_1\} \cup \{v_0 v_i / 1 \leq i \leq p\} \]
\[ \cup \{v_i u_i / 1 \leq i \leq p\}. \]

Here \( V(H_p) = 2p + 1, \ E(H_p) = 3p. \) Now define a function \( f : V(H_p) \to \{1, 2, \ldots, p^2 - 1\} \) by \( f(v_0) = 2p, \) \( f(v_i) = pi - 1 \text{ for } 1 \leq i \leq p, \) \( f(u_i) = pi - 2 \text{ for } 1 \leq i \leq p. \) Then the induced edge labeling \( f : E \to \mathbb{N} \) defined by \( f(e_i) = \left\lfloor \sqrt{f(u_i)^2 + f(v_i)^2} / 2 \right\rfloor, e_i \in E(H_p) \) are all distinct. The edge sets are

\[ E_1 = \{v_i v_{i+1} / 1 \leq i \leq p - 1\}, \]
\[ E_2 = \{v_p v_1\}, \]
\[ E_3 = \{v_0 v_i / 1 \leq i \leq p\}, \]
\[ E_4 = \{v_i u_i / 1 \leq i \leq p\}. \]

Thus the resultant edges are distinct. Hence every helm graph \( H_p \) is a root quad mean graph.

**Theorem 3.6.** Every Heger graph \( M_p \) is a root quad mean graph.

**Proof.** Let \( M_p \) be a heger graph. Let \( V(M_p) \) be the vertices of the graph such that \( v_0 \) be the centre vertex and \( v_1, v_2, \ldots, v_p \) be the vertices on the rim and
$u_1, u_2, \ldots, u_p$ be the pendant vertices. Let edges be defined as

$$E(M_p) = \{v_i v_{i+1} / 1 \leq i \leq p - 1\} \cup \{v_p v_1\} \cup \{v_0 v_{2i-1} / 1 \leq i \leq \frac{p}{2}\}$$

$$\cup \{v_{2i} u_i / 1 \leq i \leq \frac{p}{2}\}.$$ 

Here $V(M_p) = \frac{3p+2}{2}$, $E(M_p) = 2p$. Now define a function $f : V(M_p) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(v_0) = p$, $f(v_i) = pi - 1$ for $1 \leq i \leq p$, $f(u_i) = 2(pi - 1)$ for $1 \leq i \leq \frac{p}{2}$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \sqrt{f(u_i)^4 + f(v_i)^4} / 2 \rfloor$, $e_i \in E(M_p)$ are all distinct. The edge sets are

$$E_1 = \{v_i v_{i+1} / 1 \leq i \leq p - 1\},$$

$$E_2 = \{v_p v_1\},$$

$$E_3 = \{v_0 v_{2i-1} / 1 \leq i \leq \frac{p}{2}\},$$

$$E_4 = \{v_{2i} u_i / 1 \leq i \leq \frac{p}{2}\}.$$ 

Thus the resultant edges are distinct. Hence every heger graph $M_p$ is a root quad mean graph.

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References


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