**Bγ-OPEN SETS IN TOPOLOGICAL SPACES**

**R. VIJAYALAKSHMI**\(^1\) AND A. VADIVEL

**ABSTRACT.** The aim of this paper is to introduce and study the notion of \(B\gamma\)-open, \(B\) pre open, \(B\) semi open and \(B\beta\)-open sets. Some characterization of these notions are presented.

1. INTRODUCTION

Levine [7] in 1963, started the study of generalized open sets with the introduction of semi-open sets. And in the year 1970, Levine [7] introduced \(g\)-closed sets in topological spaces (briefly, ts’s) as a generalization of closed sets. Levine [8], defined the notion of \(B\)-open sets in a ts and he obtained various properties. The purpose of this paper is to introduce and study the notion of \(B\gamma\)-open, \(B\) pre open, \(B\) semi open and \(B\beta\)-open sets. Also, some characterizations of these notions are presented. Throughout this paper \((X, \tau)\) and \((Y, \sigma)\) (simply \(X\) and \(Y\)) represent nonempty ts’s on which no separation axioms are assumed, unless otherwise mentioned. For a subset \(A\) of a space \((X, \tau)\), \(cl(A)\), \(int(A)\) and \(X\setminus A\) denote the closure of \(A\), the interior of \(A\) and the complement of \(A\) respectively.

\(^1\)corresponding author

2010 Mathematics Subject Classification.  54D10, 54E55.

Key words and phrases.  \(B\gamma\)-closed, \(B\) semi-closed, \(B\) pre-closed, \(B\beta\)-closed sets.
2. Preliminaries

Since, we shall require the following known definitions, notations and some properties, we recall them in this section. A subset \( A \) of a space \((X, \tau)\) is called semiopen [10] (resp. \( \beta \)-open [1] or semi-preopen [2], \( b \)-open [3] or \( \gamma \)-open [4] or sp-open [5] and preopen [11] set) if \( A \subseteq \text{cl}(\text{int}(A)) \) (resp., \( A \subseteq \text{cl}(\text{int}(A)) \), \( A \subseteq \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A)) \) and \( A \subseteq \text{int}(\text{cl}(A)) \)). The complement of a semi open (resp. \( \beta \)-open, \( \gamma \)-open and pre open) set is called semi-closed (resp. \( \beta \)-closed, \( \gamma \)-closed and pre closed) set. The intersection of all semi closed (resp. \( \beta \)-closed, \( \gamma \)-closed and pre closed) sets containing \( A \) is called the semi closure (resp. \( \beta \)-closure, \( \gamma \)-closure and pre closure) of \( A \) and is denoted by \( \text{scl}(A) \) (resp. \( \beta \text{cl}(A), \gamma \text{cl}(A) \) and \( \text{pcl}(A) \)). The union of all semi open (resp. \( \beta \)-open, \( \gamma \)-open and pre-open) sets contained in \( A \) is called the semi-interior (resp. \( \beta \)-interior, \( \gamma \)-interior and pre-interior) of \( A \) and is denoted by \( \text{sint}(A) \) (resp. \( \beta \text{int}(A), \gamma \text{int}(A) \) and \( \text{pint}(A) \)). The family of all semi-open (resp. \( \beta \)-open, \( \gamma \)-open and pre-open) is denoted by \( \text{SO}(X) \) (resp. \( \beta \text{O}(X), \gamma \text{O}(X) \) and \( \text{PO}(X) \)).

**Definition 2.1.** A subset \( A \) of a ts \((X, \tau)\) is called \( g \)-closed [9] if \( \text{cl}(A) \subseteq G \) whenever \( A \subseteq G \) and \( G \) is open in \((X, \tau)\). The complement of \( g \)-closed set is called \( g \)-open.

**Definition 2.2.** Levine [8], defined \( \tau(B) = \{O \cup (O' \cap B) : O, O' \in \tau\} \) and called it simple expansion of \( \tau \) by \( B \), where \( B \notin \tau \).

**Lemma 2.1.** [8] Let \( A \& B \) be subsets of a space \((X, \tau)\). Then

(i) \( A \) is \( B \)-open (briefly, \( B_0 \)) iff \( A = \text{Bint}(A) \).

(ii) \( \text{Bint}(A) \) is the union of all open sets of \( X \) whose closures are contained in \( A \).

(iii) For any subset \( A \) of \( X \), \( A \subseteq \text{cl}(A) \subseteq \text{Bcl}(A) \) (resp. \( \text{Bint}(A) \subseteq \text{int}(A) \subseteq A \)).

(iv) \( \text{Bint}(A \cap C) = \text{Bint}(A) \cap \text{Bint}(C) \) and \( \text{Bint}(A) \cup \text{Bint}(C) \subset \text{Bint}(A \cup C) \).

(v) \( \text{Bcl}(A \cup C) = \text{Bcl}(A) \cup \text{Bcl}(C) \) and \( \text{Bcl}(A \cap C) = \text{Bcl}(A) \cap \text{Bcl}(C) \).

**Definition 2.3.**

(i) A subset \( A \) of \( X \) is called a locally closed set [6] if \( A = U \cap F \), where \( U \in \tau \), \( F \) is closed in \( X \).
(ii) A space \((X, \tau)\) is called extremely disconnected [12] if the closure of every open set of \(X\) is open.

3. \(B\gamma\)-open sets

**Definition 3.1.** Let \((X, \tau)\) be a ts. Then a subset \(A\) of \(X\) is said to be

(i) \(B\gamma\)-open (briefly, \(B\gamma o\)) set if \(A \subseteq \operatorname{Bint}(Bcl(A)) \cup Bcl(\operatorname{Bint}(A))\).

(ii) \(B\)-semi open (briefly, \(Bso\)) set if \(A \supseteq Bcl(\operatorname{Bint}(A))\).

(iii) \(B\)-pre open (briefly, \(Bpo\)) set if \(A \supseteq \operatorname{Bint}(Bcl(A))\).

(iv) \(B\beta\)-open (briefly, \(B\beta o\)) set \(A \subseteq Bcl(Bcl(\operatorname{Bint}(Bcl(A))))\).

The complement of a \(B\gamma o\) set (resp. \(Bso\), \(Bpo\) and \(B\beta o\)) is called \(B\beta\)-closed set (resp. \(B\)-semi closed, \(B\)-pre closed and \(B\beta\) closed) (briefly, \(B\gamma c\) (resp. \(Bsc\), \(Bpc\), \(B\beta c\))). The family of all \(B\gamma o\), \(Bso\), \(Bpo\) and \(B\beta o\) (resp. \(B\gamma c\), \(Bsc\), \(Bpc\) and \(B\beta c\)) subsets of a space \((X, \tau)\) will be as always denoted by \(B\gamma O(X)\), \(BSO(X)\), \(BPO(X)\) and \(B\beta O(X)\) (resp. \(B\gamma C(X)\), \(BSC(X)\), \(BPC(X)\) and \(B\beta C(X)\)).

**Proposition 3.1.** Let \(A\) be a subset of a space \((X, \tau)\). Then

(i) Every \(Bo\) set is \(Bso\) (resp. \(Bpo\)).

(ii) Every \(Bso\) set is \(B\gamma o\).

(iii) Every \(Bpo\) set is \(B\gamma o\).

(iv) Every \(B\gamma o\) set is \(B\beta o\).

**Remark 3.1.** The converse of the above proposition is not necessarily true as shown by the following examples.

**Example 1.** Let \(X = \{a, b, c, d, e\}\) with topology \(\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}\), a non open set \(B = \{c, d\}\). Then the subset

(i) \(\{a, e\}\) of \(X\) is \(Bso\) but not \(Bo\).

(ii) \(\{c\}\) of \(X\) is \(Bpo\) (resp. \(B\gamma o\)) but not \(Bo\) (resp. \(Bso\)).

(iii) A subset \(\{a, b\}\) of \(X\) is \(B\gamma o\) but not \(Bpo\).

(iv) A subset \(\{c, e\}\) of \(X\) is \(B\beta\)-open but not \(B\gamma o\).

**Remark 3.2.** According to Definition 3.1 and Proposition 3.1, the following diagram holds for a subset \(A\) of a space \(X\):
In the following, we present some properties on the notion of $B_\gamma o$.

**Proposition 3.2.**

(i) If $A$ is a $B_\gamma o$ subset of a ts $(X, \tau)$ and $B\text{int}(A) = \phi$, then $A$ is $Bpo$.

(ii) If $A$ is a $Bso$ subset of a ts $(X, \tau)$ and $B\text{int}(A) = \phi$, then $A$ is $Bo$.

(iii) If $A$ is a $Bpo$ subset of a ts $(X, \tau)$ and $B\text{int}(A) = \phi$, then $A$ is $Bo$.

(iv) If $A$ is a $B_\beta o$ subset of a ts $(X, \tau)$ and $B\text{int}(A) = \phi$, then $A$ is $B_\gamma o$.

**Lemma 3.1.** Let $(X, \tau)$ be a ts. Then the following statements are hold.

(i) The union of arbitrary $B_\gamma o$ sets (resp. $Bso$, $Bpo$ and $B_\beta o$) is $B_\gamma o$ (resp. $Bso$, $Bpo$ and $B_\beta o$).

(ii) The intersection of arbitrary $B_\gamma c$ sets (resp. $Bsc$, $Bpc$ and $B_\beta c$) is $B_\gamma c$ (resp. $Bsc$, $Bpc$ and $B_\beta c$).

**Proof.** (i) Let $\{A_i \in I\}$ be a family of $B_\gamma o$ sets. Then $A_i \subseteq B\text{int}(Bcl(A_i)) \cup Bcl(B\text{int}(A_i))$. Hence $\bigcup_i A_i \subseteq \bigcup_i (B\text{int}(Bcl(A_i)) \cup Bcl(B\text{int}(A_i))) \subset (B\text{int}(Bcl(\bigcup_i A_i)) \cup Bcl(B\text{int}(\bigcup_i A_i)))$, for all $i \in I$. Thus $\bigcup_i A_i$ is $B_\gamma o$.

(ii) Similar to (i).

**Remark 3.3.** The intersection of any two $B_\gamma o$ sets is not $B_\gamma o$. Let $X = \{a, b, c, d, e\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $B = \{c, d\}$. Then $A = \{a, e\}$ & $B = \{b, e\}$ are $B_\gamma o$ sets, but $A \cap B = \{e\}$ is not $B_\gamma o$.

**Definition 3.2.** Let $(X, \tau)$ be a ts. Then

(i) The union of all $B_\gamma o$ sets (resp. $Bso$, $Bpo$ and $B_\beta o$) of $X$ contained in $A$ is called the $B_\gamma$-interior (resp. $B$-semi interior, $B$-pre interior and $B_\beta$-interior) of $A$ and is denoted by $B_\gamma \text{int}(A)$ (resp. $B\text{int}(A)$, $B\text{pre}(A)$ and $B_\beta \text{int}(A)$).

(ii) The intersection of all $B_\gamma c$ sets (resp. $Bsc$, $Bpc$ and $B_\beta c$) of $X$ contained in $A$ is called the $B_\gamma$-(resp. $B$-semi , $B$-pre and $B_\beta$) closure of $A$ and is denoted by $B_\gamma \text{cl}(A)$ (resp. $B\text{cl}(A)$, $B\text{pcl}(A)$ and $B_\beta \text{cl}(A)$).
Proposition 3.3. For subsets $A$, $B$ of a space $(X, \tau)$, the following statements hold:

(i) $Bsc\ell(A) = A \cup Bint(Bcl(A))$, $Bint(A) = A \cap Bcl(Bint(A))$,
(ii) $Bpcl(A) = A \cup Bint(Bcl(A))$, $Bpint(A) = A \cap Bcl(Bcl(A))$,
(iii) $Bsc\ell(X \setminus A) = X \setminus Bsint(A)$, $Bsc\ell(A \cup C) \subset Bsc\ell(A) \cup Bsc\ell(C)$,
(iv) $Bpcl(X \setminus A) = X \setminus Bpint(A)$, $Bpcl(A \cup C) \subset Bpcl(A) \cup Bpcl(C)$,
(v) $X \setminus (Bint(A)) = Bcl(X \setminus A)$ and $Bint(X \setminus A) = X \setminus Bcl(A)$.

Lemma 3.2. The following hold for a subset $H$ of a space $(X, \tau)$,

(i) $Bpcl(H) = H \cup Bcl(Bint(H))$ and $Bpint(H) = H \cap Bint(Bcl(H))$,
(ii) $Bpcl(Bpint(H)) = Bpint(H) \cup Bcl(Bint(H))$ and $Bpint(Bpcl(H)) = Bpcl(H) \cap Bint(Bcl(H))$,
(iii) $Bsint(H) = H \cap Bcl(Bint(H))$ and $Bsc\ell(H) = H \cup Bint(Bcl(H))$.

Lemma 3.3. The following hold for subsets $H$ of a space $(X, \tau)$,

(i) $Bcl(Bint(H)) = Bcl(int(H))$, and
(ii) $Bint(Bcl(H)) = Bint(Bcl(H))$.

Theorem 3.1. Let $(X, \tau)$ be a ts and $A \subset X$. Then the following statements are equivalent

(i) $A$ is a $B\gamma_0$ set,
(ii) $A = Bsint(A) \cup Bpint(A)$.

Proof. (i) $\Rightarrow$ (ii): Let $A$ be an $B\gamma_0$ set. Then $A \subseteq Bint(Bcl(A)) \cup Bcl(Bint(A))$, hence by Proposition 3.3 and Lemma 3.2

$$Bsint(A) \cup Bpint(A) = (A \cap Bint(Bcl(A))) \cup (A \cup Bcl(Bint(A)))$$

$$= A \cap (Bint(Bcl(A)) \cup Bcl(Bint(A))) = A.$$

(ii) $\Rightarrow$ (i): Suppose that $A = Bsint(A) \cup Bpint(A)$. Then by Proposition 3.3 and Lemma 3.2

$$A = (A \cap Bint(Bcl(A))) \cup (A \cup Bcl(Bint(A))) \subset Bint(Bcl(A) \cup Bcl(Bint(A))).$$

Therefore, $A$ is $B\gamma_0$. \qed

Proposition 3.4. Let $(X, \tau)$ be a ts and $A \subset X$. Then the following statements are equivalent:

(i) $A$ is an $B\gamma_c$ set,
(ii) $A = Bsc\ell(A) \cap Bpcl(A)$. 

**Theorem 3.2.** Let $A$ be a subset of a space $(X, \tau)$. Then

(i) $B_{\gamma cl}(A) = B_{scl}(A) \cap B_{pcl}(A)$,

(ii) $B_{\gamma int}(A) = B_{sint}(A) \cup B_{pint}(A)$.

**Proof.** (i) It is easy to see that $B_{\gamma cl}(A) = B_{scl}(A) \cap B_{pcl}(A)$. Also, $B_{scl}(A) \cap B_{pcl}(A) = (A \cup B_{int}(B_{cl}(A))) \cap A \cup B_{cl}(B_{int}(A))) = A \cup (B_{int}(B_{cl}(A)) \cap B_{cl}(B_{int}(A)))$. But, $B_{\gamma cl}(A)$ is $B_{\gamma c}$, hence $B_{\gamma cl}(A) \supset B_{cl}(B_{\gamma cl}(A)) \cap B_{cl}(B_{\gamma cl}(A))) \supset B_{cl}(B_{\gamma cl}(A)) \cap B_{cl}(B_{\gamma cl}(A)))$. Thus $A \cup (B_{int}(B_{cl}(A)) \cap B_{cl}(B_{int}(A))) \supset A \cup B_{\gamma cl}(A) = B_{\gamma cl}(A)$, therefore, $B_{scl}(A) \cap B_{pcl}(A) \subset B_{\gamma cl}(A)$. So, $B_{\gamma cl}(A) = B_{scl}(A) \cap B_{pcl}(A)$.

(ii) Similar to (i). □

**Theorem 3.3.** Let $A$ be a subset of a space $(x, \tau)$. Then

(i) $A$ is an $B_{\gamma o}$ set (resp. $B so$ set, $B_{po}$ set and $B_{\beta o}$ set) iff $A = B_{\gamma int}(A)$ (resp. $A = B_{sint}(A)$, $A = B_{pint}(A)$ and $A = B_{\beta int}(A)$),

(ii) $A$ is an $B_{\gamma c}$ set (resp. $B_{so}$ set, $B_{pc}$ set and $B_{\beta c}$ set) iff $A = B_{\gamma cl}(A)$ (resp. $A = B_{scl}(A)$, $A = B_{pcl}(A)$ and $A = B_{\beta cl}(A)$).

**Proof.** (i) Let $A$ be an $B_{\gamma o}$ set. Then by Theorem 3.1, $A = B_{sint}(A) \cup B_{pint}(A)$ and by Theorem 3.2, we have $A = B_{\gamma int}(A)$. Conversely, let $A = B_{\gamma int}(A)$. Then by Theorem 3.2, $A = B_{sint}(A) \cup B_{pint}(A)$ and by Theorem 3.1, $A$ is $B_{\gamma o}$.

(ii) Similar to (i). □

**Theorem 3.4.** Let $A$ and $B$ be subsets of a space $(X, \tau)$. Then the following are hold

(i) $B_{\gamma cl}(X \setminus A) = X \setminus B_{\gamma int}(A)$.

(ii) $B_{\gamma int}(X \setminus A) = X \setminus B_{\gamma cl}(A)$.

(iii) If $A \subset B$, then $B_{\gamma cl}(A) \subseteq B_{\gamma cl}(B)$ and $B_{\gamma int}(A) \subseteq B_{\gamma int}(B)$.

(iv) $x \in B_{\gamma cl}(A)$ iff there exists an $B_{\gamma o}$ set $U$ and $x \in U$ such that $U \cap A \neq \phi$.

(v) $B_{\gamma cl}(B_{\gamma cl}(A)) = B_{\gamma cl}(A)$ and $B_{\gamma int}(B_{\gamma int}(A)) = B_{\gamma int}(A)$.

(vi) $B_{\gamma cl}(A) \cup B_{\gamma cl}(B) \subset B_{\gamma cl}(A \cup B)$ and $B_{\gamma int}(A) \cup B_{\gamma int}(B) \subset B_{\gamma int}(A \cup B)$.

(vii) $B_{\gamma int}(A \cap B) \subset B_{\gamma int}(A) \cap B_{\gamma int}(B)$ and $B_{\gamma cl}(A \cap B) \subset v - cl(A) \cap B_{\gamma cl}(B)$.
Proof. (i) Since $(X \setminus A) \subseteq X$, by Theorem 3.3, $B\gamma cl(X \setminus A) = BscI(X \setminus A) \cap Bpcl(X \setminus A)$ and by Proposition 3.3,

$$B\gamma cl(X \setminus A) = (X \setminus BsI(A)) \cap (X \setminus Bpint(A)) = X \setminus (BsI(A) \cup Bpint(A)),$$

hence by Theorem 3.3, $B\gamma cl(X \setminus A) = X \setminus B\gamma int(A)$.

(ii) Similar to (i).

(iii) Since, $B\gamma cl(A) = BscI(A) \cap Bpcl(A)$ and $A \subseteq B,$

$$B\gamma cl(A) = BscI(A) \cap Bpcl(A) \subseteq BscI(B) \cap Bpcl(B) = B\gamma cl(B).$$

(iv) Since, $B\gamma cl(B\gamma cl(A)) = BscI(B\gamma cl(A)) \cap Bpcl(B\gamma cl(A))$, by Theorem 3.3 we have:

$$BscI(BscI(A) \cap Bpcl(A)) \cap Bpcl(BscI(A) \cap Bpcl(A)) \subseteq (BscI(A) \cap BscI(Bpcl(A))) \cap (Bpcl(BscI(A) \cap Bpcl(A))) = BscI(A) \cap Bpcl(A) = B\gamma cl(A),$$

hence $B\gamma cl(B\gamma cl(A)) \subseteq B\gamma cl(A)$. But, $B\gamma cl(A) \subseteq B\gamma cl(B\gamma cl(A))$. Therefore, $B\gamma cl(B\gamma cl(A)) = B\gamma cl(A)$.

(iv), (vi) and (vii) are obvious. \qed

Remark 3.4. The inclusion relation in part (vi) and (vii) of the above theorem cannot be replaced by equality as shown by the following example.

Example 2. Let $X = \{a, b, c, d, e\}$, with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ a non-open set $B = \{c, d\}$. Then

(i) If $A = \{a, b\}, B = \{a, c\}$ and $A \cup B = \{a, b, c\}$, then $B\gamma cl(A) = A$, $B\gamma cl(B) = B$ and $B\gamma cl(A \cup B) = X$. Hence, $B\gamma cl(A \cup B) \subseteq B\gamma cl(A) \cup B\gamma cl(B)$.

(ii) If $A = \{a, b\}, C = \{a, c\}$ and $A \cap B = \{b\}$, then $B\gamma cl(A) = A$, $B\gamma cl(C) = C$ and $B\gamma cl(A \cap C) = X$. Therefore, $B\gamma cl(A \cap B) \subseteq B\gamma cl(A) \cap B\gamma cl(C)$.

(iii) If $D = \{a, d\}, C = \{b, d\}$ and $D \cup C = \{a, b, d\}$, then $B\gamma int(D) = \phi$, $B\gamma int(C) = \{B\}$ and $B\gamma int(D \cup C) = \{a, b, d\}$. So, $B\gamma int(D \cup C) \subseteq B\gamma int(D) \cup B\gamma int(C)$.

Lemma 3.4. Let $A$ be a subset of a space $(X, \tau)$. Then following statement hold:

$$Bpint(Bpcl(A)) = Bpcl(A) \cap Bint(Bcl(A)) \text{ and } Bpcl(Bpint(A)) = Bpint(A) \cup Bcl(Bint(A)).$$

Proposition 3.5. Let $A$ be a subset of a space $(X, \tau)$. Then:
(i) \( B_{\gamma cl}(A) = A \cup Bpint(Bpcl(A)) \).
(ii) \( B_{\gamma int}(A) = A \cap Bpcl(Bpint(A)) \).

Proof. (i) By Lemma 3.2,

\[
A \cup Bpint(Bpcl(A)) = A \cup (Bpcl(A) \cap Bint(Bcl(A))) \\
= (A \cup Bpcl(A)) \cap (A \cup Bint(Bcl(A))) \\
= Bpcl(A) \cap Bscl(A) \\
= B_{\gamma cl}(A).
\]

(ii) Similar to (i). \( \square \)

**Theorem 3.5.** Let \( A \) be a subset of a space \((X, \tau)\). Then the following are equivalent:

(i) \( A \) is an \( B_{\gamma o} \) set,
(ii) \( A \subseteq Bpcl(Bpint(A)) \),
(iii) \( Bpcl(A) = Bpcl(Bpint(A)) \).

Proof. (i)\( \Rightarrow \) (ii): Let \( A \) be an \( B_{\gamma o} \) set. Then by Theorem 3.3, \( A = B_{\gamma int}(A) \) and by Proposition 3.5, \( A = A \cap Bpcl(Bpint(A)) \) and hence, \( A \subseteq Bpcl(Bpint(A)) \).

(ii)\( \Rightarrow \) (i): Let \( A \subseteq Bpcl(Bpint(A)) \). Then by Proposition 3.5, \( A \subseteq A \cap Bpcl(Bpint(A)) = B_{\gamma int}(A) \). So, \( A \subseteq B_{\gamma int}(A) \). Then \( A = B_{\gamma int}(A) \) and hence, \( A \) is \( B_{\gamma o} \).

(ii)\( \Rightarrow \) (iii): Let \( A \subseteq Bpcl(Bpint(A)) \). Then \( Bpcl(A) \subseteq Bpcl(Bpint(A)) \) and hence, \( Bpcl(A) = Bpcl(Bpint(A)) \).

(iii)\( \Rightarrow \) (ii): Obvious. \( \square \)

**Theorem 3.6.** Let \( A \) be a subset of a ts \( X \). Then the following are equivalent:

(i) \( A \) is an \( B_{\gamma c} \) set,
(ii) \( Bpint(Bpcl(A)) \subseteq A \),
(iii) \( Bpcl(A) = Bpint(Bpcl(A)) \).

**Theorem 3.7.** If \( A \) is a subset of an extremally disconnected space \((X, \tau)\). Then the following are equivalent:

(i) \( A \) is an open set.
(ii) \( A \) is \( B_{\gamma o} \) and locally closed.

Proof. (i)\( \Rightarrow \) (ii): Obvious from Definitions 2.2 and 3.1.
(ii)⇒(i): Let $A$ be an $B\gamma o$ and a locally closed subset of $X$. Then $A = U \cap cl(A)$ and $A \subseteq Bcl(Bint(A))U Bint(Bcl(A))$, hence:

$$A \subseteq U \cap [Bcl(Bint(A)) \cup Bint(Bcl(A))]$$

$$\subseteq (U \cap Bcl(Bint(A))) \cup Bint(U \cap Bcl(A))$$

$$\subseteq (U \cap Bcl(Bint(A))) \cup Bint(U \cap Bcl(A))$$

$$\subseteq (U \cap Bint(Bcl(A))) \cup Bint(U \cap Bcl(A))(since X is E.D)$$

$$\subseteq Bint(U \cap Bcl(A)) \cup Bint(U \cap Bcl(A)) = Bint(A) \cup Bint(A).$$

Hence $A$ is open. □

**Definition 3.3.** A subset $A$ of a ts $(X, \tau)$ is said to be

(i) locally $B\gamma c$ if $A = U \cap F$ for each $U \in \tau$ and $F \in B\gamma C(X)$.
(ii) locally $Bsc$ if $A = U \cap F$ for each $U \in \tau$ and $F \in BSC(X)$.
(iii) locally $Bpc$ if $A = U \cap F$ for each $U \in \tau$ and $F \in BPC(X)$.
(iv) locally $B\beta c$ if $A = U \cap F$ for each $U \in \tau$ and $F \in B\beta C(X)$.

**Theorem 3.8.** Let $H$ be a subset of a space $X$. Then $H$ is locally $B\gamma c$ (resp. locally $Bsc$, locally $Bpc$ and locally $B\beta c$) iff $H = U \cap B\gamma cl(H)$ (resp. $H = U \cap BS-cl(H)$, $H = U \cap BP-cl(H)$, and $H = U \cap B\beta -cl(H)$).

**Proof.** Since $H$ is a locally $B\gamma c$ set, $H = U \cap F$, for each $U \in \tau$ and $F \in B\gamma C(X)$, hence $H \subseteq B\gamma cl(H) \subseteq B\gamma cl(F) = F$, thus $H \subseteq U \cap B\gamma cl(H) \subseteq U \cap B\gamma cl(F) = H$. Therefore $H = U \cap B\gamma cl(H)$. Conversely, since $B\gamma cl(H)$ is $B\gamma c$ and $H = U \cap B\gamma cl(H)$, then $H$ is locally $B\gamma c$. □

**Theorem 3.9.** Let $A$ be a locally $B\gamma c$ subset of a space $(X, \tau)$. Then the following statements are hold:

(i) $B\gamma cl(A) \setminus A$ is an $B\gamma c$ set.
(ii) $A \cup (X \setminus B\gamma cl(A))$ is an $B\gamma o$.
(iii) $A \subseteq B\gamma int(A \cup (X \setminus B\gamma cl(A)))$. 
Proof. (i) If $A$ is locally $B\gamma c$ set, then there exist an open set $U$ such that $A = U \cap B\gamma cl(A)$. Hence,
\[
B\gamma cl(A) \setminus A = B\gamma cl(A) \setminus (U \cup B\gamma cl(A))
\]
\[
= B\gamma cl(A) \cap [X \setminus (U \cap B\gamma cl(A))]
\]
\[
= B\gamma cl(A) \cap [(X \setminus U) \cup (X \setminus B\gamma cl(A))]
\]
\[
= B\gamma cl(A) \cap (X \setminus U)
\]
which is $B\gamma c$.

(ii) From (i), $B\gamma cl(A) \setminus A$ is $B\gamma c$, then $X \setminus [B\gamma cl(A) \setminus A]$ is an $B\gamma o$ set and $X \setminus [B\gamma cl(A) \setminus A] = X \setminus B\gamma cl(A) \cup (X \setminus A) = A \cup (X \setminus B\gamma cl(A))$, hence $A \cup (X \setminus B\gamma cl(A))$ is $B\gamma o$.

(iii) It is clear that, $A \subseteq (A \cup (X \setminus B\gamma cl(A))) = B\gamma int(A \cup (X \setminus B\gamma cl(A)))$. $\Box$

Conclusion

In this paper is to introduced and studied the notion of $B\gamma$-open, $B$ pre open, $B$ semi open and $B\beta$-open sets. Some characterization of these notions are discussed.

References


PG and Research Department of Mathematics
Arignar Anna Government Arts College
Namakkal, Tamilnadu-637 002
Mathematics Section (FEAT)
Annamalai University
Annamalainagar, Tamil Nadu, India-608 002
E-mail address: viji_lakshmi80@rediffmail.com

PG and Research Department of Mathematics
Government Arts College (Autonomous)
Karur - 639 005
Department of Mathematics
Annamalai University
Annamalai Nagar - 608 002, Tamil Nadu, India
E-mail address: avmaths@gmail.com