A NOTE ON NANO $g^\#_\alpha$-CLOSED MAPS IN NANO TOPOLOGICAL SPACES

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ABSTRACT. The point of this article is to show separation axioms of Nano $g^\#_\alpha$ closed sets in nano topological space. We moreover present and explore nano $g^\#_\alpha$-closed maps and additionally consider their principal properties.

1. INTRODUCTION AND PRELIMINARIES

In [6], the specialists introduced a nano topological space with regard to a subset $X$ of an universe which is characterized in terms of lower approximation, upper approximation and boundary region. He has also presented nano closed sets (in brief N-CS) and nano open sets (in brief N-OS). In [8], the experts presented the concept of $g^\#_\alpha$-closed sets to explore a few topological properties. V. Kokilavani et al [5] introduced $Ng^\#_\alpha$-closed sets in nano topological space (in brief nts). The essential expected of this paper is to present separation axioms of nano $g^\#_\alpha$ closed sets. We likewise present the concept of $Ng^\#_\alpha$-closed maps and study their properties in nts.

Definition 1.1. A subset $H$ of a nts $(U, \tau_R(G))$ is called

$\begin{align*}
(1) & \text{ N}_a-CS [6] \text{ if } N\text{int}(N\text{cl}(N\text{int}(H))) \subseteq H. \\
(2) & \text{ N}_g-CS [1] \text{ if } N\text{cl}(H) \subseteq G, \text{ whenever } H \subseteq G \text{ and } G \text{ is N-OS.} \\
(3) & \text{ N}_g\text{s-CS [2]} \text{ if } N\text{sc}(H) \subseteq G \text{ whenever } H \subseteq G, \text{ G is N-OS.}
\end{align*}$

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(4) \( N\alpha g-CS \) [12] if \( \text{N}\alpha \text{cl}(H) \subseteq G \) whenever \( H \subseteq G \) and \( G \) is \( N\)-OS.
(5) \( Ng^*\)-CS [10] if \( \text{N}\text{cl}(H) \subseteq G \) whenever \( H \subseteq G \) and \( G \) is \( Ng \)-OS.
(6) \( N\alpha g\)-CS [9] if \( \text{N}\text{cl}(H) \subseteq G \) whenever \( H \subseteq G \) and \( G \) is \( N\alpha g \)-OS.

The complements of the above sets are called their particular \( N\)-OS.

**Definition 1.2.** A subset \( H \) of \((U, \tau)\) is called nano \( g^#\alpha\)-closed set [5] (in brief \( Ng^#\alpha\)-CS) if \( \text{N}\alpha \text{cl}(H) \subseteq V \) whenever \( H \subseteq V \) and \( V \) is \( Ng\)-OS in \((U, \tau)\). The complements of \( Ng^#\alpha\)-CS is \( Ng^#\alpha\)-OS in \((U, \tau)\).

**Definition 1.3.** Let \((U, \tau)\) and \((V, \sigma)\) be nts. Then the map

\[ f : (U, \tau) \to (V, \sigma) \]

is called:

(1) nano continuous (in brief \( N\)-continuous) [13] if \( f^{-1}(j) \) is a \( N\)-OS (resp \( N\)-CS) in \((U, \tau)\), for each \( N\)-OS (resp \( N\)-CS) \( j \) in \((V, \sigma)\).
(2) \( N\alpha\)-continuous (in brief \( N\alpha\)-continuous) [7] if \( f^{-1}(j) \) is a \( N\alpha\)-OS (resp \( N\alpha\)-CS) in \((U, \tau)\), for each \( N\)-OS (resp \( N\)-CS) \( j \) in \((V, \sigma)\).
(3) \( Ng\)-continuous (in brief \( Ng\)-continuous) [4] if \( f^{-1}(j) \) is a \( Ng\)-OS (resp \( Ng\)-CS) in \((U, \tau)\), for each \( N\)-OS (resp \( N\)-CS) \( j \) in \((V, \sigma)\).
(4) \( N\alpha g\)-continuous (in brief \( N\alpha g\)-continuous) [7] if \( f^{-1}(j) \) is a \( N\alpha g\)-OS (resp \( N\alpha g\)-CS) in \((U, \tau)\), for each \( N\)-OS (resp \( N\)-CS) \( j \) in \((V, \sigma)\).
(5) \( Ngs\)-continuous (in brief \( Ngs\)-continuous) [3] if \( f^{-1}(j) \) is a \( Ngs\)-OS (resp \( Ngs\)-CS) in \((U, \tau)\), for each \( N\)-OS (resp \( N\)-CS) \( j \) in \((V, \sigma)\).
(6) \( N\alpha g\)-continuous (in brief \( N\alpha g\)-continuous) [9] if \( f^{-1}(j) \) is a \( N\alpha g\)-OS (resp \( N\alpha g\)-CS) in \((U, \tau)\), for each \( N\)-OS (resp \( N\)-CS) \( j \) in \((V, \sigma)\).
(7) \( Ng^*\)-continuous (in brief \( Ng^*\)-continuous) [11] if \( f^{-1}(j) \) is a \( Ng^*\)-OS (resp \( Ng^*\)-CS) in \((U, \tau)\), for each \( N\)-OS (resp \( N\)-CS) \( j \) in \((V, \sigma)\).

**Definition 1.4.** A map \( c : (U, \tau) \to (V, \sigma) \) is said to be a \( Ng^#\alpha\)-continuous [5] if \( c^{-1}(j) \) is a \( Ng^#\alpha\)-closed set in \((U, \tau)\) for each nano closed set \( j \) in \((V, \sigma)\).

**Definition 1.5.** A nts \((U, \tau)\) is said to be nano \( T_{g\alpha g} \)-space [9] (in short \( NT_{g\alpha g} \)-space) if each \( Ng\alpha g\)-CS in it is \( N\)-CS.
2. Separation Axioms in terms $Ng^\#\alpha$-closed set

**Definition 2.1.** A nts $(U, \tau_R(G))$ is said to be

(i) nano $T_{1/2}^*$-space (in brief $NT_{1/2}^*$-space) if each $Ng^\#\alpha$-CS in it is N-CS.

(ii) nano $T_{g^\#\alpha}$-space (in brief $NT_{g^\#\alpha}$-space) if each $Ng^\#\alpha$-CS in it is N-CS.

(iii) nano $\alpha^#T_{1/2}$-space (in brief $Na^#T_{1/2}$-space) if each $Ng^\#\alpha$-CS in it is N\alpha-CS.

(iv) nano $^*T_{g^\#\alpha}$-space (in brief $NT_{g^\#\alpha}$-space) if each $Ng^\#\alpha$-CS in it is $Ng^*$-CS.

(v) nano $^*^*T_{g^\#\alpha}$-space (in brief $NT_{g^\#\alpha}$-space) if each $Ng^\#\alpha$-CS in it is $Ng_{g\#\alpha}$-CS.

(vi) nano $\alpha^#T_k$-space (in brief $Na^#T_k$-space) if each $Na^#g$-CS in it is $Ng^\#\alpha$-CS.

(vii) nano $\alpha^##T_k$-space (in brief $Na^##T_k$-space) if each $Ng_{gs}$-CS in it is $Ng^\#\alpha$-CS.

**Theorem 2.1.** In a nts $(U, \tau_R(G))$, every nano $T_{g^\#\alpha}$-space are nano $T_{1/2}^*$-space, $T_{g_{g\#}}$-space, nano $^*T_{g^\#\alpha}$-space and nano $^*^*T_{g^\#}$-space.

**Proof.** Let $(U, \tau_R(G))$ be a nano $T_{g^\#\alpha}$-space and let $D$ be a nano $g^\#$-CS (resp $Ng_{g\#}$-CS, $Ng^\#\alpha$-CS) in $(U, \tau_R(G))$. Since every $Ng^\#$-closed set (resp $Ng_{g\#}$-CS) is $Ng^\#\alpha$-CS [5] we have $D$ is a nano $g^\#\alpha$-CS. Moreover $U$ is a nano $T_{g^\#\alpha}$-space, then $D$ is a N-CS in $U$. Consequently, $(U, \tau_R(G))$ is a nano $T_{1/2}^*$-space (resp $T_{g_{g\#}}$-space, nano $^*T_{g^\#\alpha}$-space and nano $^*^*T_{g^\#}$-space).

The above theorem require not be true by the following illustration.

**Example 1.** Let $U = \{\alpha, \beta, \gamma, \delta\}$ with $U/R = \{\{\alpha, \beta\}, \{\gamma\}, \{\delta\}\}$ and $G = \{\alpha, \gamma, \delta\}$. Let $\tau_{R}(G) = \{\emptyset, \{\alpha, \beta\}, \{\gamma, \delta\}, U\}$ be the nts. Then $(U, \tau_{R}(G))$ is a nano $T_{1/2}^*$-space and nano $T_{g_{g\#}}$-space but not nano $T_{g^\#\alpha}$-space. Let $U = \{a_1, a_2, a_3, a_4\}$ with $U/R = \{\{a_1\}, \{a_2, a_3\}, \{a_4\}\}$ and $G = \{a_2, a_4\}$. Let $\tau_{R}(G) = \{\emptyset, \{a_4\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}, U\}$ be the nts. $Ng^\#\alpha$-closed sets $= Ng^\#$-closed sets $= \emptyset, \{a_1\}, \{a_1, a_2\}, \{a_1, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}, \{a_1, a_3, a_4\}\}$. Then $(U, \tau_{R}(G))$ is a nano $^*T_{g^\#\alpha}$-space but not nano $T_{g^\#\alpha}$-space. Let $U = \{b, e, d\}$ with $U/R = \{\{b, e\}, \{d\}\}$ and $G = \{b, e\}$. Let $\tau_{R}(G) = \{\emptyset, \{b, e\}, U\}$ be the nts. Then $(U, \tau_{R}(G))$ is a nano $^*^*T_{g^\#\alpha}$-space but not nano $T_{g^\#\alpha}$-space.

**Remark 2.1.** Every $Na^#g$-closed (resp. $Na^#g$-open) set is $Ng_{g}$-closed (resp. $Ng_{g}$-open) set.
Theorem 2.2. Every nano $\alpha\#T_k$-space is nano $\alpha\#T_k$-space.

**Proof.** The proof is obvious. □

The converse of the above theorem need not be true shown by the following example.

**Example 2.** Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{2, 3\}, \{4\}\}$ and $G = \{2, 4\}$. Let $\tau_R(G) = \emptyset, \{4\}, \{2, 3\}, \{2, 3, 4\}, U$ be the nts. $N\alpha g$-closed sets $= \{\emptyset, \{1\}, \{2, 3\}, \{1, 4\}, \{1, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$. Then $(U, \tau_R(G))$ is a nano $\alpha\#T_k$-space but not nano $\alpha\#T_k$-space.

**Theorem 2.3.** If a nts $(U, \tau_R(G))$ is nano $T_g\#_\alpha$-space, then every singleton of $U$ is either nano g-CS or N-OS. But not conversely.

**Proof.** Let $x \in U$ and $(U, \tau_R(G))$ be a nano $T_g\#_\alpha$-space. Suppose $\{x\}$ is not a nano g-CS of $(U, \tau_R(G))$, then $X \{x\}$ is not nano g-OS. Then $X$ is the only nano g-OS containing $X \{x\}$. Then $X \{x\}$ is a $N\alpha g$-$\alpha$-CS of $(U, \tau_R(G))$. Since $(U, \tau_R(G))$ is a nano $T_g\#_\alpha$-space, $X \{x\}$ is N-CS, which implies $x$ is N-OS in $(U, \tau_R(G))$. □

**Theorem 2.4.** If a nts $(U, \tau_R(G))$ is nano $\alpha\#T_{1/2}$-space, then every singleton of $U$ is either nano g-CS or nano $\alpha$-OS. But not conversely.

**Proof.** The proof is obvious. □

**Theorem 2.5.** In a nts $(U, \tau_R(G))$, the following conditions are equivalent:

(i) $(U, \tau_R(G))$ is both $N^*T_g\#_\alpha$-space and $NT_{1/2}^*$-space.

(ii) $(U, \tau_R(G))$ is $NT_g\#_\alpha$-space.

**Proof.** (i) $\Rightarrow$ (ii): Let $C$ be a $N\alpha g$-$\alpha$-CS in $(U, \tau_R(G))$. Since $(U, \tau_R(G))$ is a $Ng\#\alpha$-CS, $C$ is $N\alpha g$-CS. Moreover since $(U, \tau_R(G))$ is a $NT_{1/2}^*$, $C$ is N-CS. Consequently, $(U, \tau_R(G))$ is a $NT_g\#\alpha$-space.

(ii) $\Rightarrow$ (i): Let $L$ be a N-CS and $M$ be a $N\alpha g$-$\alpha$-CS in $(U, \tau_R(G))$. Moreover each $Ng\#CS$ is $Ng\#\alpha$-CS in $(U, \tau_R(G))$, we have $L \subseteq M$. Moreover since $(U, \tau_R(G))$ is a $NT_g\#\alpha$-space, every $Ng\#\alpha$-CS is N-CS in $(U, \tau_R(G))$, (i,e) $M \subseteq L$. Hence $L = M$. Let $C$ be a $Ng\#CS$ in $(U, \tau_R(G))$. Since each N-CS is $Ng\#CS$ [11] and each $Ng\#CS$ is $Ng\#\alpha$-CS [5], we have $L \subseteq M \subseteq C$. But $L = M \implies M \subseteq C$. Thus $L = M = C$. (i,e) every $Ng\#\alpha$-CS is $Ng\#CS$ and every $Ng\#CS$ is N-CS in $(U, \tau_R(G))$. Consequently, $(U, \tau_R(G))$ is both $N^*T_g\#_\alpha$-space and $NT_{1/2}^*$-space. □
Theorem 2.6. In a nts \((U, \tau_R(G))\), the taking after conditions are equivalent:

(i) \((U, \tau_R(G))\) is both \(N^{**}T_{g^\#}\alpha\)-space and \(NT_{g^\#}\alpha\)-space.

(ii) \((U, \tau_R(G))\) is \(NT_{g^\#}\alpha\)-space.

Proof. (i) \(\Rightarrow\) (ii): Let \(C\) be a \(Ng^\#\alpha\)-CS in \((U, \tau_R(G))\). Since \((U, \tau_R(G))\) is a \(N^{**}T_{g^\#}\alpha\)-space, \(C\) is \(Ng^\alpha\)-CS. Moreover since \((U, \tau_R(G))\) is a \(NT_{g^\#}\alpha\)-space, \(C\) is \(N\)-CS. Consequently, \((U, \tau_R(G))\) is a \(NT_{g^\#}\alpha\)-space.

(ii) \(\Rightarrow\) (i): Let \(L\) be a \(N\)-CS and \(M\) be a \(Ng^\#\alpha\)-CS in \((U, \tau_R(G))\). Since each \(Ng\alpha\)-CS is \(Ng^\#\alpha\)-CS in \((U, \tau_R(G))\), we have \(L \subseteq M\). Moreover since \((U, \tau_R(G))\) is a \(NT_{g^\#}\alpha\)-space, every \(Ng^\#\alpha\)-CS is \(N\)-CS in \((U, \tau_R(G))\), (i.e) \(M \subseteq L\). Hence \(L = M\). Let \(C\) be a \(Ng\alpha\)-CS in \((U, \tau_R(G))\). Since each \(N\)-CS is \(Ng\alpha\)-CS \([10]\) and every \(Ng\alpha\)-CS is \(Ng^\#\alpha\)-CS \([5]\), we have \(L \subseteq M \subseteq C\). But \(L = M\). Consequently, \((U, \tau_R(G))\) is both \(N^{**}T_{g^\#}\alpha\)-space and \(NT_{g^\#}\alpha\)-space.

\(\square\)

Theorem 2.7. Let \(f : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) be a \(Ng^\#\alpha\)-continuous function. If \((U, \tau_R(G))\) is a nano \(\alpha^\#\)\(T_{1/2}\)-space, then \(f\) is \(N\alpha\)-continuous.

Proof. Let \(C\) be a \(N\)-CS in \((V, \sigma_R(H))\). Since \(f\) is \(Ng^\#\alpha\)-continuous, \(f^{-1}(C)\) could be a \(Ng^\#\alpha\)-CS. Since \((U, \tau_R(G))\) is a nano \(\alpha^\#\)\(T_{1/2}\)-space, we have \(f^{-1}(C)\) is a \(N\alpha\)-CS in \((U, \tau_R(G))\). Consequently, \(f\) is \(N\alpha\)-continuous.

\(\square\)

Theorem 2.8. Let \(f : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) be a \(Ng^\#\alpha\)-continuous function. If \((U, \tau_R(G))\) is a \(NT_{g^\#}\alpha\)-space, then \(f\) is nano continuous.

Proof. The proof is similar.

\(\square\)

Theorem 2.9. Let \((U, \tau_R(G))\) and \((V, \sigma_R(H))\) be a nts and Let \(j : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) be a map then the following statements hold:

(i) If \(j\) is \(N\alpha\)-continuous function and \(N\alpha^\#\)\(T_{k}\)-space, then \(j\) is \(Ng^\#\alpha\)-continuous.

(ii) If \(j\) is \(Ng\alpha\)-continuous function and \(N\alpha^\#\)\(T_{k}\)-space, then \(j\) is \(Ng^\#\alpha\)-continuous.

Proof. (i) Let \(D\) be a \(N\)-CS in \((V, \sigma_R(H))\). Since \(j\) is \(N\alpha\)-continuous, \(j^{-1}(D)\) is \(N\alpha\)-CS. Since \((U, \tau_R(G))\) is a \(N\alpha^\#\)\(T_{k}\)-space, we have \(j^{-1}(D)\) is a \(Ng^\#\alpha\)-CS in \((U, \tau_R(G))\). Consequently, \(j\) is \(Ng^\#\alpha\)-continuous.

(ii) The proof is similar.
Theorem 2.10. Let \((U, \tau_R(G))\) and \((V, \sigma_R(H))\) be a nts and let \(f : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) be a map then the following statements hold:

(i) If \(f\) is \(Ng^{\#}\alpha\)-continuous and \((U, \tau_R(G))\) be \(N^*T_{g^*\alpha}\)-space, then \(f\) is \(Ng^{*}\)-continuous.

(ii) If \(f\) is \(Ng^{\#}\alpha\)-continuous and \((U, \tau_R(G))\) be \(N^{**}T_{g^*\alpha}\)-space, then \(f\) is \(Ng^{*\alpha}\)-continuous.

Proof. (i) Let \(C\) be a \(N\)-CS in \((V, \sigma_R(H))\). Since \(f\) is \(Ng^{\#}\alpha\)-continuous, \(f^{-1}(C)\) is \(Ng^{\#}\alpha\)-CS. Since \((U, \tau_R(G))\) is a \(N^{*}T_{g^*\alpha}\)-space, we have \(f^{-1}(C)\) is a \(Ng^{*}\)-CS in \((U, \tau_R(G))\). Consequently, \(f\) is \(Ng^{*}\)-continuous.

(ii) The proof is similar. \(\square\)

3. \(Ng^{\#}\alpha\)-closed maps

Definition 3.1. A function \(j : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) is called \(Ng^{\#}\alpha\)-open maps if for each nano open set \(E\) of \((U, \tau_R(G))\), its image \(j(E)\) is \(Ng^{\#}\alpha\)-open maps in \((V, \sigma_R(H))\).

Example 3. Let \(U = \{b_1, b_2, b_3, b_4\}\) be the universe with \(U/R = \{\{b_1\}, \{b_3\}, \{b_2, b_4\}\}\) and let \(G = \{b_1, b_2\}\). Then the N-OS are \(\{\emptyset, \{b_1\}, \{b_2, b_4\}, \{b_1, b_2, b_4\}, U\}\). The \(Ng^{\#}\alpha\)-CS are \(\emptyset, \{b_3\}, \{b_3, b_4\}, \{b_1, b_3\}, \{b_2, b_4\}, \{b_1, b_3, b_4\}, \{b_1, b_2, b_4\}, U\). Let \(V = \{a, b, c, d\}\) be the other universe with \(V/R = \{\{a\}, \{b, c\}, \{d\}\}\) and let \(H = \{b, d\}\). Then the N-OS are \(\emptyset, \{a\}, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, V\). Define the function \(j : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) as \(j(e) = b, j(f) = d, j(g) = a, j(h) = c\). Now the images of N-OS of \(U\) which are \(Ng^{\#}\alpha\)-OS in \(V\). Thus the function \(j\) is \(Ng^{\#}\alpha\)-open map.

Definition 3.2. A function \(j : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) is called \(Ng^{\#}\alpha\)-closed maps if for every \(N\)-CS \(E\) of \((U, \tau_R(G))\), its image \(j(E)\) is \(Ng^{\#}\alpha\)-closed maps in \((V, \sigma_R(H))\).

Example 4. In example 3, \(j(U) = V, j(\emptyset) = \emptyset, j(\{g\}) = \{a\}, j(\{e, g\}) = \{a, c\}, j(\{f, g, h\}) = \{a, c, d\}\) are the images of \(N\)-CS of \(U\) which are \(Ng^{\#}\alpha\)-closed maps in \(V\). Thus the function \(j\) is \(Ng^{\#}\alpha\)-closed map.

Theorem 3.1. A function \(j : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))\) is \(Ng^{\#}\alpha\)-closed function if and only if \(Ng^{\#\alpha\text{cl}}(f(C)) \subseteq f(\text{cl}(C))\) for each subset \(C\) of \((U, \tau_R(G))\).
Proof. Let \( j : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H)) \) be a \( Ng^\#\alpha \)-closed function and \( C \subseteq U \). Then \( Ncl(C) \) is N-CS in \( (U, \tau_R(G)) \) and hence \( j(Ncl(C)) \) is \( Ng^\#\alpha \)-closed function in \( (V, \sigma_R(H)) \). Since \( C \subseteq Ncl(C) \), it implies that \( j(C) \subseteq j(Ncl(C)) \). As \( Ng^\#\alpha cl(j(Ncl(C))) \) is the \( Ng^\#\alpha \)-CS containing \( j(C) \), it follows that
\[
Ng^\#\alpha cl(j(C)) \subseteq Ng^\#\alpha cl(j(Ncl(C))) \subseteq j(Ncl(C)).
\]
Conversely, let \( C \) be any N-CS in \( (U, \tau_R(G)) \). Then \( C = Ncl(C) \) and so \( j(C) = j(Ncl(C)) \subseteq Ng^\#\alpha cl(j(C)) \) by the given hypothesis. Also, it follows that \( j(C) \subseteq Ng^\#\alpha cl(j(C)) \). Hence \( j(C) = Ng^\#\alpha cl(j(C)) \). i.e., \( j(C) \) is \( Ng^\#\alpha \)-CS and hence \( j : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H)) \) is \( Ng^\#\alpha \)-closed function. \( \square \)

Theorem 3.2. A function \( f : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H)) \) is \( Ng^\#\alpha \)-CS if and only if for each subset \( Y \) of \( (V, \sigma_R(H)) \) and for each N-OS \( A \) of \( (U, \tau_R(G)) \) containing \( f^{-1}(Y) \), there is a \( Ng^\#\alpha \)-OS \( B \) of \( (V, \sigma_R(H)) \) such that \( Y \subseteq B \) and \( f^{-1}(B) \subseteq A \).

Proof. Let \( Y \) be the subset of \( (V, \sigma_R(H)) \) and \( A \) be a N-OS of \( (U, \tau_R(G)) \) such that \( f^{-1}(Y) \subseteq A \). Now \( V - f(U-A) \), say \( B \), is a \( Ng^\#\alpha \)-OS containing \( Y \) in \( V \) such that \( f^{-1}(B) \subseteq A \). Conversely, let \( F \) be a N-OS of \( U \), then \( f^{-1}(V - f(F)) \) \( U - F \) and \( U - F \) is N-OS. Now, there is a \( Ng^\#\alpha \)-OS \( B \) of \( (V, \sigma_R(H)) \) such that \( V - f(F) \subseteq B \) and \( f^{-1}(B) \subseteq U - F \). Hence \( F \subseteq U - f^{-1}(B) \) and thus \( V - B \subseteq f(F) \subseteq f(U - f^{-1}(B)) \subseteq V - B \) which implies \( f(F) = V - B \). Since \( V - B \) is \( Ng^\#\alpha \)-CS, \( f(F) \) is a \( Ng^\#\alpha \)-CS in \( (V, \sigma_R(H)) \) for each N-CS \( F \) in \( (U, \tau_R(G)) \). Hence \( f : (U, \tau_R(G)) \rightarrow (V, \sigma_R(H)) \) is a \( Ng^\#\alpha \)-closed function. \( \square \)

Remark 3.1. The following illustration shows that the composition of two \( Ng^\#\alpha \)-closed function require not be \( Ng^\#\alpha \)-closed.

Example 5. Let \( U = \{i, j, k, l\} \) be the universe with \( U/R = \{\{i\}, \{k\}, \{j, l\}\} \) and let \( G = \{i, j\} \). Then the N-OS are \( \{\emptyset, \{i\}, \{j, l\}, \{i, j, l\}, U\} \) and the N-CS are \( \{\emptyset, \{i\}, \{k\}, \{j, k, l\}, U\} \). The \( Ng^\#\alpha \)-CS are \( \{\emptyset, \{k\}, \{j, k, l\}, \{i, k\}, \{j, l\}, \{i, j, l\}, U\} \). The \( Ng^\#\alpha \)-OS are \( \{\emptyset, \{k\}, \{j\}, \{i\}, \{i, k\}, \{j, l\}, \{i, j, l\}, U\} \). Let \( V = \{a, b, c, d\} \) be another universe with \( V/R = \{\{a\}, \{b, c\}, \{d\}\} \) and let \( H = \{b, d\} \). Then the N-OS are \( \{\emptyset, \{d\}, \{b, c\}, \{b, c, d\}, V\} \) and the N-CS are \( \{\emptyset, \{a\}, \{a, d\}, \{a, b, c\}, V\} \). The \( Ng^\#\alpha \)-CS are \( \{\emptyset, \{a\}, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, V\} \) and \( Ng^\#\alpha \)-OS are \( \{\emptyset, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, V\} \).

Also, \( W = \{t, u, v\} \) with \( W/R = \{\{t, u\}, \{v\}\} \) and \( I = \{t, u\} \). Let \( \emptyset, \{t, u\}, W \) be the N-OS and \( \{\emptyset, \{v\}, W\} \) be the N-CS. The \( Ng^\#\alpha \)-CS are \( \{\emptyset, \{v\}, \{u, v\}, \} \).
\{t, v\}, W\} and the \(N^*_\alpha\)-OS are \(\emptyset, \{t\}, \{u\}, \{t, u\}, W\). Define the function \(f : (U, \tau_R(G)) \to (V, \sigma_R(H))\) as \(f(i) = b, f(j) = d, f(k) = a, f(l) = c\). Also define the map \(h : (V, \sigma_R(H)) \to (W, \eta_R(I))\) be \(h(a) = t = h(d), h(b) = u, h(c) = v\). Then both \(f\) and \(h\) are \(N^*_\alpha\)-closed functions but their composition \(h \circ f : (U, \tau_R(G)) \to (W, \eta_R(I))\) is not \(N^*_\alpha\)-closed functions since for the N-CS \(\{j, k, l\}\) in \((U, \tau_R(G))\), \(h \circ f(\{j, k, l\}) = h[f(\{j, k, l\})] = h[\{a, c, d\}] = \{t, v\}\) is not \(N^*_\alpha\)-CS in \((W, \eta_R(I))\). Consequently, the composition of two \(N^*_\alpha\)-closed functions require not be \(N^*_\alpha\)-CS.

**Theorem 3.3.** If \(f : (U, \tau_R(G)) \to (V, \sigma_R(H))\) be a nano closed map and \(h : (V, \sigma_R(H)) \to (W, \eta_R(I))\) be \(N^*_\alpha\)-closed mapping then their composition \(h \circ f : (U, \tau_R(G)) \to (W, \eta_R(I))\) is \(N^*_\alpha\)-closed mapping.

**Proof.** Let \(C\) be a N-CS in \((U, \tau_R(G))\). Then \(f(C)\) is nano closed in \((V, \sigma_R(H))\). Then \(h \circ f(C) = h(f(C))\) is \(N^*_\alpha\)-closed since \(h : (V, \sigma_R(H)) \to (W, \eta_R(I))\) is \(N^*_\alpha\)-closed map. Hence their composition is \(N^*_\alpha\)-closed mapping. \(\square\)

**Theorem 3.4.** If \(f : (U, \tau_R(G)) \to (V, \sigma_R(H))\) and \(h : (V, \sigma_R(H)) \to (W, \eta_R(I))\) be two mappings such that their composition \(h \circ f : (U, \tau_R(G)) \to (W, \eta_R(I))\) is \(N^*_\alpha\)-closed mapping. If \(f\) is nano continuous and surjective then \(h\) is \(N^*_\alpha\)-closed.

**Proof.** Let \(D\) be a N-CS in \((V, \sigma_R(H))\). Since \(f : (U, \tau_R(G)) \to (V, \sigma_R(H))\) is nano continuous, it follows that \(f^{-1}(D)\) is nano closed in \((U, \tau_R(G))\). Since \(h \circ f : (U, \tau_R(G)) \to (W, \eta_R(I))\) is \(N^*_\alpha\)-closed mapping, \((h \circ f)[f^{-1}(D)]\) is \(N^*_\alpha\)-closed in \((W, \eta_R(I))\). i.e., \(h(D)\) is \(N^*_\alpha\)-closed in \((W, \eta_R(I))\) as the function \(f : (U, \tau_R(G)) \to (V, \sigma_R(H))\) is surjective. Hence the image of a nano closed set in \((V, \sigma_R(H))\) is \(N^*_\alpha\)-closed in \((W, \eta_R(I))\). Thus \(h : (V, \sigma_R(H)) \to (W, \eta_R(I))\) is \(N^*_\alpha\)-closed. \(\square\)

**Theorem 3.5.** Let \((U, \tau_R(G))\) and \((V, \sigma_R(H))\) be any two nts. Let \(j : (U, \tau_R(G)) \to (V, \sigma_R(H))\) be nano closed map. Then \(j\) is \(N^*_\alpha\)-closed map but not conversely.

**Proof.** Let \(j : (U, \tau_R(G)) \to (V, \sigma_R(H))\) be a nano closed map. Let \(E\) be a N-CS in nts \((U, \tau_R(G))\). Then the image under the map \(j\) is nano closed in the nts \((V, \sigma_R(H))\). Since every N-CS is \(N^*_\alpha\)-closed set. \(j(E)\) is \(N^*_\alpha\)-CS. Hence \(j\) is \(N^*_\alpha\)-closed. \(\square\)

The subsequent illustration shows that the reverse implication is not true.
Example 6. In example 3, the set $\{e, g\}$ is $N_g^\#\alpha$-closed map but it is not nano closed map.

4. Conclusion

In this article we examined separation axioms of $N_g^\#\alpha$-CS and $N_g^\#\alpha$-homeomorphism. Additionally we analyzed their fundamental properties. In future it makes a difference to apply the concept in mappings and compactness.

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