ON CONTRA SOFT $A_R S$ CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT. In this paper we introduce soft $A_R S$ closed on soft topological spaces and study some of their properties. We also investigate the concepts of contra soft $A_R S$ closed mappings, contra soft $A_R S$ open mappings and also discuss their relationship with other soft mappings. Counter examples are given to show the non coincidence of these functions.

1. INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. Molodtsov, see [1] shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. In 2010 Muhammad Shabir and Munazza Naz used soft sets to define a topology namely soft topology. soft generalized closed set was introduced by K. Kannan in 2012. The investigation of generalized closed sets has led to several new and interesting concepts like new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. In this paper we defined soft ARS - closed mapping, soft $A_R S$ - open mapping and a detailed study of some of its properties in soft

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topological spaces. With the help of counter examples, we show that the non coincidence of these various types of mappings.

2. Preliminaries

Throughout this paper \((X, \tau, E)\) or \(\tilde{X}\) denotes the soft topological spaces. For a subset \((A, E)\) of \(\tilde{X}\), the closure, the interior and the complement of \((A, E)\) are denoted by \(cl(A, E)\), \(int(A, E)\) and \((A, E)^C\) respectively. We recall some basic definitions that are used in the sequel.

**Definition 2.1.** Let \((X, \tau, E)\) be a soft topological space. A soft set \((F, E)\) is called soft \(A_{RS}\) closed set if \(\beta cl(F, E) \subseteq (U, E)\) and \((U, E)\) is soft \(\omega\)-open.

The set of all soft \(A_{RS}\) closed sets is denoted by \(SA_{RS}(X)\).

**Definition 2.2.** A map \(f : (X, \tau, E) \to (Y, \sigma, K)\) is said to be a soft \(A_{RS}\) continuous if inverse image of every soft closed set in \((Y, \sigma, K)\) is soft \(A_{RS}\) closed in \((X, \tau, E)\).

**Definition 2.3.** A map \(f : (X, \tau, E) \to (Y, \sigma, K)\) is said to be soft \(A_{RS}\) open map if image of each soft open set in \((X, \tau, E)\) is soft \(A_{RS}\) open in \((Y, \sigma, K)\).

**Definition 2.4.** A map \(f : (X, \tau, E) \to (Y, \sigma, K)\) is said to be soft slightly \(A_{RS}\) continuous if the inverse image of every soft closed set in \((Y, \sigma, K)\) is soft \(A_{RS}\) open in \((X, \tau, E)\).

**Proposition 2.1.**

1. If the map \(f : (X, \tau, E) \to (Y, \sigma, K)\) is soft continuous (or soft semi continuous or soft \(\alpha\)-continuous or soft \(JP\)-continuous) then it is a soft \(A_{RS}\) continuous.

2. If the map \(f : (X, \tau, E) \to (Y, \sigma, K)\) is soft \(A_{RS}\) continuous function then it is soft \(gsp\)-continuous.

3. Contra soft \(A_{RS}\) continuous functions

**Definition 3.1.** A map \(f : (X, \tau, E) \to (Y, \sigma, K)\) is said to be contra soft \(A_{RS}\) continuous if the inverse image of every soft open set in \(Y\) is soft \(A_{RS}\) closed in \(X\).

**Example 1.** Let \(X = \{x_1, x_2\}\), \(Y = \{y_1, y_2\}\) and \(E = \{e_1, e_2\}\), \(K = \{k_1, k_2\}\) where \(\tau = \{F_3, F_11, F_15, F_16\}\), \(\tau^c = \{F_9, F_5, F_15, F_16\}\), \(A_{RS}(Y, \sigma, K) = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}\) and \(\sigma = \{F_5, F_{12}, F_{15}, F_{16}\}\), \(\sigma^c = \{F_{11}, F_4, F_{15}, F_{16}\}\), is
defined as \( f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16} \). Clearly \( f \) is contra soft \( A_{R S} \) continuous.

**Theorem 3.1.** Every contra soft continuous map is a contra soft \( A_{R S} \) continuous.

**Proof.** Let \( f : (X, \tau, E) \rightarrow (Y, \sigma, K) \) be a contra soft continuous function and \((U, K)\) be a soft open set in \((Y, \sigma, K)\), \( f^{-1}(U, K) \) is soft closed in \((X, \tau, E)\). Since every soft closed set is soft \( A_{R S} \) closed, \( f^{-1}(U, K) \) is soft \( A_{R S} \) closed in \((X, \tau, E)\). Therefore \( f \) is contra soft \( A_{R S} \) continuous. \( \square \)

The converse statement is not true.

**Example 2.** Let \( \tau = (F_3, F_{11}, F_{15}, F_{16}) \), \( \tau^c = \{F_6, F_9, F_{15}, F_{16}\} \), \( A_{R S} C(Y, \sigma, K) = (F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}) \) and \( \sigma = (F_3, F_4, F_5, F_{16}), \sigma^c = \{F_{11}, F_4, F_{15}, F_{16}\} \), is defined as \( f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16} \). Clearly \( f \) is contra soft \( A_{R S} \) continuous. But \( F_{12} \) is not in \( \tau^c \) of \((X, \tau, E)\). Hence \( f \) is not contra soft continuous.

**Theorem 3.2.** Every contra soft semi continuous map is a contra soft \( A_{R S} \) continuous.

**Proof.** Obvious from the definition and proposition 2.1. \( \square \)

The converse statement is not true.

**Example 3.** Let \( \tau = (F_1, F_4, F_2, F_3, F_{13}, F_{15}, F_{16}) \), \( \tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}\} \), \( A_{R S} C(X, \tau, E) = (F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}) \) \((X, \tau, E) = (F_{14}, F_6, F_{10}, F_9, F_5, F_4, F_2, F_{15}, F_{16}) \) and \( \sigma = (F_4, F_{12}, F_{15}, F_{16}), \sigma^c = \{F_{11}, F_4, F_{15}, F_{16}\} \), is defined as \( f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16} \). Clearly \( f \) is contra soft \( A_{R S} \) continuous. But \( F_{12} \) is not in soft semi closed of \( X \). Hence \( f \) is not contra soft semi continuous.

**Theorem 3.3.** Every contra soft alpha-continuous map is a contra soft \( A_{R S} \) continuous.

**Proof.** Obvious from the definition and proposition 2.1. \( \square \)
The converse statement is not true.

**Example 4.** Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$, $f : (X, \tau, E) \rightarrow (Y, \sigma, K)$ where $\tau = \{F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$, $\tau^c = \{F_{10}, F_{12}, F_{15}, F_{16}\}$, $A_{RSC}(X, \tau, E) = \{F_1, F_2, F_3, F_4, F_5, F_6, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$, $\alpha(X, \tau, E) = \{F_{12}, F_3, F_{10}, F_8, F_5, F_2, F_1, F_{15}, F_{16}\}$ and $\sigma = \{F_3, F_{11}, F_{15}, F_{16}\}$, $\sigma^c = \{F_6, F_5, F_{15}, F_{16}\}$, is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly $f$ is contra soft $A_{RS}$ continuous. But $F_{11}$ is not in soft $\alpha$ closed of $X$. Hence $f$ is not contra soft $\alpha$ continuous.

**Theorem 3.4.** Every contra soft $JP$ continuous map is a contra soft $A_{RS}$ continuous.

*Proof.* Obvious from the definition and proposition 2.1. \(\Box\)

The converse statement is not true.

**Example 5.** Let $f : (X, \tau, E) \rightarrow (Y, \sigma, K)$ where $\tau = \{F_1, F_4, F_7, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$, $A_{RSC}(X, \tau, E) = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and $\sigma = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\sigma^c = \{F_{14}, F_{12}, F_{15}, F_{16}\}$, is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly $f$ is contra soft $A_{RS}$ continuous. But $F_1$ is not in soft $\alpha$ closed of $X$. Hence $f$ is not contra soft $JP$ continuous.

**Theorem 3.5.** Every contra soft $A_{RS}$ continuous map is a contra soft $gsp$ continuous.

*Proof.* Obvious from the definition and proposition 2.1. \(\Box\)

The converse statement is not true.

**Example 6.** Let $f : (X, \tau, E) \rightarrow (Y, \sigma, K)$ where $\tau = \{F_1, F_4, F_7, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$, $A_{RS}(X, \tau, E) = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and $\sigma = \{F_1, F_{14}, F_{12}, F_{10}, F_2, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$, $\sigma^c = \{F_{14}, F_{12}, F_{10}, F_2, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$, is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$. Clearly $f$ is contra soft $A_{RS}$ continuous. But $F_1$ is not in soft $\alpha$ closed of $X$. Hence $f$ is not contra soft $JP$ continuous.
Proof. \( f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16} \). Clearly \( f \) is contra soft gsp continuous. But \( F_7, F_{13} \) is not in soft \( A_R S \) closed of \( X \). Hence \( f \) is not contra soft \( A_R S \) continuous.

**Remark 3.1.** The composition of two contra soft \( A_R S \) continuous maps need not be contra soft \( A_R S \) continuous.

**Example 7.** In the soft topological space \((X, \tau, E), (Y, \sigma, K), (Z, \eta, R) \) and \( X = \{x_1, x_2\}, Y = \{y_1, y_2\}, Z = \{z_1, z_2\}, \) and \( E = \{e_1, e_2\}, K = \{k_1, k_2\}, R = \{r_1, r_2\} \)

\( f : (X, \tau, E) \rightarrow (Y, \sigma, K), \ g : (Y, \sigma, K) \rightarrow (Z, \eta, R) \) where \( \tau = \{F_4, F_7, F_{15}, F_{16}\}, \ \tau^e = \{F_{10}, F_{12}, F_{15}, F_{16}\} \)

\( A_R S(C(X, \tau, E)) = \{F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\} \) and \( \sigma = \{F_3, F_{11}, F_{15}, F_{16}\}, \ \sigma^e = \{F_9, F_{13}, F_{14}, F_{15}, F_{16}\}, \ A_R S(C(Y, \sigma, K)) = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\} \)

\( \eta^e = \{F_{11}, F_4, F_{15}, F_{16}\} \) is defined as \((g \circ f) F_1 = F_{1}, (g \circ f) F_2 = F_{2}, (g \circ f) F_3 = F_3, (g \circ f) F_4 = F_4, (g \circ f) F_5 = F_5, (g \circ f) F_6 = F_6, (g \circ f) F_7 = F_7, (g \circ f) F_8 = F_8, (g \circ f) F_9 = F_9, (g \circ f) F_{10} = F_{10}, (g \circ f) F_{11} = F_{11}, (g \circ f) F_{12} = F_{12}, (g \circ f) F_{13} = F_{13}, (g \circ f) F_{14} = F_{14}, (g \circ f) F_{15} = F_{15}, (g \circ f) F_{16} = F_{16} \). Clearly \( f \) and \( g \) is contra soft \( A_R S \) continuous. But \((g \circ f)^{-1}(x_1) = F_5, F_4 \) is not soft \( A_R S \) closed in \((X, \tau, E) \). Hence \((g \circ f)^{-1}(x_1) \) is not contra soft \( A_R S \) continuous.

**Definition 3.2.** A space \( X \) is said to be locally soft \( A_R S \) indiscrete if every soft \( A_R S \) open set of \( X \) is soft closed in \( X \).

**Theorem 3.6.** If \( f : (X, \tau, E) \rightarrow (Y, \sigma, K) \) is soft \( A_R S \) continuous and if \( Y \) is soft locally indiscrete, then \( f \) is contra soft \( A_R S \) continuous.

**Proof.** Let \((G, F) \) be an soft open set of \((Y, \sigma, K) \). Since \( Y \) is soft locally discrete, \((G, F) \) is soft closed. Since \( f \) is soft \( A_R S \) continuous, \((f^{-1})(G, F) \) is soft \( A_R S \) closed in \( X \). Therefore, \( f \) is contra soft \( A_R S \) continuous. \( \square \)

**Theorem 3.7.** If \( f : (X, \tau, E) \rightarrow (Y, \sigma, K) \) is soft continuous and \( X \) is a locally indiscrete space, then \( f \) is contra soft \( A_R S \) continuous.

**Proof.** Let \((G, F) \) be any soft open set of \((Y, \sigma, K) \). Since \( f \) is continuous \((f^{-1})(G, F) \) is soft open in \( X \). Since \( X \) is soft locally discrete, \((f^{-1})(G, F) \) is soft closed in
Every soft closed set is soft $A_R S$ closed, $(f^{-1})((G, F))$ is soft $A_R S$ closed in $X$. Therefore, $f$ is contra soft $A_R S$ continuous.

**Theorem 3.8.** Let $f : (X, \tau, E) \to (Y, \sigma, K)$ be soft $A_R S$ irresolute map with $Y$ as locally soft $A_R S$ indiscrete space and $g : (Y, \sigma, K) \to (Z, \eta, R)$ be contra soft $A_R S$ continuous, then $(g \circ f)$ is soft $A_R S$ continuous.

**Proof.** Let $(G, F)$ be any soft closed set in $(Z, \eta, R)$. Since $g$ is contra soft $A_R S$ continuous, $g^{-1}((G, F))$ is soft $A_R S$ open in $Y$. But $Y$ is locally soft $A_R S$ indiscrete, $g^{-1}((G, F))$ is soft closed in $Y$. Hence, $g^{-1}((G, F))$ is soft $A_R S$ closed in $Y$. Since, $f$ is soft $A_R S$ irresolute, $f^{-1}(g^{-1}((G, F))) = (g \circ f)^{-1}((G, F))$ is soft $A_R S$ closed in $X$. Therefore, $(g \circ f)$ is soft $A_R S$ continuous.

**References**