ON INTUITIONISTIC FUZZY ABSOLUTE C-CENTRED STRUCTURES $\omega_C(\mathbb{R})$

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ABSTRACT. In this paper, an intuitionistic fuzzy C-irreducible function on intuitionistic fuzzy C-Hausdorff spaces is defined and several properties of it are discussed. Moreover, the intuitionistic fuzzy absolute C-centred structures are coined and proved that they are homeomorphic under the natural mapping, $\pi_\mathbb{R}$.

1. INTRODUCTION

Problem of uncertainties had been resolved through the concept of fuzziness, defined by L. A. Zadeh [14]. Applications of fuzzy sets are in many fields such as information [10], control [11], robotics [8, 9], e.t.c. C. L. Chang [4] coined a structure using fuzzy set, named as fuzzy topological space. K. K. Azad [3] insisted that every regular open set is open. Notion of intuitionistic fuzziness was defined by Krassimir Atanassov [1, 2]. Later Dogan Coker [5, 6] established the intuitionistic fuzzy topological spaces. One of the most challenging task in medicine is the nervous system which deals with the help of centred systems, introduced by S. Iliadis, S. Fomin [7]. Properties based on the intuitionistic fuzzy C-centred systems have been investigated by T. Yogalakshmi, Oscar Castillo [12, 13].

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In this paper, an intuitionistic fuzzy $C$-irreducible function on the intuitionistic fuzzy $C$-Hausdorff spaces is defined and some of the properties of it are discussed. Moreover, the intuitionistic fuzzy absolute $C$-centred structures are coined and proved that they are homeomorphic under the natural mapping, $\pi_R$.

2. Preliminaries

**Definition 2.1.** [14] A fuzzy set, $\delta : X \to [0, 1]$ is a mapping from a non-empty set $X$ into $[0, 1]$. $\delta' = 1 - \delta$ is said to be the complement of $\delta$.

**Definition 2.2.** [1, 6] Let the fuzzy sets $\lambda_C$ and $\mu_C$ be the degrees of membership (namely $\lambda_C(x)$) and non-membership (namely $\mu_C(x)$) respectively to the non-empty set $C$ such that $0 \leq \mu_C(x) + \lambda_C(x) \leq 1$, for all $x \in X$. An intuitionistic fuzzy set (inshort. IFS) $C$ is of the form $C = \{\langle x, \lambda_C(x), \mu_C(x) \rangle : x \in X \}$. The symbol $\overline{C} = \langle X, \mu, \lambda \rangle$ for the IFS $\{\langle x, \lambda_C(x), \mu_C(x) \rangle : x \in X \}$ shall be used for the sake of simplicity. The complement of an intuitionistic fuzzy set, $C$ is defined as $\overline{C} = \langle X, \mu, \lambda \rangle$.

**Definition 2.3.** [5] Let $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ such that $0 < \alpha(x) + \beta(x) \leq 1$. An intuitionistic fuzzy point (inshort. IFP) $x_{(\alpha, \beta)}$ of $X$ is an IFS of $X$ defined by

$$x_{(\alpha, \beta)}(y) = \begin{cases} \langle x, \alpha(x), \beta(x) \rangle, & \text{if } x = y; \\ \langle x, 0, 1 \rangle, & \text{if } x \neq y. \end{cases}$$

Then, $x$, $\alpha$ and $\beta$ is called the support, value and non-value of $x_{(\alpha, \beta)}$ respectively.

**Definition 2.4.** [6] An intuitionistic fuzzy topology (inshort. IFT) is a collection $\tau$ of IFSs of a non-empty set $X$ having the axioms:

(i) $0, 1, 1, \in \tau$.
(ii) $C_1 \cap C_2 \in \tau$, for any $C_1, C_2 \in \tau$.
(iii) $C_1 \cup C_2 \in \tau$, for any $C_i \in \tau$.

Now, $(X, \tau)$ is said to be an intuitionistic fuzzy topological space (inshort. IFTS) and each member of $\tau$ is called as an intuitionistic fuzzy open set (inshort. IFOS) of $X$. The complement of IFOS is an intuitionistic fuzzy closed set (inshort. IFCS).

**Definition 2.5.** [3] Let $f : X \to Y$ be any function. The pre-image of $C = \langle Y, \delta, \gamma \rangle$ is defined as $f^{-1}(C) = \langle X, f^{-1}(\delta), f^{-1}(\gamma) \rangle$ and the image of $A = \langle X, \lambda, \mu \rangle$ is
defined as $f(A) = \langle Y, f(\lambda), f(\mu) \rangle$

where

$$f(\lambda)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \lambda(x), & \text{if } f^{-1}(y) \neq \emptyset; \\ 0, & \text{otherwise} \end{cases}$$

and

$$f(\mu)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \emptyset; \\ 1, & \text{otherwise} . \end{cases}$$

**Definition 2.6.** [6] Let $(X, \tau)$ and $(Y, \sigma)$ be any IFTSs. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be an intuitionistic fuzzy continuous function, if the inverse image of every IFOS in $(Y, \sigma)$ is an IFOS in $(X, \tau)$.

**Definition 2.7.** [12] The intuitionistic fuzzy $C$-interior and intuitionistic fuzzy $C$-closure of an IFS $P$ are respectively defined as

$$\text{IFint}_c(P) = \bigcup \{ N : N \text{ is an IFcOS in } X \text{ and } N \subseteq P \}$$

and

$$\text{IFcl}_c(P) = \bigcap \{ L : L \text{ is an IFcCS in } X \text{ and } L \supset P \} .$$

**Definition 2.8.** [12] Let $J$ be an indexed set. An intuitionistic fuzzy $C$-centred system is a system $\mathcal{S}_C = \{ A_i \}_{i \in J}$ of intuitionistic fuzzy $C$-open sets in an intuitionistic fuzzy Hausdorff space $\mathcal{R}$ such that $\bigcap_{i=1}^n A_i \neq 0_\sim$. The system $\mathcal{S}_C$ is called as an intuitionistic fuzzy $C$-end if it is maximal.

**Proposition 2.1.** [12] Let $\mathcal{S}_C$ be the intuitionistic fuzzy $C$-end. Then,

1. If $A_i \in \mathcal{S}_C$, for $i=1,2,...,n$, then $\bigcap_{i=1}^n A_i \in \mathcal{S}_C$.
2. If $0_\sim \neq A \in \mathcal{S}_C$ and $P$ is an intuitionistic fuzzy $C$-open set such that $A \subseteq P$, then $P \in \mathcal{S}_C$.
3. If $\mathcal{S}_C$ is the intuitionistic fuzzy $C$-end and $P$ is an intuitionistic fuzzy $C$-open set, then $P \notin \mathcal{S}_C$ iff there is an IFS $D \in \mathcal{S}_C$ with $P \cap D = 0_\sim$.
4. If $P \cup Q \in \mathcal{S}_C$ and $P$, $Q$ are the IFcOSs such that $P \cap Q = 0_\sim$, then either $P \in \mathcal{S}_C$ or $Q \in \mathcal{S}_C$.
5. If $\text{IFcl}_c(A) = 1_\sim$, then $A \in \mathcal{S}_C$, for any intuitionistic fuzzy $C$-end $\mathcal{S}_C$.

**Definition 2.9.** [12] Let $\mathcal{E}_C(\mathcal{R})$ be the collection of all intuitionistic fuzzy $C$-ends belonging to $\mathcal{R}$. If $\mathcal{S}_C(A)$ is the set of all intuitionistic fuzzy $C$-ends which includes IFcOS $A$ of $\mathcal{R}$ as a member of it, then the collection of intuitionistic fuzzy neighbourhoods of each intuitionistic fuzzy $C$-end contained in $\mathcal{S}_C(A)$ forms an intuitionistic fuzzy topology $\%$ in $\mathcal{E}_C(\mathcal{R})$. Thus, the pair $(\mathcal{E}_C(\mathcal{R}), \%)$ or $\mathcal{E}_C(\mathcal{R})$ is called as an intuitionistic fuzzy $C$-centred space.
Remark 2.1. [12] For each intuitionistic fuzzy $C$-open set $B$ of $R$, there corresponds an intuitionistic fuzzy neighbourhood $\mathcal{S}_C(B)$ in $E_C(R)$. That is, $\mathcal{S}_C(B)$ is an intuitionistic fuzzy open subset of $E_C(R)$ and its complement is said to be an intuitionistic fuzzy closed subset of $E_C(R)$, denoted by $E_C(R) - \mathcal{S}_C(B)$.

Definition 2.10. [12] Let $R$ be an intuitionistic fuzzy Hausdorff space and $E_C(R)$ be an intuitionistic fuzzy $C$-centred space. If $G$ is any intuitionistic fuzzy $C$-open subset of $R$, then intuitionistic fuzzy interior of $E_C(R)$ and intuitionistic fuzzy closure of $E_C(R)$ are defined as $\text{Int}_{E_C(R)}(\mathcal{S}_C(G)) = \bigcup\{\mathcal{S}_C(P) : \mathcal{S}_C(P) \text{ is an intuitionistic fuzzy open subset of } E_C(R) \text{ and } G \supseteq P\}$ and $\text{Cl}_{E_C(R)}(\mathcal{S}_C(G)) = \bigcap\{\mathcal{S}_C(Q) : \mathcal{S}_C(Q) \text{ is an intuitionistic fuzzy closed subset of } E_C(R) \text{ and } G \subseteq Q\}$ respectively.

Proposition 2.2. [12] The intuitionistic fuzzy $C$-centred space, $E_C(R)$ is an intuitionistic fuzzy $C$-compact space and has a base of intuitionistic fuzzy $C$-neighbourhoods that are both intuitionistic fuzzy $C$-open and intuitionistic fuzzy $C$-closed sets.

3. On intuitionistic fuzzy absolute $C$-centred structures $\omega_C(R)$

Definition 3.1. Let $\mathbb{R}_1$ and $\mathbb{R}_2$ be any two intuitionistic fuzzy $C$-Hausdorff spaces. A function $f: \mathbb{R}_1 \rightarrow \mathbb{R}_2$ is said to be an intuitionistic fuzzy $C$-irreducible function if there is no proper intuitionistic fuzzy $C$-closed set $B$ of $\mathbb{R}_1$ such that $f(B) = 1_\infty$.

Definition 3.2. Let $\mathbb{R}_1$ and $\mathbb{R}_2$ be any two intuitionistic fuzzy $C$-Hausdorff spaces. A function $f: \mathbb{R}_1 \rightarrow \mathbb{R}_2$ is said to be an intuitionistic fuzzy $C$-perfect function if the image of an intuitionistic fuzzy $C$-closed set is intuitionistic fuzzy $C$-closed and the inverse image of each intuitionistic fuzzy point is intuitionistic fuzzy $C$-compact.

Definition 3.3. Let $\mathbb{R}_1$ and $\mathbb{R}_2$ be any two intuitionistic fuzzy $C$-Hausdorff spaces. A function $f: \mathbb{R}_1 \rightarrow \mathbb{R}_2$ is said to be an intuitionistic fuzzy $C$-compact function if the inverse image of each intuitionistic fuzzy set is intuitionistic fuzzy $C$-compact.

Definition 3.4. The intuitionistic fuzzy absolute $C$-centred structures of $\mathbb{R}$, $\omega_C(\mathbb{R})$ is the collection of all intuitionistic fuzzy $C$-ends containing all intuitionistic fuzzy $C$-open sets, $C_i$ such that $C_i \supseteq x(\alpha,\beta)$. 
Definition 3.5. An intuitionistic fuzzy natural mapping \( \pi_\mathbb{R} \) of \( \omega_\mathcal{C}(\mathbb{R}) \) onto \( \mathbb{R} \) if the image of each intuitionistic fuzzy \( \mathcal{C} \)-end is an intuitionistic fuzzy point of \( \mathbb{R} \).

Proposition 3.1. The intuitionistic fuzzy natural mapping \( \pi_\mathbb{R} \) of \( \omega_\mathcal{C}(\mathbb{R}) \) onto \( \mathbb{R} \) is intuitionistic fuzzy \( \mathcal{C} \)-irreducible and intuitionistic fuzzy \( \mathcal{C} \)-compact.

Proof. Let \( x_{(\alpha,\beta)} \) be an intuitionistic fuzzy point of \( \mathbb{R} \). If \( \pi_\mathbb{R}(\mathcal{G}_\mathcal{C}) = x_{(\alpha,\beta)} \), \( \pi_\mathbb{R}^{-1}(x_{(\alpha,\beta)}) \) is a set of all intuitionistic fuzzy \( \mathcal{C} \)-ends, \( \mathcal{G}_\mathcal{C} \) which contain all the intuitionistic fuzzy \( \mathcal{C} \)-open sets, \( C_i \) such that \( C_i \supseteq x_{(\alpha,\beta)} \). Since \( \mathcal{E}_\mathcal{C}(\mathbb{R}) \) has a base of intuitionistic fuzzy \( \mathcal{C} \)-neighbourhoods that are both intuitionistic fuzzy \( \mathcal{C} \)-open and intuitionistic fuzzy \( \mathcal{C} \)-closed, \( \pi_\mathbb{R}^{-1}(x_{(\alpha,\beta)}) \) is an IFcCS in \( \mathcal{E}_\mathcal{C}(\mathbb{R}) \). Since \( \mathcal{E}_\mathcal{C}(\mathbb{R}) \) is intuitionistic fuzzy \( \mathcal{C} \)-compact space, \( \pi_\mathbb{R}^{-1}(x_{(\alpha,\beta)}) \) is an intuitionistic fuzzy \( \mathcal{C} \)-compact. Therefore, \( \pi_\mathbb{R} \) is an intuitionistic fuzzy \( \mathcal{C} \)-compact function. It is clear that \( \pi_\mathbb{R} \) is an intuitionistic fuzzy \( \mathcal{C} \)-irreducible function, because each \( \mathcal{G}_\mathcal{C}(A) \) includes \( A \) with \( x_{(\alpha,\beta)} \subseteq A \) and \( \{\mathcal{G}_\mathcal{C}(A)\} \) is a base in intuitionistic fuzzy \( \mathcal{C} \)-centred space, \( \mathcal{E}_\mathcal{C}(\mathbb{R}) \). \( \square \)

Proposition 3.2. An intuitionistic fuzzy \( \mathcal{C} \)-irreducible and \( \mathcal{C} \)-closed image of every intuitionistic fuzzy \( \mathcal{C} \)-open set is an intuitionistic fuzzy \( \mathcal{C} \)-open set.

Proof. Proof is obvious. \( \square \)

Proposition 3.3. If \( f \) is an intuitionistic fuzzy \( \mathcal{C} \)-continuous, intuitionistic fuzzy \( \mathcal{C} \)-irreducible and intuitionistic fuzzy \( \mathcal{C} \)-closed functions of \( \mathbb{R}_1 \) onto \( \mathbb{R}_2 \), then \( \mathcal{C} - \text{int} (f^{-1}(A)) \neq 0_\sim \), for every intuitionistic fuzzy \( \mathcal{C} \)-open set \( A \neq 0_\sim \) in \( \mathbb{R}_2 \).

Proof. Proof is obvious. \( \square \)

Proposition 3.4. Let \( \mathbb{R}_1 \) and \( \mathbb{R}_2 \) be any two intuitionistic fuzzy \( \mathcal{C} \)-Hausdorff spaces. Let \( f: \mathbb{R}_1 \to \mathbb{R}_2 \) be an intuitionistic fuzzy \( \mathcal{C} \)-continuous, intuitionistic fuzzy \( \mathcal{C} \)-irreducible and intuitionistic fuzzy \( \mathcal{C} \)-perfect functions. Then there exists an intuitionistic fuzzy \( \mathcal{C} \)-homeomorphism \( \Phi \) of \( \omega_\mathcal{C}(\mathbb{R}_1) \) onto \( \omega_\mathcal{C}(\mathbb{R}_2) \) such that \( f \circ \pi_{\mathbb{R}_1} = \pi_{\mathbb{R}_2} \circ \Phi \).

\[
\begin{array}{c}
\mathbb{R}_1 \xrightarrow{f} \mathbb{R}_2 \\
\pi_{\mathbb{R}_1} \downarrow \quad \downarrow \pi_{\mathbb{R}_2} \\
\omega_\mathcal{C}(\mathbb{R}_1) \xrightarrow{\Phi} \omega_\mathcal{C}(\mathbb{R}_2)
\end{array}
\]
Proof. Let \( \{A_i\} \) be a collection of intuitionistic fuzzy \( C \)-open sets of \( \mathbb{R}_1 \) such that \( \bigcap_{i=1}^{n} (A_i) \neq 0_\omega \) which would be a maximal intuitionistic fuzzy \( C \)-centred system. Since \( f \) is intuitionistic fuzzy \( C \)-irreducible and intuitionistic fuzzy \( C \)-perfect functions, the system \( \{ C - \text{int}(f(A_i)) \} \) is intuitionistic fuzzy \( C \)-open with \( C - \text{int}(f(A)) \neq 0_\omega \). Clearly the system becomes an intuitionistic fuzzy \( C \)-centred system. Let us extend this intuitionistic fuzzy \( C \)-centred system to an intuitionistic fuzzy \( C \)-end and prove that it is unique. Suppose that there exists two intuitionistic fuzzy \( C \)-open sets \( A_1 \) and \( A_2 \) in \( \mathbb{R}_1 \) with \( A_1 \cap A_2 = 0_\omega \), such that \( A_1 \cap C - \text{int}(f(A_i)) \neq 0_\omega \) and \( A_2 \cap C - \text{int}(f(A_i)) \neq 0_\omega \). Now by the Proposition : 3.2. and the Proposition: 3.3., it is easy to see that, \( \{ C - \text{int} (f(A_i)) \} \) can be extended in only one way to a maximal intuitionistic fuzzy \( C \)-centred system in \( \mathbb{R}_2 \). Now, consider the system \( \{ C - \text{int} (f^{-1}(K_i)) \} \) of intuitionistic fuzzy \( C \)-open sets in \( \mathbb{R}_1 \) and \( \{K_i\} \in \omega_C(\mathbb{R}_2) \). This implies that the system is an intuitionistic fuzzy \( C \)-centred system and extend it to a maximal intuitionistic fuzzy \( C \)-centred system in \( \mathbb{R}_2 \). Now consider the function \( \Phi : \omega_C(\mathbb{R}_1) \rightarrow \omega_C(\mathbb{R}_2) \) with \( \Phi(\{A_i\}) \). It is clear that \( \Phi \) is one-to-one and onto. To show that \( \Phi \) is an intuitionistic fuzzy \( C \)-homeomorphism it is sufficient to prove that \( \Phi \) is an intuitionistic fuzzy \( C \)-continuous, since \( \mathcal{E}_C(\mathbb{R}_1) \) is an intuitionistic fuzzy \( C \)-compact. Let \( \mathcal{E}_C = \{A_i\} \) be an arbitrary intuitionistic fuzzy \( C \)-end in \( \mathbb{R}_1 \), that is, an element of \( \mathcal{E}_C(\mathbb{R}_1) \) and let \( \Phi(\mathcal{E}_C) = \{K_i\} \). It is claim that \( \Phi(\mathcal{E}_C(A_i)) \subseteq \mathcal{E}_C(K_i) = \mathcal{E}_C(C - \text{int}(f(A_i))) \). If \( \mathcal{E}_C \in \mathcal{E}_C(A_i) \), then \( A_i \in \mathcal{E}_C \), so \( C - \text{int}(f(A_i)) \in \mathcal{E}_C \) which means that \( \Phi(\mathcal{E}_C) \in \mathcal{E}_C(C - \text{int}(f(A_i))) \). This proves that \( \Phi \) is an intuitionistic fuzzy \( C \)-homeomorphism. From the construction of \( \Phi \), it follows that, \( \Phi(\pi_{\mathbb{R}_1}^{-1}(x_{[\alpha,\beta]})) \subseteq \pi_{\mathbb{R}_2}^{-1}(f(x_{[\alpha,\beta]})) \). Hence, \( f \circ \pi_{\mathbb{R}_1} = \pi_{\mathbb{R}_2} \circ \Phi \). \( \square \)

**Proposition 3.5.** The intuitionistic fuzzy \( C \)-absolute of \( \mathbb{R}_1 \) and \( \mathbb{R}_2 \) are the intuitionistic fuzzy \( C \)-homeomorphic if there exists an intuitionistic fuzzy topological space on \( \mathbb{R} \) such that \( \mathbb{R} \) can be mapped onto both \( \mathbb{R}_1 \) and \( \mathbb{R}_2 \) by intuitionistic fuzzy \( C \)-irreducible, intuitionistic fuzzy \( C \)-continuous and intuitionistic fuzzy \( C \)-perfect functions.


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Proof. Proof is obvious.

□

4. Conclusion

Centred systems have many applications in the fields of medical sciences such as nervous system, muscular system, etc. In section:1 and section:2, introduction and preliminaries of [12] were provided. The intuitionistic fuzzy absolute $C$-centred structures on intuitionistic fuzzy $C$-Hausdorff spaces were coined and established that they were homeomorphic to each other in section:3.

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