\(\delta^*g\alpha\)-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT. In this paper, the authors introduce a new class of sets called \(\delta^*g\alpha\)-closed set in Topological spaces. Some of their properties and characterizations are investigated. Also we introduce and study a new class of space namely \(\alpha\delta T^*_{1/2}g\alpha\)-space, \(\delta T^*_{c}\)-space, \(\delta^*T_{1/2}\)-space and \(\delta\alpha T^*_{c}\)-space.

1. INTRODUCTION

Levine [13], Mashhour et al. [2], Njastad [15] and Velicko [14] introduced semi-open sets, pre-open sets, \(\alpha\)-open sets and \(\delta\)-closed sets respectively. Levine [12] introduced generalized closed (briefly g-closed) sets and studied their basic properties. Bhattacharya and Lahiri [16], Arya and Nour [17], Maki et al [6,7], Dontchev and Ganster [8] introduced generalized semi-closed (briefly gs-closed) sets, \(\alpha\)-generalized closed (briefly \(\alpha\)g-closed) sets and \(\delta\)-generalized closed (briefly \(\delta\)g-closed) sets respectively. M.Vigneshwaran and R.Devi [10] introduced \(*\)generalized \(\alpha\)-closed (briefly \(*\)g\(\alpha\)-closed) sets. The purpose of this paper is to define a new class of closed sets called \(\delta^*g\alpha\)-closed sets and also we obtain some basic properties of \(\delta^*g\alpha\) closed sets in topological spaces. Applying this set, we obtain some new spaces such as \(\alpha\delta T^*_1g\alpha\)-space, \(\delta T^*_c\)-space, \(\delta^*T_{1/2}\)-space and \(\delta\alpha T^*_c\)-space.

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Throughout this paper \((X, \tau)\) (or simply \(X\)) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset \(A\) of \(X\), \(\text{cl}(A)\), \(\text{int}(A)\) and \(X - A\) denote the closure of \(A\), the interior of \(A\) and the complement of \(A\) respectively. Let us recall the following definitions, which are useful in the sequel.

**Definition 2.1.** A subset \(A\) of \((X, \tau)\) is said to be

(i) semi-open set \([13]\) if \(A \subseteq \text{cl}(\text{int}(A))\).

(ii) pre-open set \([2]\) if \(A \subseteq \text{int}(\text{cl}(A))\).

(iii) semi-preopen set \([1]\) if \(A \subseteq \text{cl}(\text{int}(\text{cl}(A)))\).

(iv) \(\alpha\)-open set \([15]\) if \(A \subseteq \text{int}(\text{cl}(\text{int}(A)))\).

(v) regular open set \([11]\) if \(A = \text{int}(\text{cl}(A))\).

The complement of a semi-open (resp. pre-open, \(\alpha\)-open, regular open) set is called semi-closed (resp. semi-closed, \(\alpha\)-closed, regular closed).

**Definition 2.2.** The \(\delta\)-interior \([14]\) of a subset \(A\) of \(X\) is the union of all regular open set of \(X\) contained in \(A\) and is denoted by \(\text{Int}_\delta(A)\). The subset \(A\) is called \(\delta\)-open \([14]\) if \(A = \text{Int}_\delta(A)\), i.e. a set is \(\delta\)-open if it is the union of regular open sets. The complement of a \(\delta\)-open is called \(\delta\)-closed. Alternatively, a set \(A \subseteq (X, \tau)\) is called \(\delta\)-closed \([14]\) if \(A = \text{cl}_\delta(A)\), where

\[
\text{cl}_\delta(A) = \{ x \in X : \text{int} (\text{cl} (U)) \neq \emptyset, U \in \tau \text{ and } x \in U \}.
\]

**Definition 2.3.** A subset \(A\) of \((X, \tau)\) is called

(i) a generalized closed (briefly \(g\)-closed) set \([12]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open set in \((X, \tau)\).

(ii) a generalized semi-closed (briefly \(gs\)-closed) set \([17]\) if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open set in \((X, \tau)\).

(iii) a \(\alpha\)-generalized closed (briefly \(\alpha g\)-closed) set \([6]\) if \(\alpha \text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open set in \((X, \tau)\).

(iv) a \(\delta\)-generalized closed (briefly \(\delta g\)-closed) set \([8]\) if \(\text{cl}_\delta(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open set in \((X, \tau)\).

(v) a generalized preclosed (briefly \(gp\)-closed) set \([5]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open set in \((X, \tau)\).

(vi) a generalized semi-preclosed (briefly \(gsp\)-closed) set \([3]\) if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open set in \((X, \tau)\).
(vii) a *generalized α-closed (briefly *gα-closed) set [10] if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( g\alpha \)-open set in \((X, \tau)\).

(viii) a generalized-δ closed (briefly \( g\delta \)-closed) set [4] if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \delta \)-open set in \((X, \tau)\).

(ix) a \( \delta \)-generalized*-closed (briefly \( \delta g^* \)-closed) set [21] if \( \text{cl}_{\delta}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \delta \)-open set in \((X, \tau)\).

(x) a \( \delta \)-generalized-δ semi closed (briefly \( g\delta s \)-closed) set [9] if \( \text{scl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \delta \)-open set in \((X, \tau)\).

(xi) a \( \delta \)-generalized b-closed (briefly \( \delta gb \)-closed) set [19] if \( \text{bcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \delta \)-open set in \((X, \tau)\).

The complement of a \( g \)-closed (resp. \( gs \)-closed, \( \alpha g \)-closed, \( \delta g \)-closed, \( gp \)-closed, \( gsp \)-closed, \( g\delta \)-closed, \( g\delta^* \)-closed, \( g\delta s \)-closed and \( \delta gb \)-closed) set is called \( g \)-open (resp. \( gs \)-open, \( \alpha g \)-open, \( \delta g \)-open, \( gp \)-open, \( gsp \)-open, \( g\delta \)-open, \( g\delta^* \)-open, \( g\delta s \)-open and \( \delta gb \)-open).

Definition 2.4. A space \((X, \tau)\) is called a

(i) \( T_{1/2} \)-space [12] if every \( g \)-closed set in it is closed.

(ii) \( T_{3/4} \)-space [8] if every \( \delta g \)-closed set in it is \( \delta \)-closed.

(iii) \( \delta T_{3/4} \)-space [18] if every \( g\delta s \)-closed set in it is \( \delta \)-closed.

(iv) \( \delta T_{\delta gb} \)-space [20] if every \( \delta gb \)-closed set in it is \( \delta \)-closed.

(v) \( \alpha T_{\delta} \)-space [7] if every \( \alpha g \)-closed set in it is \( g \)-closed.

3. Properties of \( \delta^*g\alpha \)-closed sets in Topological Spaces

Definition 3.1. A subset \( A \) of a space \((X, \tau)\) is called \( \delta^*g\alpha \)-closed if \( \text{cl}_{\delta}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is a \( \ast g\alpha \)-open set in \((X, \tau)\).

Theorem 3.1. Every \( \delta \)-closed set is \( \ast g\alpha \)-closed.

Proof. Let \( A \) be \( \delta \)-closed and \( U \) be any \( g\alpha \)-open set containing \( A \). Since \( A \) is \( \delta \)-closed, \( \text{cl}_{\delta}(A) = A \). Therefore \( \text{cl}_{\delta}(A) \subseteq A \subseteq U \). We know that \( \text{cl}(A) \subseteq \text{cl}_{\delta}(A) \subseteq U \). Hence \( A \) is \( \ast g\alpha \)-closed. \( \square \)

Theorem 3.2. Every \( \delta \)-closed set is \( \delta^*g\alpha \)-closed set. Converse is not true is showed through an example.
Proof. Let $A \subseteq U$ and $U$ is $^{*}g_{\alpha}$-open set. Since $A$ is $\delta$-closed $\text{cl}_{\delta}(A) = A$, then $\text{cl}_{\delta}(A) \subseteq U$ therefore $A$ is $^{*}g_{\alpha}$-closed set. □

**Example 1.** Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\} , \{b\} , \{a, b\} , \{b, c\} \}$; Here $\{a, c\}$ is $^{*}g_{\alpha}$-closed but not $\delta$-closed in $(X, \tau)$.

**Theorem 3.3.** Every $^{*}g_{\alpha}$-closed set is $g_{\alpha}$-closed. Converse is not true is showed through an example.

Proof. Let $A \subseteq U$ and $U$ is open set. Since every open set is $^{*}g_{\alpha}$-open[9], then $U$ is $^{*}g_{\alpha}$-open set. Since $A$ is $^{*}g_{\alpha}$-closed, then $\text{cl}_{\delta}(A) \subseteq U$. But $\text{cl}(A) \subseteq \text{cl}_{\delta}(A)$, then $\text{cl}(A) \subseteq U$, Therefore $A$ is $g_{\alpha}$-closed set. □

**Example 2.** Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\} , \{a, c\} \}$; Here $\{a\}$ is $g_{\alpha}$-closed but not $^{*}g_{\alpha}$-closed in $(X, \tau)$.

**Theorem 3.4.** Every $^{*}g_{\alpha}$-closed set is $\alpha g$-closed. Converse is not true is showed through an example.

Proof. Let $A \subseteq U$ and $U$ is open set. Since every open set is $^{*}g_{\alpha}$-open, then $U$ is $^{*}g_{\alpha}$-open set. Since $A$ is $^{*}g_{\alpha}$-closed, then $\text{cl}_{\delta}(A) \subseteq U$. But $\text{cl}(A) \subseteq \text{cl}_{\delta}(A)$, then $\text{cl}(A) \subseteq U$, Therefore $A$ is $\alpha g$-closed set. □

**Example 3.** Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\} , \{a, b\} \}$; Here $\{b\}$ is $\alpha g$-closed but not $^{*}g_{\alpha}$-closed in $(X, \tau)$.

**Theorem 3.5.** Every $^{*}g_{\alpha}$-closed set is $g_{\alpha}$sp-closed. Converse is not true is showed through an example.

Proof. Let $A \subseteq U$ and $U$ is open set. Since every open set is $^{*}g_{\alpha}$-open, then $U$ is $^{*}g_{\alpha}$-open set. Since $A$ is $^{*}g_{\alpha}$-closed, then $\text{cl}_{\delta}(A) \subseteq U$. But $\text{spcl}(A) \subseteq \text{cl}_{\delta}(A)$, then $\text{spcl}(A) \subseteq U$, Therefore $A$ is $g_{\alpha}$sp-closed set. □

**Example 4.** Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\} , \{b\} , \{a, b\} \}$; Here $\{a\}$ and $\{b\}$ are $g_{\alpha}$sp-closed but not $^{*}g_{\alpha}$-closed in $(X, \tau)$.

**Theorem 3.6.** Every $^{*}g_{\alpha}$-closed set is $g_{\alpha}$-closed. Converse is not true is showed through an example.

Proof. Let $A \subseteq U$ and $U$ is open set. Since every open set is $^{*}g_{\alpha}$-open, then $U$ is $^{*}g_{\alpha}$-open set. Since $A$ is $^{*}g_{\alpha}$-closed, then $\text{cl}_{\delta}(A) \subseteq U$. But $\text{pcl}(A) \subseteq \text{cl}_{\delta}(A)$, then $\text{pcl}(A) \subseteq U$, Therefore $A$ is $g_{\alpha}$-closed. □
Example 5. Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{b, c\}\} \); Here \( \{b\} \) and \( \{c\} \) are \( \delta^*g\alpha \)-closed but not \( \delta^*g\alpha \)-closed in \( (X, \tau) \).

Theorem 3.7. Every \( \delta^*g\alpha \)-closed set is \( \delta gp \)-closed. Converse is not true is showed through an example.

Proof. Let \( A \subseteq U \) and \( U \) is \( \delta \)-open set. Since every \( \delta \)-open set is \( \ast g\alpha \)-open, then \( U \) is \( \ast g\alpha \)-open set. Since \( A \) is \( \delta^*g\alpha \)-closed, then \( \text{cl}_\delta(A) \subseteq U \). But \( \text{pcl}(A) \subseteq \text{cl}_\delta(A) \), then \( \text{pcl}(A) \subseteq U \), Therefore \( A \) is \( \delta gp \)-closed. \( \square \)

Example 6. Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{a, b\}\} \); Here \( \{a\} \), \( \{b\} \) and \( \{a, b\} \) is \( \delta gp \)-closed but not \( \delta^*g\alpha \)-closed in \( (X, \tau) \).

Theorem 3.8. Every \( \delta^*g\alpha \)-closed set is \( g\delta \)-closed. Converse is not true is showed through an example.

Proof. Let \( A \subseteq U \) and \( U \) is \( \delta \)-open set. Since every \( \delta \)-open set is \( \ast g\alpha \)-open, then \( U \) is \( \ast g\alpha \)-open set. Since \( A \) is \( \delta^*g\alpha \)-closed, then \( \text{cl}_\delta(A) \subseteq U \). Hence \( A \) is \( g\delta \)-closed. \( \square \)

Example 7. Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{a\}\} \); Here \( \{a\} \) is \( g\delta^* \)-closed but not \( \delta^*g\alpha \)-closed in \( (X, \tau) \).

Theorem 3.9. Every \( \delta^*g\alpha \)-closed set is \( \delta g^* \)-closed. Converse is not true is showed through an example.

Proof. Let \( A \subseteq U \) and \( U \) is \( \delta \)-open set. Since \( \delta \)-every open set is \( \ast g\alpha \)-open, then \( U \) is \( \ast g\alpha \)-open set. Since \( A \) is \( \delta^*g\alpha \)-closed, then \( \text{cl}_\delta(A) \subseteq U \). Hence \( A \) is \( g\delta^* \)-closed. \( \square \)

Example 8. Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{a, b\}\} \); Here \( \{a\}, \{b\} \) and \( \{a, b\} \) are \( g\delta^* \)-closed but not \( \delta^*g\alpha \)-closed in \( (X, \tau) \).

Theorem 3.10. Every \( \delta^*g\alpha \)-closed set is \( g\delta s \)-closed. Converse is not true is showed through an example.

Proof. Let \( A \subseteq U \) and \( U \) is \( \delta \)-open set. Since every \( \delta \)-open set is \( \ast g\alpha \)-open, then \( U \) is \( \ast g\alpha \)-open set. Since \( A \) is \( \delta^*g\alpha \)-closed, then \( \text{cl}_\delta(A) \subseteq U \). But \( \text{scl}(A) \subseteq \text{cl}_\delta(A) \), then \( \text{scl}(A) \subseteq U \), Therefore \( A \) is \( g\delta s \)-closed. \( \square \)

Example 9. Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{a\}, \{a, b\}\} \); Here \( \{a\}, \{b\} \) and \( \{a, b\} \) are \( g\delta s \)-closed but not \( \delta^*g\alpha \)-closed in \( (X, \tau) \).
Theorem 3.11. Every $\delta^*g\alpha$-closed set is $\delta gb$-closed. Converse is not true is showed through an example.

Proof. Let $A \subseteq U$ and $U$ is $\delta$-open set. Since every $\delta$-open set is $^*g\alpha$-open, then $U$ is $^*g\alpha$-open set. Since $A$ is $\delta^*g\alpha$-closed, then $\text{cl}_{\delta}(A) \subseteq U$. But $\text{bcl}(A) \subseteq \text{cl}_{\delta}(A)$, then $\text{bcl}(A) \subseteq U$, Therefore $A$ is $\delta gb$-closed. □

Example 10. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, c\}, \{a, c\}\}$; Here $\{a\}, \{c\}$ and $\{a, c\}$ are $\delta gb$-closed but not $\delta^*g\alpha$-closed in $(X, \tau)$.

Theorem 3.12. The finite union of $\delta^*g\alpha$-closed sets is $\delta^*g\alpha$-closed.

Proof. Let $\{A_i/ i = 1, 2, ... n\}$ be a finite class of $\delta^*g\alpha$-closed subsets of a space $(X, \tau)$. Then for each $^*g\alpha$-open set $U_i$ in $X$ containing $A_i, \text{cl}_{\delta}(A_i) \subseteq \cap U_i, i \in \{1, 2, ... n\}$. Hence $\cup A_i \subseteq \cup U_i = V$. Since arbitrary union of $^*g\alpha$-open sets in $(X, \tau)$ is also $^*g\alpha$-open set in $(X, \tau)$, $V$ is $^*g\alpha$-open in $(X, \tau)$. Also $\cup \text{cl}_{\delta}(A_i) = \text{cl}_{\delta}(\cup A_i) \subseteq V$. Therefore $U_i A_i$ is $\delta^*g\alpha$-closed in $(X, \tau)$. □

Remark 3.1. Intersection of any two $\delta^*g\alpha$-closed sets in $(X, \tau)$ need not be $\delta^*g\alpha$-closed in $(X, \tau)$, it can be seen by the following example.

Example 11. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$; $\{b, c\}$ and $\{b, d\}$ are $\delta^*g\alpha$-closed sets but their intersection $\{b\}$ is not $\delta^*g\alpha$-closed.

Theorem 3.13. Let $A$ be a $\delta^*g\alpha$-closed set of $(X, \tau)$, then $\text{cl}_{\delta}(A)-A$ does not contain a non-empty $^*g\alpha$-closed set.

Proof. Suppose that $A$ is $\delta^*g\alpha$-closed, let $F$ be a $^*g\alpha$-closed set contained in $\text{cl}_{\delta}(A)-A$. Now $F^c$ is $^*g\alpha$-open set of $(X, \tau)$ such that $A \subseteq F^c$. Since $A$ is $\delta^*g\alpha$-closed set of $(X, \tau)$, then $\text{cl}_{\delta}(A) \subseteq F^c$. Thus $F \subseteq (\text{cl}_{\delta}(A))^c$. Also $F \subseteq \text{cl}_{\delta}(A) - A$. Therefore $F \subseteq \text{cl}_{\delta}(A) \cap (\text{cl}_{\delta}(A) = \phi$. Hence $F = \phi$. □

Theorem 3.14. If $A$ is $^*g\alpha$-open and $\delta^*g\alpha$-closed subset of $(X, \tau)$ then $A$ is an $\delta$-closed subset of $(X, \tau)$.

Proof. Since $A$ is $g\alpha$-open and $\delta^*g\alpha$-closed, $\text{cl}_{\delta}(A) \subseteq A$. Hence $A$ is $\delta$-closed. □

Theorem 3.15. The intersection of a $\delta^*g\alpha$-closed set and a $\delta$-closed set is always $\delta^*g\alpha$-Closed.
Proof. Let A be $\delta^*g\alpha$-Closed and let F be $\delta$-closed. If U is an $^\ast g\alpha$-open set with $A \cap F \subseteq U$, then $A \subseteq U \cap F^c$ and so $cl_\delta(A) \subseteq U \cap F^c$. Now $cl_\delta(A \cap F) \subseteq cl_\delta(A) \cap F \subseteq U$. Hence $A \cap F$ is $\delta^*g\alpha$-closed. □

Theorem 3.16. In a $T_{3/4}$-space every $\delta^*g\alpha$-closed set is $\delta$-closed.

Proof. Let X be a $T_{3/4}$-space. Let A be a $\delta^*g\alpha$-closed set of X. We know that every $\delta^*g\alpha$-closed set is $\delta g\alpha$-closed. Since X is a $T_{3/4}$-space, A is $\delta$-closed. □

Theorem 3.17. If A is a $\delta^*g\alpha$-closed set in a space $(X, \tau)$ and $A \subseteq B \subseteq cl_\delta(A)$, then B is also a $\delta^*g\alpha$-closed set.

Proof. Let U be a $^\ast g\alpha$-open set of $(X, \tau)$ such that $B \subseteq cl_\delta(A)$, then $A \subseteq U$. Since A is a $\delta^*g\alpha$-closed set, $cl_\delta(A) \subseteq U$. Also since $B \subseteq cl_\delta(A)$, $cl_\delta(B) \subseteq cl_\delta(cl_\delta(A)) = cl_\delta(A) \subseteq U$. Implies $cl?(B) \subseteq U$. Therefore B is also a $\delta^*g\alpha$-closed set. □

Theorem 3.18. Let A be a $\delta^*g\alpha$-closed of $(X, \tau)$, then A is $\delta$-closed iff $cl_\delta(A) - A$ is $^\ast g\alpha$-closed.

Proof. Necessity. Let A be a $\delta$-closed subset of X. Then $cl_\delta(A) = A$ and so $cl_\delta(A) - A = \phi$ which is $^\ast g\alpha$-closed.

Sufficiency. Since A is $\delta^*g\alpha$-closed, by proposition, $cl_\delta(A) - A$ does not contain a non-empty $^\ast g\alpha$-closed set. But $cl_\delta(A) - A = \phi$. That is $cl_\delta(A) = A$. Hence A is $\delta$-closed. □

4. SOME SPACES USING $\delta^*g\alpha$-CLOSED SETS

We introduce the following definition.

Definition 4.1. A space $(X, \tau)$ is called $\alpha_\delta T_{3^*4}^\ast g\alpha$-space if every $\delta^*g\alpha$-closed set is an $\delta$-closed.

Theorem 4.1. For a topological space $(X, \tau)$, the following conditions are equivalent.

(i) $(X, \tau)$ is a $\alpha_\delta T_{3^*4}^\ast g\alpha$-space.

(ii) Every singleton $\{x\}$ is either $^\ast g\alpha$-closed or $\delta$-open.
**Theorem 4.4.** Every $δ$-space is $\alpha g$-closed. Since every $δgα$-closed set is $δ$-closed, then $A$ is $gδsC$ closed. Since $(X, τ)$ is $αδT_{3}^{∗∗}$-space, then $A$ is $δ$-closed.

Therefore $(X, τ)$ is $αδT_{3}^{∗∗}$-space. □

**Theorem 4.2.** Every $δT_{3/4}^{∗}$-space is a $αδT_{3}^{∗∗}$-space. Converse is not true is showed through an example.

**Proof.** Let $A$ be a $δ$-closed set of $(X, τ)$. Since every $δgα$-closed set is $gδ$-closed, then $A$ is $gδsC$ closed. Since $(X, τ)$ is $δT_{3/4}^{∗}$-space, then $A$ is $δ$-closed. Therefore $(X, τ)$ is $αδT_{3}^{∗∗}$-space. □

**Example 12.** Let $X = \{a, b, c\}$, $τ = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$; Here it is $αδT_{3}^{∗∗}$-space but not $δT_{3/4}^{∗}$-space, Since $\{a\}$ is $gδ$-closed set but not $δ$-closed set.

**Theorem 4.3.** Every $δT_{δgb}$-space is a $αδT_{3}^{∗∗}$-space. Converse is not true is showed through an example.

**Proof.** Let $A$ be a $δgα$-Closed set of $(X, τ)$. Since every $δgα$-Closed set is $δgb$-closed, then $A$ is $δgb$-closed. Since $(X, τ)$ is $δT_{δgb}$-space, then $A$ is $δ$-closed.

Therefore $(X, τ)$ is $αδT_{3}^{∗∗}$-space. □

**Example 13.** Let $X = \{a, b, c\}$, $τ = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$; Here it is $αδT_{3}^{∗∗}$-space but not $δT_{δgb}$-space, Since $\{a\}$ is $δgb$-closed set but not $δ$-closed set.

**Definition 4.2.** A space $(X, τ)$ is called $δT_{c}^{∗∗}$-space if every $gs$-Closed set in it is an $δ^∗gα$-closed.

**Theorem 4.4.** Every $δT_{c}^{∗∗}$-space is a $αT_{d}$-space. Converse is not true is showed through an example.

**Proof.** Let $A$ be a $αg$-Closed set of $(X, τ)$. Since $αg$-Closed set is $gs$-closed, then $A$ is $gs$-closed. Since $(X, τ)$ is a $δT_{c}^{∗∗}$-space in $(X, τ)$, then $A$ is $δ^∗gα$-closed.
Since every $\delta^* g^0$-closed set is $g$-closed, then $A$ is $g$-closed. Therefore $(X, \tau)$ is $\alpha T_d$-space.

Example 14. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$; Here it is $\alpha T_d$-space but not $\delta^* T_c$-space.

Definition 4.3. A space $(X, \tau)$ is called $\delta \alpha T_c$-space if every $\alpha g$-Closed set in it is an $\delta^* g^0$-closed.

Theorem 4.5. Every $\delta T_c^*$-space is a $\delta \alpha T_c^*$-space. Converse is not true is showed through an example.

Proof. Let $A$ be a $\alpha g$-Closed set of $(X, \tau)$. Since every $\alpha g$-Closed set is $gs$-closed, then $A$ is $gs$-closed. Since $(X, \tau)$ is $\delta T_c^*$-space, then $A$ is $\delta^* g^0$-closed. Therefore $(X, \tau)$ is an $\delta \alpha T_c^*$-space.

Example 15. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$; Here it is $\delta \alpha T_c^*$-space but not $\delta^* T_c^*$-space, Since $\{a\}$ is gs-closed set but not $\delta^* g^0$-closed set.

Definition 4.4. A space $(X, \tau)$ is called $\delta \alpha T_c^*$-space if every $g$-Closed set in it is an $\delta^* g^0$-closed.

Theorem 4.6. Every $\delta T_c^*$-space is a $\delta \alpha T_c^*$-space. Converse is not true is showed through an example.

Proof. Let $A$ be a $g$-Closed set of $(X, \tau)$. Since every $g$-Closed set is $\alpha g$-closed, then $A$ is $\alpha g$-closed. Since $(X, \tau)$ is $\alpha T_c^*$-space, then $A$ is $\delta^* g^0$-closed. Therefore $(X, \tau)$ is an $\delta \alpha T_c^*$-space.

Example 16. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$; Here it is $\delta \alpha T_c^*$-space but not $\delta T_c^*$-space, Since $\{b\}$ is gs-closed set but not $\delta^* g^0$-closed set.

Theorem 4.7. Every $\delta \alpha T_c^*$-space is a $\delta \alpha T_2$-space. Converse is not true is showed through an example.

Proof. Let $A$ be a $g$-Closed set of $(X, \tau)$. Since every $g$-Closed set is $\alpha g$-closed, then $A$ is $\alpha g$-closed. Since $(X, \tau)$ is $\delta \alpha T_c^*$-space, then $A$ is $\delta^* g^0$-closed. Therefore $(X, \tau)$ is an $\delta \alpha T_2$-space.
5. Conclusion

This article defined $\delta^*g\alpha$-closed set in Topological Spaces and relation with other exciting sets in topology were studied. Along with that some of there properties were discussed. Also $\alpha_0T^{**}_c$-space, $\delta T^{**}_c$-space, $^{**}_cT^{**}_{1/2}$-space and $\delta_0T^{**}_c$-space of a set were introduced and discussed their properties. This set can be used to derive few more functions such as $\delta^*g\alpha$-continuous and $\delta^*g\alpha$-irresolute functions. In addition to that it can be extended to homeomorphisms of topological spaces.

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