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A TOPOLOGICAL APPROACH IN CORDIAL GRAPHS

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ABSTRACT. The focus of this paper is to introduce CI-Lower and CI-Upper approximation in cordial incidence topological space. Some effective characterizations and properties are studied. Furthermore, results are proved with some counter examples and are discussed using graphs.

1. INTRODUCTION AND PRELIMINARIES

Z. Pawlak [3] initiated Rough set theory in 1982. Some basic ideas of rough sets and many applications have been presented recently by Z.Pawlak and Skowron [4, 5]. Lower and upper approximations of Pawlak's definitions originally developed with the reference to an equivalence relation. In 2013, M.E. Abd Ei-Monsef, A.M. Kozae and M.J. Iqelan [1] initiated near approximation in topological space. Pawlak and Skowron [4, 5] have been derived many lower and upper approximations properties. Our work depends on some ideas in terms of cordial incidence topology. We hope that, cordial incidence topological space will play a pivotal role for knowledge modification extraction and processing.

The following summary of definitions are used in the subsequent sequel.

Definition 1.1. *[2] A mapping $f : V(G) \to \{0, 1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f. The induced edge*

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labeling $f^* : E(G) \to \{0,1\}$ *is given by* $f^*(e = uv) = |f(u) - f(v)|$. *Let us denote* $v_f(0), v_f(1)$ *be the number of vertices of* G *having labels 0 and 1 respectively under* f *and* $e_f(0), e_f(1)$ *be the number of edges of* G *having labels 0 and 1 respectively under* f^*.

Definition 1.2. *[2] A binary vertex labeling of a graph* G *is called a cordial labeling if* $|v_f(0) - v_f(1)| \leq 1$ *and* $|e_f(0) - e_f(1)| \leq 1$. *A graph* G *is called cordial if it admits labeling.*

Definition 1.3. *[2] A binary vertex labeling of a graph* G *with induced edge labeling* $f^* : E(G) \to \{0,1\}$ *defined by* $f^*(uv) = |f(u) + f(v)|(mod 2)$ *is called sum cordial labeling if* $|v_f(0) - v_f(1)| \leq 1$ *and* $|e_f(0) - e_f(1)| \leq 1$. *A graph* G *is called sum cordial if it admits sum cordial labeling.*

Definition 1.4. *[6] Let* $G = (V(G), E(G))$ *be a simple graph with sum cordial labeling and with out isolated vertex. Define* S_{0G} *and* S_{1G} *as follows.* $S_{0G} = \{A_{v(0)} | v \in V\}$ *and* $S_{1G} = \{A_{v(1)} | v \in V\}$ *such that* $A_{v(0)}$ *and* $A_{v(1)}$ *is the set of all vertices adjacent to* v *of* G *having label* 0 *and* 1 *respectively. Since* G *has no isolated vertex,* $S_{0G} \cup S_{1G}$ *forms a subbasis for a topology* τ_{CG} *on* V *is called Cordial graphic topology of* G *and it is denoted by* (V, τ_{CG}).

Definition 1.5. *[6] Let* $G = (V(G), E(G))$ *be a simple graph with sum cordial labeling and with out isolated vertex. Define* $S_{E(0G)}$ *and* $S_{E(1G)}$ *as follows.* $S_{E(0G)} = \{I_{e(0)} | e \in E\}$ *and* $S_{E(1G)} = \{I_{e(1)} | e \in E\}$ *such that* $I_{e(0)}$ *and* $I_{e(1)}$ *is the incidence vertices having label* 0 *and* 1 *respectively. Since* G *has no isolated vertex,* $S_{E(0G)} \cup S_{E(1G)}$ *forms a subbasis for a topology* τ_{CI} *on* V *is called cordial incidence topology of* G *and it is denoted by* (V, τ_{CI}).

Definition 1.6. *[1] Let* $A \subseteq X$, *then the upper approximation (resp.the lower approximation) of* A *is given by:*

$$\overline{R}A = \{x \in X : R_x \cap A \neq \emptyset\},$$
$$\underline{R}A = \{x \in X : R_x \subseteq A\},$$

where $R_x \subseteq X$ *to denote the equivalence class containing* $x \in X$ *and* X/R *to denote the set of all elementary set of* R.

Proposition 1.1. *[1] In an approximation space* $K = (X, R)$ *if* A *and* B *are two subsets of* X *then:*

(1) $\underline{R}A \subseteq A \subseteq A\overline{R}A$
(2) $\underline{R}\emptyset = \overline{R}\emptyset = \emptyset$, $\underline{R}X = \overline{R}X = X$
(3) $\overline{R}(A \cup B) = \overline{R}A \cup \overline{R}B$
(4) $\underline{R}(A \cap B) = \underline{R}A \cap \underline{R}B$
(5) *If* $A \subseteq B$, *then* $\underline{R}A \subseteq \underline{R}B$
(6) *If* $A \subseteq B$, *then* $\overline{R}A \subseteq \overline{R}B$
(7) $\underline{R}(A \cup B) \supseteq \underline{R}A \cup \underline{R}B$
(8) $\overline{R}(A \cap B) \subseteq \overline{R}A \cap \overline{R}B$
(9) $\underline{R}A^c = [\overline{R}A]^c$
(10) $\overline{R}A^c = [\underline{R}A]^c$
(11) $\underline{R}(\underline{R}A) = \overline{R}\underline{R}A = \underline{R}A$
(12) $\overline{R}(\overline{R}A) = \underline{R}\overline{R}A = \overline{R}A$

Lemma 1.1. *[1] Let* (X, τ) *be a topological space, then* $int(A^c) = [cl(A)]^c$, *for all* $A \subseteq X$.

Lemma 1.2. *[1] Let* A *and* B *be two subsets of* X *in a topological space* (X, τ). *If* A *is open then* $A \cap cl(B) \subseteq cl(A \cap B)$.

2. CI-lower and CI-upper Approximation

Definition 2.1. *Let* $G = (V(G), E(G))$ *be a sum cordial graph and admits cordial incidence topology* τ_{CI} *induced by* V *and* H *be the subgraph of* G, *then the interior and closure of* H *has the following form,*

$$int_{CI}(V(H)) = \cup\{U \in \tau_{CI} | U \subseteq V(H)\},$$
$$cl_{CI}(V(H)) = \cap\{U \in \tau_{CI}^c | V(H) \subseteq U\}.$$

Example 1. *Let us consider the sum cordial graph with* $V = \{v_1, v_2, v_3, v_4\}$ *and* $E = \{e_1, e_2, e_3\}$.

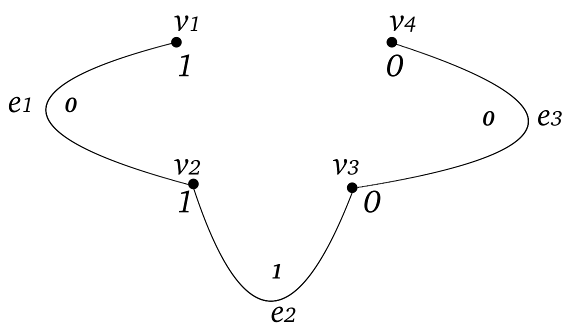

Figure 1

From the figure 1, $I_{e_1}(0) = \{v_1, v_2\}$, $I_{e_2}(1) = \{v_2, v_3\}$, $I_{e_3}(0) = \{v_3, v_4\}$, $S_{E(0G)} = \{\{v_1, v_2\}, \{v_3, v_4\}\}$ *and* $S_{E(1G)} = \{\{v_2, v_3\}$.

Thus $S_{E(0G)} \cup S_{E(1G)} = \{\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}\}$
$\tau_{CI} = \{V, \emptyset, \{v_1, v_2\}, \{v_3, v_4\}, \{v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_2\}, \{v_3\}, \{v_1, v_2, v_3\}\}$.

$\tau_{CI}^c = \{V, \emptyset, \{v_3, v_4\}, \{v_1, v_4\}\}, \{v_1, v_2\}, \{v_4\}, \{v_1\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_4\}\}$.

Now let us consider the subgraph H of G as follows,

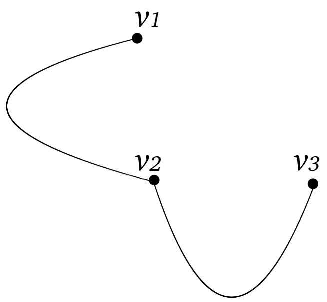

FIGURE 2

From figure 2 we have, $V(H) = \{v_1, v_2, v_3\}$, $int_{CI}\{V(H)\} = \{v_1, v_2, v_3\}$ and $cl_{CI}\{V(H)\} = \{v_1, v_2, v_3, v_4\} = V$.

Definition 2.2. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V and let H be the subgraph of G, then CI-lower (resp. CI-upper) approximation of H is defined by:*

$$L_{CI}[V(H)] = int_{CI}(V(H)),$$
$$U_{CI}[V(H)] = cl_{CI}(V(H)).$$

Definition 2.3. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V and let H be the subgraph of G, then CI-boundary region of H is defined by,*

$$B_{CI}[V(H)] = U_{CI}[V(H)] - L_{CI}[V(H)].$$

3. PROPERTIES OF CI-LOWER AND CI-UPPER APPROXIMATION

Proposition 3.1. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V. If H and S are two subgraphs of G then:*

(1) $L_{CI}[V(H)] \subseteq V(H) \subseteq U_{CI}[V(H)]$
(2) $L_{CI}[V(\emptyset)] = U_{CI}[V(\emptyset)]$ *and* $L_{CI}[V(G)] = U_{CI}[V(G)]$
(3) $V(H) \subseteq V(S)$ *then* $L_{CI}[V(H)] \subseteq L_{CI}[V(S)]$
(4) $V(H) \subseteq V(S)$ *then* $U_{CI}[V(H)] \subseteq U_{CI}[V(S)]$.

Proof.

(1) Proof is obvious.
(2) Obviously the result is true, since $V(G)$ and $[V(\emptyset)]$ are exact sets in τ_{CI}.
(3) Let $v_1 \in L_{CI}[V(H)]$, then there exists $U \in \tau_{CI}$, such that $v_1 \in U \subseteq V(H)$. Hence $v_1 \in U \subseteq V(S)$, since $V(H) \subseteq V(S)$ such that $U \in \tau_{CI}$ and $v_1 \in L_{CI}[V(S)]$.
$\Rightarrow L_{CI}[V(H)] \subseteq L_{CI}[V(S)]$.
(4) Similarly, $U_{CI}[V(H)] \subseteq U_{CI}[V(S)]$.

□

Proposition 3.2. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V. If H is a subgraph of G then:*

(1) $L_{CI}[V(H)^c] = [U_{CI}(V(H))]^c$,
(2) $U_{CI}[V(H)^c] = [L_{CI}(V(H))]^c$.

Proof.

(1) Since $L_{CI}[V(H)^c] = \cup\{U \in \tau_{CI} | U \subseteq V(H)^c\}$, $U \in \tau_{CI}$ then $V - U \in \tau_{CI}^c$,

$$\begin{aligned}\Rightarrow L_{CI}[V(H)^c] &= \cup\{V - [V - U] \in \tau_{CI} | V - (V - U) \subseteq V - V(H)\} \\ &= V - \cap\{V - U \in \tau_{CI}^c | V(H) \subseteq (V - U)\} \\ &= V - U_{CI}[V(H)] \\ &= [U_{CI}(V(H))]^c.\end{aligned}$$

Hence, $L_{CI}[V(H)^c] = [U_{CI}(V(H))]^c$
(2) Similarly, $U_{CI}[V(H)^c] = [L_{CI}(V(H))]^c$.

□

Proposition 3.3. *Let $G = (V(G), E(G))$ be approximation space and τ_{CI} be the cordial incidence topology induced by V. If H and S are two subgraphs of G then:*

(1) $L_{CI}[V(H) \cap V(S)] = L_{CI}[V(H)] \cap L_{CI}[V(S)]$,
(2) $U_{CI}[V(H) \cup V(S)] = U_{CI}[V(H)] \cup U_{CI}[V(S)]$,
(3) $L_{CI}[V(H) \cup V(S)] \supseteq L_{CI}[V(H)] \cup L_{CI}[V(S)]$,
(4) $U_{CI}[V(H) \cap V(S)] \subseteq U_{CI}[V(H)] \cap U_{CI}[V(S)]$.

Proof.

(1) Since $V(H) \cap V(S) \subseteq V(H)$ and $V(H) \cap V(S) \subseteq V(S)$

$\Rightarrow L_{CI}[V(H) \cap V(S)] \subseteq L_{CI}[V(H)]$ and $L_{CI}[V(H) \cap V(S)] = L_{CI}[V(S)]$

(3.1) $$\Rightarrow L_{CI}[V(H) \cap V(S)] \subseteq L_{CI}[V(H)] \cap L_{CI}[V(S)]$$

Since $L_{CI}[V(H)] \in \tau_{CI}$ and $L_{CI}[V(S)] \in \tau_{CI}$ then $L_{CI}[V(H)] \cap L_{CI}[V(S)] \in \tau_{CI}$,
$\Rightarrow L_{CI}[V(H)] \cap L_{CI}[V(S)]$ is contained in $V(H) \cap V(S)$. Thus we have,

(3.2) $$L_{CI}[V(H)] \cap L_{CI}[V(S)] \subseteq L_{CI}[V(H) \cap V(S)]$$

From (3.1) and (3.2) we have,
$L_{CI}[V(H) \cap V(S)] = L_{CI}[V(H)] \cap L_{CI}[V(S)]$

(2) Similarly, $U_{CI}[V(H) \cup V(S)] = U_{CI}[V(H)] \cup U_{CI}[V(S)]$

(3) Since $V(H) \subseteq V(H) \cup V(S)$ and $V(S) \subseteq V(H) \cup V(S)$
$\Rightarrow L_{CI}[V(H)] \subseteq L_{CI}[V(H) \cup V(S)]$ and $L_{CI}[V(S)] \subseteq L_{CI}[V(H) \cup V(S)]$
$\Rightarrow L_{CI}[V(H)] \cup L_{CG}[V(S)] \subseteq L_{CI}[V(H) \cup V(S)]$

(4) Similarly, $U_{CI}[V(H) \cap V(S)] \subseteq U_{CI}[V(H)] \cap U_{CI}[V(S)]$.

□

Remark 3.1. *The following results are not true:*

(1) $L_{CI}\{L_{CI}[V(H)]\} = U_{CI}\{L_{CI}[V(H)]\} = L_{CI}[V(H)]$,
(2) $U_{CI}\{U_{CI}[V(H)]\} = L_{CI}\{U_{CI}[V(H)]\} = U_{CI}[V(H)]$.

Example 2. *Let G be a simple graph with $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3\}$.*

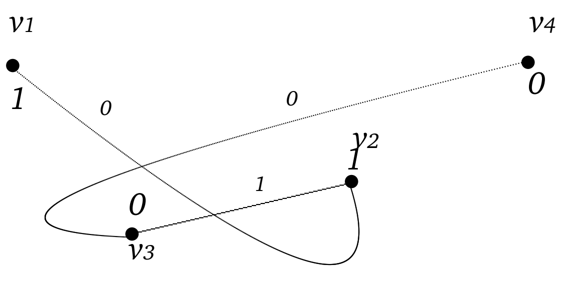

FIGURE 3

(1) *Let us consider $V(H) = \{v_1, v_3\}$, from the figure 3 then*
$L_{CI}[V(H)] = \{v_3\}$
$L_{CI}(L_{CI}[V(H)]) = \{v_3\}$
$U_{CI}(L_{CI}[V(H)]) = \{v_3, v_4\}$

From the above three equations we have,
$L_{CI}\{L_{CI}[V(H)]\} = L_{CI}[V(H)] \neq U_{CI}\{L_{CI}[V(H)]\}$

(2) $U_{CI}[V(H)] = \{v_1, v_3, v_4\}$
$U_{CI}(U_{CI}[V(H)]) = \{v_1, v_3, v_4\}$
$L_{CI}(U_{CI}[V(H)]) = \{v_3, v_4\}$
From the above three equations we have,

$$U_{CI}[V(H)] = U_{CI}\{U_{CI}[V(H)]\} \neq L_{CI}\{U_{CI}[V(H)]\}$$

Proposition 3.4. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V. If H and S are two subgraphs of G, then the following results are true:*

(1) $L_{CI}[V(H)) - V(S))] \subseteq L_{CI}[V(H)] - L_{CI}[V(S)]$,
(2) $U_{CI}[V(H)) - V(S))] \supseteq U_{CI}[V(H)] - U_{CI}[V(S)]$.

Proof.

(1) Since $V(H) - V(S) = V(H) \cap V(S)^c$

$$\begin{aligned}
\Rightarrow int_{CI}[V(H) - V(S)] &= int_{CI}[V(H) \cap V(S)^c] \\
&= int_{CI}(V(H)) \cap int_{CI}(V(S)^c). \\
&= int_{CI}(V(H)) \cap [cl_{CI}(V(S))]^c \\
&= int_{CI}[V(H)] - cl_{CI}[V(S)]
\end{aligned}$$

$$\Rightarrow int_{CI}[V(H) - V(S)] \subseteq int_{CI}(V(H)) - int_{CI}(V(S))$$
$$\Rightarrow L_{CI}[V(H) - V(S)] \subseteq L_{CI}[V(H)] - L_{CI}[V(S)]$$

(2) Since $cl_{CI}(V(H)) - cl_{CI}(V(S)) = cl_{CI}(V(H)) \cap [cl_{CI}(V(S))]^c$

$$\begin{aligned}
&= cl_{CI}[V(H)] \cap int_{CI}[V(S)^c] \\
&\subseteq cl_{CI}[V(H) \cap int_{CI}(V(S)^c)] \\
&= cl_{CI}[V(H) \cap [cl_{CI}(V(S))]^c] \\
&= cl_{CI}[V(H) - cl_{CI}(V(S))]
\end{aligned}$$

$$cl_{CI}[V(H)] - cl_{CI}[V(S)] \subseteq cl_{CI}[V(H) - V(S)]$$
$$U_{CI}[V(H)) - V(S))] \supseteq U_{CI}[V(H)] - U_{CI}[V(S)]$$

□

Theorem 3.1. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V. If H and S are two subgraphs of G then the results are true:*

(1) *If $V(S) \in \tau_{CI}$ contained in V(H) then $V(S) \subseteq L_{CI}[V(H)]$,*

(2) *If* $V(S) \in \tau_{CI}^c$ *containing V(H) then* $U_{CI}[V(H)] \subseteq V(S)$.

Proof.

(1) Let $v_1 \in V(S)$. Since $V(S) \in \tau_{CI}$ contained in $V(H)$, which gives that $v_1 \in int_{CI}[V(H)]$, hence $v_1 \in L_{CI}[V(H)]$.

(2) We have $U_{CI}[V(H)] = \cap\{U \in \tau_{CI}^c | V(H) \subseteq U\}$, therefore $U_{CI}[V(H)]$ is contained in every $U \in \tau_{CI}^c$ containing $V(H)$. Since $V(S) \in \tau_{CI}^c$ and containing $V(H)$, so $U_{CI}[V(H)] \subseteq V(S)$.

□

Theorem 3.2. *Let* $G = (V(G), E(G))$ *be an approximation space and* τ_{CI} *be the cordial incidence topology induced by* V. *If* H *and* S *are two subgraphs of* G *and* $V(H) \cap V(S) = V(\emptyset)$*, then* $L_{CI}[V(H)] \cap L_{CI}[V(S)] = V(\emptyset)$.

Proof. Suppose let us assume that $L_{CI}[V(H)] \cap L_{CI}[V(S)] \neq V(\emptyset)$, therefore $v_1 \in L_{CI}[V(H)]$ and $v_1 \in L_{CI}[V(S)]$ which implies that $v_1 \in int_{CI}V(H)$ and $v_1 \in int_{CI}V(S)$ then there exists $U, W \in \tau_{CI}$ such that $v_1 \in U \subseteq V(H)$ and $v_1 \in W \subseteq V(S)$, so $v_1 \in U \cap W \subseteq V(H)$ and $v_1 \in U \cap W \subseteq V(S)$, hence $v_1 \in V(H) \cap V(S)$, thus $V(H) \cap V(S) \neq V(\emptyset)$. This is contradiction to our assumption $V(H) \cap V(S) = V(\emptyset)$. □

Theorem 3.3. *Let* $G = (V(G), E(G))$ *be an approximation space and* τ_{CI} *be the cordial incidence topology induced by* V. *If* H *and* S *are two subgraphs of* G *and* $U_{CI}[V(H)] \cap U_{CI}[V(S)] = V(\emptyset)$*, then* $V(H) \cap V(S) = V(\emptyset)$.

Proof. Let us assume that $U_{CI}[V(H)] \cap U_{CI}[V(S)] = V(\emptyset)$, suppose $v_1 \in V(H) \cap V(S)$ which implies that $v_1 \in V(H)$ and $v_1 \in V(S)$. So $v_1 \in cl_{CI}(V(H))$ and $v_1 \in cl_{CI}(V(S))$. Hence $v_1 \in U_{CI}[V(H)]$ and $v_1 \in U_{CI}[V(S)]$. Therefore $v_1 \in U_{CI}[V(H)] \cap U_{CI}[V(S)]$. This is contradiction to our assumption $U_{CI}[V(H)] \cap U_{CI}[V(S)] = V(\emptyset)$. Hence $V(H) \cap V(S) = V(\emptyset)$. □

Theorem 3.4. *Let* $G = (V(G), E(G))$ *be an approximation space and* τ_{CI} *be the cordial incidence topology induced by* V. *For any* $v_1 \in U_{CI}[V(H)]$ *if and only if for every* $U \in \tau_{CI}$ *containing* v_1 *such that* $U \cap V(H) \neq V(\emptyset)$.

Proof. Suppose let us assume that there exists $U \in \tau_{CI}$ containing v_1 such that $U \cap V(H) = V(\emptyset)$. Then $V(H) \subset V(G) - U$ and $V(G) - U \in \tau_{CI}^c$, so hence $U_{CI}[V(H)] \subset V(G) - U$. Therefore $U_{CI}[V(H)] \cap U = V(\emptyset)$, thus $v_1 \notin U_{CI}[V(H)]$, which is contradiction to $v_1 \in U_{CI}[V(H)]$. Thus $U \cap V(H) \neq V(\emptyset)$ for every $U \in \tau_{CI}$ containing v_1.

Conversely, let us assume that $U \cap V(H) \neq V(\emptyset)$ for every $U \in \tau_{CI}$ containing v_1. Suppose let $v_1 \notin U_{CI}[V(H)]$ then there exists $U \in \tau_{CI}$ containing v_1 such that $U \cap V(H) = V(\emptyset)$, which is contradiction to $U \cap V(H) \neq V(\emptyset)$. Hence $v_1 \in U_{CI}[V(H)]$. □

Theorem 3.5. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V. If H is subgraph of G then:*

(1) $V(G) - U_{CI}[V(H)] = L_{CI}[V(G) - V(H)]$,
(2) $V(G) - L_{CI}[V(H)] = U_{CI}[V(G) - V(H)]$.

Proof.

(1) Let $v_1 \in V(G) - U_{CI}[V(H)]$, which implies $v_1 \notin U_{CI}[V(H)]$ then there exists $U \in \tau_{CI}$ containing v_1 such that $V(H) \cap U = V(\emptyset)$. So $v_1 \in U \subset V(G) - V(H)$ thus $v_1 \in L_{CI}[V(G) - V(H)]$. Therefore,

$$V(G) - U_{CI}[V(H)] \subseteq L_{CI}[V(G) - V(H)]. \tag{3.3}$$

Let $v_1 \in L_{CI}[V(G) - V(H)]$. Then there exists $U \in \tau_{CI}$ such that $v_1 \in U \subset V(G) - V(H)$, so $v_1 \notin U_{CI}[V(H)]$, which implies that $v_1 \in V(G) - U_{CI}[V(H)]$. Therefore,

$$L_{CI}[V(G) - V(H)] \subseteq V(G) - U_{CI}[V(H)]. \tag{3.4}$$

From (3.3) and (3.4), $V(G) - U_{CI}[V(H)] = L_{CI}[V(G) - V(H)]$

(2) Suppose let us take, $v_1 \in V(G) - L_{CI}[V(H)]$, then $v_1 \notin L_{CI}[V(H)]$ then for every $U \in \tau_{CI}$ containing v_1 such that $U \not\subset V(H)$, which implies that $U \cap V(H)^c \neq V(\emptyset)$ then $v_1 \in U_{CI}[V(G) - V(H)]$. Therefore,

$$V(G) - L_{CI}[V(H)] \subseteq U_{CI}[V(G) - V(H)]. \tag{3.5}$$

Let $v_1 \in U_{CI}[V(G) - V(H)]$, then for every $U \in \tau_{CI}$ containing v_1 such that $U \cap V(H)^c \neq V(\emptyset)$, so $U \not\subset V(H)$, which implies that $v_1 \notin L_{CI}[V(H)]$, hence $v_1 \in V(G) - L_{CI}[V(H)]$. Therefore,

$$U_{CI}[V(G) - V(H)] \subseteq V(G) - L_{CI}[V(H)]. \tag{3.6}$$

From (3.5) and (3.6), $V(G) - L_{CI}[V(H)] = U_{CI}[V(G) - V(H)]$.

□

Theorem 3.6. *Let $G = (V(G), E(G))$ be an approximation space and τ_{CI} be the cordial incidence topology induced by V. If H is subgraph of G then:*

(1) $U_{CI}[V(H)] = V(G) - L_{CI}[V(G) - V(H)]$,
(2) $L_{CI}[V(H)] = V(G) - U_{CI}[V(G) - V(H)]$.

Proof.

(1) Suppose $v_1 \in U_{CI}[V(H)]$, then for every $U \in \tau_{CI}$ containing v_1 such that $U \cap V(H) \neq V(\emptyset)$, which implies that $U \not\subset V(G) - V(H)$. So $v_1 \notin L_{CI}[V(G) - V(H)]$, $v_1 \in V(G) - L_{CI}[V(G) - V(H)]$. Therefore,

$$U_{CI}[V(H)] \subseteq V(G) - L_{CI}[V(G) - V(H)]. \tag{3.7}$$

Let $v_1 \in V(G) - L_{CI}[V(G) - V(H)]$ then $v_1 \notin L_{CI}[V(G) - V(H)]$, hence for every $U \in \tau_{CI}$ containing v_1 such that $U \not\subset V(G) - V(H)$. which means $U \cap V(H) \neq V(\emptyset)$ so $v_1 \in U_{CI}[V(H)]$.Therefore,

$$V(G) - L_{CI}[V(G) - V(H)] \subseteq U_{CI}[V(H)]. \tag{3.8}$$

From (3.7) and (3.8), $U_{CI}[V(H)] = V(G) - L_{CI}[V(G) - V(H)]$.

(2) Let $v_1 \in L_{CI}[V(H)]$, then there exists $U \in \tau_{CI}$ such that $v_1 \in U \subset V(H)$. Hence $v_1 \notin U_{CI}[V(G) - V(H)]$, which implies that $v_1 \in V(G) - U_{CI}[V(G) - V(H)]$. Therefore,

$$L_{CI}[V(H)] \subseteq V(G) - U_{CI}[V(G) - V(H)]. \tag{3.9}$$

Let $v_1 \in V(G) - U_{CI}[V(G) - V(H)]$ then $v_1 \notin U_{CI}[V(G) - V(H)]$, then there exists $U \in \tau_{CI}$ such that $v_1 \in U \cap [V(G) - V(H)] = V(\emptyset)$, which means $v_1 \in U \subset V(H)$. So $v_1 \in L_{CI}[V(H)]$. Therefore,

$$V(G) - U_{CI}[V(G) - V(H)] \subseteq L_{CI}[V(H)]. \tag{3.10}$$

From (3.9) and (3.10), $L_{CI}[V(H)] = V(G) - U_{CI}[V(G) - V(H)]$.

□

Conclusion

In this paper, we have defined interior and closure for subgraph H of G in cordial incidence topology. This sort of study would help to determine the blood path way in the human heart and kidney.

REFERENCES

[1] M. E. ABD EI-MONSEF, A. M. KOZAE, M. J. IQELAN: *Near Approximations in Topological Spaces*, Int. J. Math. Anal., **4**(6) (2010), 279–290.

[2] M. S. BOSMIA, V. R. VISAVALIYA, B. M. PATEL: *Further Results on sum cordial graphs*, Malaya J. Matematik, **3**(2) (2015), 175–181.

[3] Z. PAWLAK: *Rough Sets,* Int. J. Comp. Informat. Sci., **11**(5) (1982), 341–356.

[4] Z. PAWLAK, A. SKOWRON: *Rough sets:some extensions*, Inform. Sci., **177**(1) (2007), 28–40.

[5] Z. PAWLAK, A. SKOWRON: *Rudiments of rough sets*, Inform. Sci., **177**(1) (2007), 3–27.

[6] D. SASIKALA, A. DIVYA: *An Alexandroff topological space on the vertex set of sum cordial graphs,* J. Advanc. Resear. Dyn. Cont. Syst., **11**(2) (2019), 1551–1555.

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