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FUZZY ECONOMIC ORDER QUANTITY INVENTORY MODEL USING LAGRANGIAN METHOD

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ABSTRACT. In this paper, an economic order quantity (EOQ) inventory model with shortage has been considered in a fuzzy environment in order to determine the optimal total cost and the optimal length value for the proposed inventory model. Further to achieve this optimal length value, used extension of the Lagrangian method. An algorithm for optimizing this model is developed. We have illustrated crisp and fuzzy value in numerical example.

1. INTRODUCTION

A century ago, when the first economic order quantity (EOQ) inventory model was suspected by Harris [1]. His inventory model minimizes total costs. An alteration of EOQ inventory model is the economic production quantity (EPQ) model suspected by Taft [2]. This paper considers an inventory system for a single product. By suitable nutrition, new-born items grow and reach the ideal weight for satisfying customers demand.

In this paper, Nobil [3] addressed an inventory system with permissible shortage. The proposed principle for the arithmetic operation of fuzzy number is Chen's [4] function principle and for optimization, it is a Lagrangian method. Also for defuzzifying the annual integrated total cost for EPQ, The Signed Distance Method is used. we have provided the numerical example is to illustrate crisp and fuzzy sense. This is followed by conclusion.

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2. Mathematical Model

2.1. **Notations And Assumption.**

D: Demand rate per time unit.
G: Growing rate per chick per time unit.
W_0**:** Approximated weight of new-born items (gr).
W_1**:** Approximated weight at the moment of slaughtering (gr).
Q: Total weight of inventory at t.
P_1**:** Growing period.
P_2**:** Consumption period.
P_3**:** Shortage period.
L: Length of each period (decision variable).
O: Number of ordered items per period.
S: Shortage quantity per period (gr).
c: Purchasing cost per weight unit $ gr.
r: Feeding cost per weight unit per time unit.
H: Holding cost per weight unit per time unit.
F: Shortage cost per weight unit per time unit.
A: Setup cost (ordering cost),

Inventory total cost per time unit is

$$TC = \frac{Dcw_0}{w_1} + \frac{Dr(w_1 - w_0)^2}{2Gw_1} + \frac{A}{L} + \left(\frac{(H+F)}{2D}\right)\left(\frac{S^2}{L}\right) + \frac{HDL}{2} - H(S)\,.$$

By minimizing the total cost we obtain the optimal length value

$$L = \sqrt{\frac{2DA + (H+F)S^2}{HD^2}}\,.$$

3. Methodology

3.1. The Signed Distance Method. Let $\tilde{A} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ be a trapezoidal fuzzy numbers. Then the signed distance representation

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 (A_L(\alpha) + A_R(\alpha))d\alpha = \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{4}\,.$$

3.2. Fuzzy arithmetical operations under function principle. Suppose $\tilde{A} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $\tilde{B} = (\beta_1, \beta_2, \beta_3, \beta_4)$ are two trapezoidal fuzzy numbers. Then

(1) $\tilde{A} \oplus \tilde{B} = (\alpha_1 + \beta_1, \alpha_2 + \beta_2, \alpha_3 + \beta_3, \alpha_4 + \beta_4)$
(2) $\tilde{A} \otimes \tilde{B} = (\alpha_1\beta_1, \alpha_2\beta_2, \alpha_3\beta_3, \alpha_4\beta_4)$
(3) $\tilde{A} \ominus \tilde{B} = (\alpha_1 - \beta_4, \alpha_2 - \beta_3, \alpha_3 - \beta_2, \alpha_4 - \beta_1)$
(4) $\frac{1}{\tilde{B}} = \tilde{B}^1 = \left(\frac{1}{\beta_4}, \frac{1}{\beta_3}, \frac{1}{\beta_2}, \frac{1}{\beta_1}\right) \quad \tilde{A} \oslash \tilde{B} = \left(\frac{\alpha_1}{\beta_4}, \frac{\alpha_2}{\beta_3}, \frac{\alpha_3}{\beta_2}, \frac{\alpha_4}{\beta_1}\right)$
(5) For any $\alpha \in R$,
 (i) If $\beta \geq 0$, Then $\beta \otimes \tilde{A} = (\beta\alpha_1, \beta\alpha_2, \beta\alpha_3, \beta\alpha_4)$
 (ii) If $\beta \leq 0$, Then $\beta \otimes \tilde{A} = (\beta\alpha_4, \beta\alpha_3, \beta\alpha_2, \beta\alpha_1)$.

4. Fuzzy Inventory Models

4.1. Fuzzy integrated inventory model for crisp order quantity value. The annual integrated total cost for this model

$$
\begin{aligned}
TC = {} & \frac{D_1cw_0}{w_1} + \frac{D_1r(w_1 - w_0)^2}{2Gw_1} + \frac{A_1}{L} + \left(\frac{(H+F)}{2D_1}\right)\left(\frac{S_1^2}{L}\right) + \frac{HD_1L}{2} - H(S_1), \\
& \frac{D_2cw_0}{w_1} + \frac{D_2r(w_1 - w_0)^2}{2Gw_1} + \frac{A_2}{L} + \left(\frac{(H+F)}{2D_2}\right)\left(\frac{S_2^2}{L}\right) + \frac{HD_2L}{2} - H(S_2), \\
& \frac{D_3cw_0}{w_1} + \frac{D_3r(w_1 - w_0)^2}{2Gw_1} + \frac{A_3}{L} + \left(\frac{(H+F)}{2D_3}\right)\left(\frac{S_3^2}{L}\right) + \frac{HD_3L}{2} - H(S_3), \\
& \frac{D_4cw_0}{w_1} + \frac{D_4r(w_1 - w_0)^2}{2Gw_1} + \frac{A_4}{L} + \left(\frac{(H+F)}{2D_4}\right)\left(\frac{S_4^2}{L}\right) + \frac{HD_4L}{2} - H(S_4),
\end{aligned}
$$

we defuzzify the Total cost by the signed distance representation of $\tilde{T}C$ is

$$
\tilde{T}C = \frac{1}{4}\left(\begin{array}{c}
\frac{D_1cw_0}{w_1} + \frac{D_1r(w_1-w_0)^2}{2Gw_1} + \frac{A_1}{L} + \left(\frac{(H+F)}{2D_1}\right)\left(\frac{S_1^2}{L}\right) + \frac{HD_1L}{2} - H(S_1) + \\
\frac{D_2cw_0}{w_1} + \frac{D_2r(w_1-w_0)^2}{2Gw_1} + \frac{A_2}{L} + \left(\frac{(H+F)}{2D_2}\right)\left(\frac{S_2^2}{L}\right) + \frac{HD_2L}{2} - H(S_2) + \\
\frac{D_3cw_0}{w_1} + \frac{D_3r(w_1-w_0)^2}{2Gw_1} + \frac{A_3}{L} + \left(\frac{(H+F)}{2D_3}\right)\left(\frac{S_3^2}{L}\right) + \frac{HD_3L}{2} - H(S_3) + \\
\frac{D_4cw_0}{w_1} + \frac{D_4r(w_1-w_0)^2}{2Gw_1} + \frac{A_4}{L} + \left(\frac{(H+F)}{2D_4}\right)\left(\frac{S_4^2}{L}\right) + \frac{HD_4L}{2} - H(S_4)
\end{array}\right).
$$

By minimizing the total cost $\tilde{T}C$. We obtain the optimal length value $\frac{\delta \tilde{T}C}{\delta L} = 0$

$$L = \sqrt{\frac{2((A_1+A_2+A_3+A_4)(D_1+D_2+D_3+D_4))((H+F)(S_1^2+S_2^2+S_3^2+S_4^2))}{H(D_1^2+D_2^2+D_3^2+D_4^2)}}.$$

4.2. **Fuzzy integrated inventory model for fuzzy order quantity.** Suppose fuzzy order Length value L be a trapezoidal fuzzy number $\tilde{L} = (L_1, L_2, L_3, L_4)$ with $0 < L_1 \leq L_2 \leq L_3 \leq L_4$.

The annual integrated total cost for this model

$$\begin{aligned} TC = {} & \frac{D_1 c w_0}{w_1} + \frac{D_1 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_1}{L} + \left(\frac{(H+F)}{2D_1}\right)\left(\frac{S_1^2}{L}\right) + \frac{H D_1 L}{2} - H(S_1), \\ & \frac{D_2 c w_0}{w_1} + \frac{D_2 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_2}{L} + \left(\frac{(H+F)}{2D_2}\right)\left(\frac{S_2^2}{L}\right) + \frac{H D_2 L}{2} - H(S_2), \\ & \frac{D_3 c w_0}{w_1} + \frac{D_3 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_3}{L} + \left(\frac{(H+F)}{2D_3}\right)\left(\frac{S_3^2}{L}\right) + \frac{H D_3 L}{2} - H(S_3), \\ & \frac{D_4 c w_0}{w_1} + \frac{D_4 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_4}{L} + \left(\frac{(H+F)}{2D_4}\right)\left(\frac{S_4^2}{L}\right) + \frac{H D_4 L}{2} - H(S_4), \end{aligned}$$

we defuzzify the Total cost by the signed distance representation of $\tilde{T}C$ is

$$P\tilde{T}C = \frac{1}{4}\begin{pmatrix} \frac{D_1 c w_0}{w_1} + \frac{D_1 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_1}{L} + \left(\frac{(H+F)}{2D_1}\right)\left(\frac{S_1^2}{L}\right) + \frac{H D_1 L}{2} - H(S_1) + \\ \frac{D_2 c w_0}{w_1} + \frac{D_2 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_2}{L} + \left(\frac{(H+F)}{2D_2}\right)\left(\frac{S_2^2}{L}\right) + \frac{H D_2 L}{2} - H(S_2) + \\ \frac{D_3 c w_0}{w_1} + \frac{D_3 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_3}{L} + \left(\frac{(H+F)}{2D_3}\right)\left(\frac{S_3^2}{L}\right) + \frac{H D_3 L}{2} - H(S_3) + \\ \frac{D_4 c w_0}{w_1} + \frac{D_4 r (w_1 - w_0)^2}{2 G w_1} + \frac{A_4}{L} + \left(\frac{(H+F)}{2D_4}\right)\left(\frac{S_4^2}{L}\right) + \frac{H D_4 L}{2} - H(S_4) \end{pmatrix},$$

with $0 < L_1 \leq L_2 \leq L_3 \leq L_4$ into the following inequality $L_2 - L_1 \geq 0$, $L_3 - L_2 \geq 0, L_4 - L_3 \geq 0$, we used Lagrangian method. We obtain the optimal order Length.

Step 1 : Solve the unconstraint problem. To find the min $\tilde{T}C$

Let $\frac{\delta P}{\delta L_1} = 0$ then $L_1 = \sqrt{\frac{2A_4 D_4 + ((H+F)S_4^2)}{H(D_1 D_4)}}$.

Let $\frac{\delta P}{\delta L_2} = 0$ then $L_2 = \sqrt{\frac{2A_3 D_3 + ((H+F)S_3^2)}{H(D_2 D_3)}}$.

Let $\dfrac{\delta P}{\delta L_3} = 0$ then $L_3 = \sqrt{\dfrac{2A_2D_2 + ((H+F)S_2^2)}{H(D_2D_3)}}$.

Let $\dfrac{\delta P}{\delta L_4} = 0$ then $L_4 = \sqrt{\dfrac{2A_1D_1 + ((H+F)S_1^2)}{H(D_4D_1)}}$.

The above result shows $L_1 > L_2 > L_3 > L_4$, it does not satisfy the constraint $0 \leq L_1 \leq L_2 \leq L_3 \leq L_4$. set $A = 1$ and go to Step 2.

Step 2 : Convert the inequality constraint $L_2 - L_1 \geq 0$ into equality constraint $L_2 - L_1 = 0$. The Lagrangian function as

$$l(L_1, L_2, L_3, L_4, \lambda) = P(TC - \lambda(L_2 - L_1).$$

To find the minimization of $l(L_1, L_2, L_3, L_4, \lambda)$

$$\begin{aligned} L_1 = L_2 &= \sqrt{\frac{2(A_3D_3 + A_4D_4) + ((H+F)(S_3^2 + S_4^2)}{H(D_1D_4 + D_2D_3)}} \\ L_3 &= \sqrt{\frac{2A_2D_2 + ((H+F)S_2^2)}{H(D_2D_3)}} \\ L_4 &= \sqrt{\frac{2A_1D_1 + ((H+F)S_1^2)}{H(D_4D_1)}}. \end{aligned}$$

The results show that $L_3 > L_4$, it does not satisfy the constraint $0 \leq L_1 \leq L_2 \leq L_3 \leq L_4$ set $A = 2$ and go to Step 3.

Step 3 : Convert the inequality constraints $L_2 - L_1 \geq 0, L_3 - L_2 \geq 0$ into equality constraints $L_2 - L_1 = 0$ and $L_3 - L_2 = 0$. The Lagrangean method is

$$l(L_1, L_2, L_3, L_4, \lambda_1, \lambda_2) = P(TC - \lambda_1(L_2 - L_1) - \lambda_2(L_3 - L_2).$$

To find the minimization of $l(L_1, L_2, L_3, L_4, \lambda_1, \lambda_2)$

$$L_1 = L_2 = L_3 = \sqrt{\frac{2(A_2D_2 + A_3D_3 + A_4D_4) + ((H+F)(S_2^2 + S_3^2 + S_4^2)}{H(D_1D_4 + D_2D_3 + D_3D_2)}}$$

$$L_4 = \sqrt{\frac{2A_1D_1 + ((H+F)S_1^2)}{H(D_4D_1)}}$$

The results show that $L_1 > L_4$, it does not satisfy the constraint $0 \leq L_1 \leq L_2 \leq L_3 \leq L_4$ set $A = 3$ and go to Step 4.

Step 4 : Convert the inequality constraints $L_2 - L_1 \geq 0, L_3 - L_2 \geq 0, L_4 - L_3 \geq 0$ into equality constraints $L_2 - L_1 = 0, L_3 - L_2 = 0$ and $L_4 - L_3 = 0$.

The Lagrangian function is given by

$$l(L_1, L_2, L_3, L_4, \lambda_1, \lambda_2, \lambda_3) = P(TC - \lambda_1(L_2 - L_1) - \lambda_2(L_3 - L_2) - \lambda_3(L_4 - L_3).$$

To find the minimization of $l(L_1, L_2, L_3, L_4, \lambda_1, \lambda_2, \lambda_3)$

$$L_1 = L_2 = L_3 = L_4$$
$$= \sqrt{\frac{2(A_1D_1 + A_2D_2 + A_3D_3 + A_4D_4) + ((H + F)(S_1^2 + S_2^2 + S_3^2 + S_4^2)}{H(D_1D_4 + D_2D_3 + D_3D_2 + D_4D_1)}} \cdot$$

The solution $\tilde{L} = (L_1, L_2, L_3, L_4)$ satisfies all inequality constraints,

Let $L_1 = L_2 = L_3 = L_4 = (\tilde{L^*})$ Then the optimal fuzzy production quantity value is $(\tilde{L^*}) = (L^*, L^*, L^*, L^*)$

$$L^* = \sqrt{\frac{2(A_1D_1 + A_2D_2 + A_3D_3 + A_4D_4) + ((H + F)(S_1^2 + S_2^2 + S_3^2 + S_4^2)}{H(D_1D_4 + D_2D_3 + D_3D_2 + D_4D_1)}}\,.$$

5. Algorithm for finding fuzzy total cost and fuzzy optimal order quantity

Step 1 : Calculate $L = \sqrt{\dfrac{2DA + (H + F)S^2}{HD^2}}$.

Step 2 : Now, to find a fuzzy total cost using fuzzy operations on fuzzy setup cost and demand rate , taken as trapezoidal fuzzy numbers.

Step 3 : we find the fuzzy order length value $\tilde{L}$ Using signed distance Method.

Step 4: we find the fuzzy optimal order length value $\tilde{L}^* = (L^*, L^*, L^*, L^*)$ To find the min $P(\tilde{T}C)$, extension of the Lagrangian Method is Used,

Step 5: To check whether, the economic order quantity obtained by signed distance Method is same as the crisp order quantity.

6. Numerical Example

Example 1. ***Crisp Model:***

Thus, we consider a real inventory system for the growing items with

$D = 100,000$ *g/year,*

$H = 0.4$*/gram/year,*

$A = \$1000$*/gram/year,*

$F = \$2$*/gram/year.*

$S = 4082.4$

By minimizing the total cost we obtain the optimal length value

$$L = \sqrt{\frac{2DA + (H + F)S^2}{HD^2}}\,.$$

Fuzzy Model (trapezoidal number):

Let us take the following data;

$S = (3490.6, 3849, 4480, 4510)$

$A = (750, 850, 1150, 1250)$

$D = (80500, 95500, 109000, 115000)$

$H = 0.4$ */gram/year*

$F = 2$ */gram/year*

$L^* = 0.24$

7. Conclusion

In this 'Integrated Inventory EPQ Model' with fuzzy input parameters are represented as a trapezoidal fuzzy number. Lagrangian method has been applied and. Also for defuzzified using signed distance method. we have provided the Numerical example.

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