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RETRIEVABILITY IN INTERVAL NEUTROSOPHIC AUTOMATA

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ABSTRACT. In this paper, we introduce retrievability, quasi-retrievability, submachine using the concept of interval neutrosophic automaton and discuss their properties.

1. INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh in 1965 [5] as a generalizations of crisp sets. After these fuzzy sets, Attanasov introduced the concept of intuitionistic fuzzy sets in 1986 [1] which is an extension of fuzzy set. The concept of neutrosophy and neutrosophic set(NS) was introduced by Florentin Smarandache [3]. Wang et al., [4] introduced the notion of interval-valued neutrosophic sets. The interval neutrosophic set are characterized by an interval memebership degree, interval indeterminacy degree, and interval nonmembership degree. The concept of interval neutrosophic finite state machine was introduced by Tahir Mahmood [2]. In this paper, we introduced the concept of retrievability, quasi retrievability of interval neutrosophic automata and discussed their properties. Also, we provide the relation between retrievable and quasi-retrievable of interval neutrosophic automaton.

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2. Preliminaries

Definition 2.1. *[3] Let U be the universe of discourse. A neutrosophic set (NS) N in U is characterized by a truth membership function T_N, an indeterminacy membership function I_N and a falsity membership function F_N, where T_N, I_N, and F_N are real standard or non-standard subsets of $]0^-, 1^+[$. That is*

$$N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle, x \in U,\ T_A, I_A, F_A \in\ ]0^-, 1^+[\ \}$$

and with the condition $0^- \leq sup\ T_N(x) + sup\ I_N(x) + sup\ F_N(x) \leq 3^+$ we need to take the interval $[0,1]$ for technical applications instead of $]0^-, 1^+[$.

Definition 2.2. *[4] Let U be a universal set. An interval neutrosophic set (INS for short) is of the form*

$$\begin{aligned} N &= \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle \,|x \in U\} \\ &= \{\langle x, [\textit{inf}\ \alpha_N(x), \textit{sup}\ \alpha_N(x)], [\textit{inf}\ \beta_N(x), \textit{sup}\ \beta_N(x)], \\ &\quad [\textit{inf}\ \gamma_N(x), \textit{sup}\gamma_N(x)]\rangle \,|x \in U\}, \end{aligned}$$

where $\alpha_N(x), \beta_N(x), and \gamma_N(x)$ are the truth-membership, indeterminacy-membership and falsity membership functions for each $x \in U$.

$$\alpha_N(x),\ \beta_N(x),\ \gamma_N(x) \subseteq [0,1]$$

and

$$0 \leq \textit{sup}\ \alpha_N(x) +\ \textit{sup}\ \beta_N(x) +\ \textit{sup}\ \gamma_N(x) \leq\ 3.$$

Definition 2.3. *[4] An INS N is empty if inf $\alpha_N(x) =$ sup $\alpha_N(x) = 0$, inf $\beta_N(x) =$ sup $\beta_N(x) = 1$, inf $\gamma_N(x) =$ sup $\gamma_N(x) = 1$ forall $x \in U$.*

3. Interval Neutrosophic Automata

Definition 3.1. *[2] $M = (Q,\ \Sigma,\ N)$ is called interval neutrosophic automaton ($INAforshort$), where Q and Σ are non-empty finite sets called the set of states and input symbols respectively, and $N = \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle\}$ is an INS in $Q \times \Sigma \times Q$. The set of all words of finite length of Σ is denoted by Σ^*. The empty word is denoted by ϵ, and the length of each $x \in \Sigma^*$ is denoted by $|x|$.*

Definition 3.2. *[2] $M = (Q,\ \Sigma,\ N)$ be an INA. Define an INS $N^* = \{\langle \alpha_{N^*}(x), \beta_{N^*}(x), \gamma_{N^*}(x)\rangle\}$ in $Q \times \Sigma^* \times Q$ by*

$$\alpha_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [1,1] & \text{if } q_i = q_j \\ [0,0] & \text{if } q_i \neq q_j \end{cases}$$

$$\beta_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \text{if } q_i = q_j \\ [1,1] & \text{if } q_i \neq q_j \end{cases}$$

$$\gamma_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \text{if } q_i = q_j \\ [1,1] & \text{if } q_i \neq q_j \end{cases}$$

$$\alpha_{N^*}(q_i, w, q_j) = \alpha_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\alpha_{N^*}(q_i, x, q_r) \cup \alpha_{N^*}(q_r, y, q_j)],$$
$$\beta_{N^*}(q_i, w, q_j) = \beta_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\beta_{N^*}(q_i, x, q_r) \cup \beta_{N^*}(q_r, y, q_j)],$$
$$\gamma_{N^*}(q_i, w, q_j) = \gamma_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\gamma_{N^*}(q_i, x, q_r) \cup \gamma_{N^*}(q_r, y, q_j)],$$

$\forall q_i, q_j \in Q, w = xy, x \in \Sigma^*$ *and* $y \in \Sigma$.

Definition 3.3. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton and let $q_i, q_j \in Q$. Then q_i is called a immediate successor of q_j if the following condition holds. $\exists x \in \Sigma$ such that $\alpha_{N^*}(q_j, x, q_i) > [0,0],\ \beta_{N^*}(q_j, x, q_i) < [1,1],$ and $\gamma_{N^*}(q_j, x, q_i) < [1,1]$. We say that q_i is a successor of q_j if the following condition holds. $\exists w \in \Sigma^*$ such that $\alpha_{N^*}(q_j, w, q_i) > [0,0],\ \beta_{N^*}(q_j, w, q_i) < [1,1],$ and $\gamma_{N^*}(q_j, w, q_i) < [1,1]$. We denote by $S(q_j)$ the set of all successors of q_j. For any subset Q' of Q, the set of all successors of Q' denoted by $S(Q')$ is defined to be the set $S(Q') = \cup\{S(q_j) \mid q_j \in Q'\}$.*

Definition 3.4. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Let $Q' \subseteq Q$. Let $N_{Q'}$ be an interval neutrosophic subset of $Q' \times \Sigma \times Q$ and let $M' = (Q', \Sigma, N')$. The interval neutrosophic automaton M' is called submachine of M if*

(i) $\alpha_N|_{(Q' \times \Sigma \times Q')} = \alpha'_N$;
(ii) $\beta_N|_{(Q' \times \Sigma \times Q')} = \beta'_N$;
(iii) $\gamma_N|_{(Q' \times \Sigma \times Q')} = \gamma'_N$;
(iv) $S(Q') \subseteq Q'$.

Definition 3.5. *Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton. Let $M' = (Q', \Sigma, N') \neq \phi$ be a submachine of M. Then M' is separated if $S(Q \backslash Q') = (Q \backslash Q')$.*

Definition 3.6. *Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton. Then M is called strongly connected if $\forall\ q_i, q_j \in Q,\ q_i \in S(q_j)$.*

4. Retrievability in Interval Neutrosophic Automata

Definition 4.1. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. M is said to be retrievable if $\forall q_i \in Q,\ \forall y \in \Sigma^*$ and $\exists\ q_j \in Q$ such that $\alpha_{N^*}(q_i, y, q_j) > [0,0],\ \beta_{N^*}(q_i, y, q_j) < [1,1]$, and $\gamma_{N^*}(q_i, y, q_j) < [1,1]$ then $\exists\ x \in \Sigma^*$ such that $\alpha_{N^*}(q_j, x, q_i) > [0,0],\ \beta_{N^*}(q_j, x, q_i) < [1,1]$ and $\gamma_{N^*}(q_j, x, q_i) < [1,1]$.*

Definition 4.2. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. M is called quasi-retrievable if $\forall\ q_i \in Q, \forall\ y \in \Sigma^*$ and $\exists\ q_j \in Q$ such that $\alpha_{N^*}(q_i, y, q_j) > [0,0],\ \beta_{N^*}(q_i, y, q_j) < [1,1]$, and $\gamma_{N^*}(q_i, y, q_j) < [1,1]$ then $\exists\ x \in \Sigma^*$ such that $\alpha_{N^*}(q_i, yx, q_i) > [0,0],\ \beta_{N^*}(q_i, yx, q_i) < [1,1],\ \gamma_{N^*}(q_i, yx, q_i) < [1,1]$.*

Definition 4.3. *Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton. Let $q_i, q_j, q_k \in Q$. q_j and q_k are said to be q_i-related if $\exists\ y \in \Sigma^*$ such that $\alpha_{N^*}(q_i, y, q_j) > [0,0], \beta_{N^*}(q_i, y, q_j) < [1,1],\ \gamma_{N^*}(q_i, y, q_j) < [1,1],\ \alpha_{N^*}(q_i, y, q_k) > [0,0],\ \beta_{N^*}(q_i, y, q_k) < [1,1]$,and $\gamma_{N^*}(q_i, y, q_k) < [1,1]$. If q_j and q_k are q_i-related, then q_j and q_k are said to be q_i-twins if $S(q_k) = S(q_j)$.*

Definition 4.4. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. We say that M satisfies the exchange property if the following condition holds: Let $q_i, q_j \in Q$ and let $Q' \subseteq Q$. Suppose that if $q_i \in S(Q' \cup \{q_j\}),\ q_i \notin S(Q')$, then $q_j \in S(Q' \cup \{q_i\})$.*

5. Properties of Retrievability in Interval Neutrosophic Automata

Lemma 5.1. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Then the following conditions are equivalent.*

(i) $\forall\ q_i, q_j, q_k \in Q,\ \forall x, y \in \Sigma^*$ *if* $\alpha_{N^*}(q_i, y, q_j) > [0,0],\ \beta_{N^*}(q_i, y, q_j) < [1,1],\ \gamma_{N^*}(q_i, y, q_j) < [1,1],\ \alpha_{N^*}(q_i, yx, q_k) > [0,0],\ \beta_{N^*}(q_i, yx, q_k) < [1,1],\ \gamma_{N^*}(q_i, yx, q_k) < [1,1]$, *then* $q_k \in S(q_j)$.

(ii) $\forall q_i, q_j, q_l \in Q$, *if* q_j *and* q_l *are* q_i*-related, then* q_j *and* q_l *are* q_i*-twins.*

Proof.

$(i) \Rightarrow (ii)$: Let $q_i, q_j, q_k \in Q$ be such that q_j and q_k are q_i-related. Then $\exists\, y \in \Sigma^*$ such that $\alpha_{N^*}(q_i, y, q_j) > [0,0], \beta_{N^*}(q_i, y, q_j) < [1,1], \gamma_{N^*}(q_i, y, q_j) < [1,1]$ and

$$\alpha_{N^*}(q_i, y, q_k) > [0,0], \beta_{N^*}(q_i, y, q_k) < [1,1], \gamma_{N^*}(q_i, y, q_k) < [1,1]. \tag{5.1}$$

Let $q_l \in S(q_k)$. Then $\exists\, x \in \Sigma^*$ such that $\alpha_{N^*}(q_k, x, q_l) > [0,0],\ \beta_{N^*}(q_k, x, q_l) < [1,1]$, and $\gamma_{N^*}(q_k, x, q_l) < [1,1]$.

Then by (5.1) we have

$$\alpha_{N^*}(q_i, yx, q_l) > [0,0],\ \beta_{N^*}(q_i, yx, q_l) < [1,1],\ \text{and } \gamma_{N^*}(q_i, yx, q_l) < [1,1]. \tag{5.2}$$

From (5.1), (5.2) and by (i), $q_l \in S(q_j)$. Similarly if $q_l \in S(q_j)$, then $q_l \in S(q_k)$. Therefore $S(q_j) = S(q_k)$. Hence q_j and q_k are q_i-twins.

$(ii) \Rightarrow (i)$: Let $q_i,\ q_j,\ q_l \in Q$ and $x, y \in \Sigma^*$ be such that $\alpha_{N^*}(q_i, y, q_j) > [0,0]$, $\beta_{N^*}(q_i, y, q_j) < [1,1],\ \gamma_{N^*}(q_i, y, q_j) < [1,1]\ \alpha_{N^*}(q_i, yx, q_l) < [0,0],\ \beta_{N^*}(q_i, yx, q_l) < [1,1]$, and $\gamma_{N^*}(q_i, yx, q_l) < [1,1]$, where

$\alpha_{N^*}(q_i, yx, q_l) = \vee_{q_j\, \in\, Q} \{\alpha_{N^*}(q_i,\ y,\ q_j) \wedge \alpha_{N^*}(q_j, x, q_l)\} > [0,0]$,

$\beta_{N^*}(q_i, yx, q_l) = \wedge_{q_j\, \in\, Q} \{\beta_{N^*}(q_i,\ y,\ q_j) \vee \beta_{N^*}(q_j, x, q_l)\} < [1,1]$,

$\gamma_{N^*}(q_i, yx, q_l) = \wedge_{q_j\, \in\, Q} \{\gamma_{N^*}(q_i,\ y,\ q_j) \vee \gamma_{N^*}(q_j, x, q_l)\} < [1,1]$.

Hence $\exists\, q_k \in Q$ such that

$\alpha_{N^*}(q_i, y, q_k) > [0,0],\ \beta_{N^*}(q_i, y, q_k) < [1,1]$, and $\gamma_{N^*}(q_i, y, q_k) < [1,1]$. This implies $q_l \in S(q_k)$. Thus by the hypothesis $q_l \in S(q_j)$ $[Since\ S(q_j) = S(q_k)]$.

□

Lemma 5.2. *Let* $M = (Q, \Sigma, N)$ *be an interval neutrosophic finite automaton. Then the following conditions are equivalent:*

(i) M *is retrievable;*

(ii) M *is quasi-retrievable and* $\forall q_i, q_j, q_k \in Q$, *if* q_j *and* q_k *are* q_i*-related, then* q_j *and* q_k *are* q_i*-twins.*

Proof.

$(i) \Rightarrow (ii)$: It is immediate that retrievability implies quasi retrievability. Let $q_i, q_j, q_k \in Q$ and let $q_j,\ q_k$ are q_i-related. Then there exists $y \in \Sigma^*$ such that

$$\alpha_{N^*}(q_i, y, q_k) > [0,0],\ \beta_{N^*}(q_i, y, q_k) < [1,1],\ \gamma_{N^*}(q_i, y, q_k) < [1,1]. \tag{5.3}$$

Let $q_l \in S(q_k)$. Then there exists $x \in \Sigma^*$ such that $\alpha_{N^*}(q_k, x, q_l) > [0,0]$, $\beta_{N^*}(q_k, x, q_l) < [1,1], \gamma_{N^*}(q_k, x, q_l) < [1,1]$. Now,

(5.4) $\alpha_{N^*}(q_i, yx, q_l) > [0,0],\ \beta_{N^*}(q_i, yx, q_l) < [1,1],$ and $\gamma_{N^*}(q_i, yx, q_l) < [1,1]$.

From (5.3) and (5.4), and Lemma 5.1 $q_l \in S(q_j)$. Therefore,

$$S(q_k) \subseteq S(q_j). \tag{5.5}$$

Similarly we can prove that

$$S(q_j) \subseteq S(q_k)\,. \tag{5.6}$$

From (5.5) and (5.6), $S(q_k) = S(q_j)$. Hence q_j and q_k are q_i-twins.

$(ii) \Rightarrow (i)$: Let $q_i \in Q$ and $y \in \Sigma^*$. Suppose $\exists\ q_m \in Q$ such that $\alpha_{N^*}(q_i, y, q_m) > [0,0]$, $\beta_{N^*}(q_i, y, q_m) < [1,1]$, and $\gamma_{N^*}(q_i, y, q_m) < [1,1]$. Then $\exists x \in X^*$ such that $\alpha_{N^*}(q_i, yx, q_i) > [0,0],\ \beta_{N^*}(q_i, yx, q_i) < [1,1],$ and $\gamma_{N^*}(q_i, yx, q_i) < [1,1]$. Since M is quasi retrievable by Lemma 5.1, $q_i \in S(q_m)$. That is, there exists $z \in \Sigma^*$ such that $\alpha_{N^*}(q_m, z, q_i) > [0,0]\ \beta_{N^*}(q_m, z, q_i) < [1,1]$, and $\gamma_{N^*}(q_m, z, q_i) < [1,1]$. Hence M is retrievable. □

Theorem 5.1. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Then the following conditions are equivalent: (i) M is retrievable; (ii) M is the union of strongly connected submachines; (iii) M satisfies the exchange property.*

Proof.
$(i) \Rightarrow (ii)$: Let $q_j \in Q$ and let $q_k,\ q_l \in S(q_j)$. Then $\exists y, y_1 \in \Sigma^*$ such that $\alpha_{N^*}(q_j, y, q_k) > [0,0],\ \beta_{N^*}(q_j, y, q_k) < [1,1],$ and $\gamma_{N^*}(q_j, y, q_k) < [1,1]$. Also, $\alpha_{N^*}(q_j, y_1, q_l) > [0,0],\ \beta_{N^*}(q_j, y_1, q_l) < [1,1],$ and $\gamma_{N^*}(q_j, y_1, q_l) < [1,1]$. Therefore, $q_j \in S(q_k)$. Hence $\langle q_j \rangle$ is strongly connected. Hence $M = \cup_{q_j \in Q} \langle q_j \rangle$.

$(ii) \Rightarrow (i)$: $M = \cup_{i=1}^m M_i$, and each $M_i = (Q_i, \Sigma^*, N_i)$ is strongly connected. Let $q_j \in Q, y \in \Sigma^*$ then $\exists q_l \in Q$ such that $\alpha_{N^*}(q_j, y, q_l) > [0,0],\ \beta_{N^*}(q_j, y, q_l) < [1,1]$, and $\gamma_{N^*}(q_j, y, q_l) < [1,1]$. Now, $q_j \in Q_i$ for some i. Therefore, $q_l \in S(q_j) \subseteq S(Q_i)$. Since M_i is strongly connected, $q_j \in S(q_l)$. Hence, $\exists x \in \Sigma^*$ such that $\alpha_{N^*}(q_l, y, q_j) > [0,0],\ \beta_{N^*}(q_l, y, q_j) < [1,1],$ and $\gamma_{N^*}(q_l, y, q_j) < [1,1]$. Therefore, M is retrievable.

$(ii) \Rightarrow (iii)$: $M = \cup_{i=1}^m M_i$, and each $M_i = (Q_i, \Sigma^*, N_i)$ is strongly connected. Let $q_i, q_j \in Q$. Suppose $q_i \in S(q_j)$. Now, $q_j \in Q_i$ for some i. Then $q_i \in S(q_j) \subseteq S(Q_i) = Q_i$. Therefore, $q_i, q_j \in Q_i$. Since $M_i$is strongly connected,

$q_j \in S(q_i)$. Hence M satisfies the exchange property.

$(iii) \Rightarrow (ii)$: Let M satisfies the exchange property. Then for any $q_i, q_j \in Q$ and $Q' \subseteq Q$, if $q_i \in S(Q' \cup \{q_j\}),\ q_i \notin S(Q')$, then $q_j \in S(Q' \cup \{q_i\})$. Therefore $q_i \notin S(Q' \backslash q_i), \forall q_i \in Q'$. Hence, $M = \cup_{i=1}^{m} \langle q_{i'} \rangle$ is a basis of M. Also, $S(q_{i'}) \cap S(q_{j'}) = \phi$ if $i \neq j$. suppose $q_i, q_j \in S(q_{i'})$,then $q_{i'} \in S(q_i)$. Therefore, $q_j \in S(q_i)$. Hence, $\langle q_{i'} \rangle$ is strongly connected. □

Theorem 5.2. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Then the following conditions are equivalent:*

(i) *M is strongly connected;*
(ii) *M is connected and retrievable;*
(iii) *Every submachine of M is strongly connected.*

Proof.

$(i) \Rightarrow (ii)$: Let M is strongly connected interval neutrosophic automaton. Then $\forall\ q_i, q_j \in Q,\ q_i \in S(q_j)$. Since, M is strongly connected it has no proper submachines. Also it has no proper separated submachines. Hence interval neutrosophic automaton M is connected. Now, let $q_j, q_l \in Q$ and $y \in \Sigma^*$ such that $\alpha_{N^*}(q_j, y, q_l) > [0,0],\ \beta_{N^*}(q_j, y, q_l) < [1,1]$ and $\gamma_{N^*}(q_j, y, q_l) < [1,1]$. Since, M is strongly connected $q_j \in S(q_l)$. Then there exits $x \in \Sigma^*$ such that $\alpha_{N^*}(q_l, x, q_j) > [0,0],\ \beta_{N^*}(q_l, x, q_j) < [1,1]$ and $\gamma_{N^*}(q_l, y, q_j) < [1,1]$. Hence, M is retrievable interval neutrosophic automaton.

$(ii) \Rightarrow (iii)$: Let $M' = (Q',\ \Sigma,\ N')$ be a submachine of interval neutrosophic automaton M. If $q_i, q_j \in Q'$ and $q_i \notin S(q_i)$ then $S(q_j) \neq Q$. Thus $M_1 = (Q'', \Sigma, N|_{S(q_j)\times\Sigma\times S(q_j)}),\ Q'' = S(q_j)$ is a proper submachine of M. Also, by the hypothesis M is connected $S(Q\backslash S(q_j)) \cap S(q_j) \neq \phi$.

Let $q_k \in S(Q\backslash S(q_j)) \cap S(q_j)$. Then $q_k \in S(q_l)$ for some $q_l \in Q\backslash S(q_j)$ and $q_k \in S(q_j)$. Now, $\exists y \in \Sigma^*$ such that $\alpha_{N^*}(q_l, y, q_k) > [0,0],\ \beta_{N^*}(q_l, y, q_k) < [1,1]$ and $\gamma_{N^*}(q_l, y, q_k) < [1,1]$. Since M retrievable, $\exists x_1 \in \Sigma^*$ such that $\alpha_{N^*}(q_k, y, q_l) > [0,0],\ \beta_{N^*}(q_k, y, q_l) < [1,1]$ and $\gamma_{N^*}(q_k, y, q_l) < [1,1]$. Therefore, $q_l \in S(q_j)$. Thus,$q_l \in S(q_j) \subseteq S(q_j)$, which is a contradiction. Thus $q_i \in S(q_j), \forall p_i, q_j \in Q$. Hence M' is strongly connected.

$(iii) \Rightarrow (i)$: Every submachine of M is strongly connected. Then every submachine of M is connected and retrievable. Let $q_i \in Q$ and $q_k,\ q_l \in S(q_j)$. Then $\exists y, y_1 \in \Sigma^*$ such that $\alpha_{N^*}(q_j, y, q_k) > [0,0],\ \beta_{N^*}(q_j, y, q_k) < [1,1]$, and

$\gamma_{N^*}(q_j, y, q_k) < [1,1]$. Also, $\alpha_{N^*}(q_j, y_1, q_l) > [0,0]$, $\beta_{N^*}(q_j, y_1, q_l) < [1,1]$, and $\gamma_{N^*}(q_j, y_1, q_l) < [1,1]$. Therefore, $q_j \in S(q_k)$. Hence $\langle q_j \rangle$ is strongly connected. Hence $M = \cup_{q_j \in Q} \langle q_j \rangle$. Therefore M is strongly connected. □

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