SOME BISTAR RELATED EDGE BIMAGIC HARMONIOUS GRAPHS

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ABSTRACT. A graph \( G = (V, E) \) with \( p \) vertices and \( q \) edges is said to be edge bimagic harmonious if there exists a bijection \( f: V \cup E \rightarrow \{1, 2, 3, \ldots, p + q\} \) such that for each edge \( xy \) in \( E(G) \), the value of \( ([f(x) + f(y)](mod \, q) + f(xy)) \) is equal to \( k_1 \) or \( k_2 \), where \( k_1 \) and \( k_2 \) are two distinct magic constants. In this paper we prove that the \( \langle B_{m,n} : 2 \rangle \), restricted square graph of \( B_{n,n} \) and duplication of apex vertex of \( B_{n,n} \) are edge bimagic harmonious graphs.

1. INTRODUCTION


\( B_{m,n} \) is a \( (m, n) \) bistar obtained from two disjoint copies of \( K_{1,m} \) and \( K_{1,n} \) by joining the central vertices by an edge [8]. The graph \( \langle B_{m,n} : 2 \rangle \) obtained from the graph \( B_{m,n} \) by subdividing the middle edge with a new vertex [8]. The restricted square [4] of \( B_{n,n} \) is a graph \( G \) with vertex set \( V(G) = V(B_{n,n}) \) and edge set \( E(G) = E(B_{n,n}) \cup \{uv_i, vu_i/1 \leq i \leq n\} \).

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2010 Mathematics Subject Classification. 05C78.

Key words and phrases. Graph, Bijection, Harmonious, Labeling, Magic, Bimagic, Bistar graphs.
Duplication [4] of a vertex \( v \) of a graph \( G \) produces a new graph \( G' \) by adding a vertex \( v' \) with \( N(v') = N(v) \). In other words a vertex \( v' \) is said to be a duplication of \( v \) if all the vertices which are adjacent to \( v \) are now adjacent to \( v' \). A graph \( G = (V, E) \) with \( p \) vertices and \( q \) edges is said to be edge bimagic harmonious if there exists a bijection \( f : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., p + q\} \) such that for each edge \( xy \) in \( E(G) \), the value of \( [(f(x) + f(y))(mod \ q) + f(xy)] = k_1 \) or \( k_2 \), called two distinct magic constants [9]. In this paper we prove that the \( \langle B_{m,n} : 2 \rangle \), restricted square graph of \( B_{n,n} \) and duplication of apex vertex of \( B_{n,n} \) are edge bimagic harmonious graphs.

2. Main Results

Theorem 2.1. The graph \( \langle B_{m,n} : 2 \rangle \) admits an edge bimagic harmonious labeling for all \( m \) and \( n \).

Proof. Let \( V = \{u, v, w, u_i, v_j/1 \leq i \leq m, 1 \leq j \leq n\} \) and
\[
E = \{uu_i, vv_j, uw, wv/1 \leq i \leq m, 1 \leq j \leq n\}
\]
be the vertex set and the edge set of the graph \( \langle B_{m,n} : 2 \rangle \). The graph \( \langle B_{m,n} : 2 \rangle \) has \((m + n + 3)\) vertices and \((m + n + 2)\) edges. Define a bijection \( f : V \cup E \rightarrow \{1, 2, 3, ..., 2m + 2n + 5\} \) such that
\[
\begin{align*}
f(u) &= 1 \\
f(v) &= m + n + 2 \\
f(u_i) &= i + 1, 1 \leq i \leq m \\
f(v) &= m + n + 2 \\
f(v_j) &= m + j + 1, 1 \leq j \leq n \\
f(w) &= m + n + 3 \\
f(uu_i) &= 2m + 2n - i + 4, 1 \leq i \leq m \\
f(uw) &= 2m + 2n + 4 \\
f(wv) &= 2m + 2n + 5 \\
f(vv_j) &= m + 2n - j + 4, 1 \leq j \leq n.
\end{align*}
\]

Using these labelings, there exist two magic constants for each edge \( xy \in E \), \([ (f(x) + f(y))(mod \ q) + f(xy) ] \) yields any one of the magic constants \( k_1 = 2(m + n + 3) \) and \( k_2 = 2m + 2n + 5 \). Therefore, the graph \( \langle B_{m,n} : 2 \rangle \) admits an edge bimagic harmonious labeling for all \( m \) and \( n \). \( \square \)
Example 1. Bimagic harmonious labeling of ($B_{5,6} : 2$) is given in figure 1.

![Figure 1: ($B_{5,6} : 2$) with $k_1 = 28$ and $k_2 = 27$.](image)

Theorem 2.2. The restricted square of bistar $B_{n,n}$ admits an edge bimagic harmonious labeling for all $n$.

Proof. Let $V = \{u, v, u_i, v_i / 1 \leq i \leq n\}$ and $E = \{uu_i, vv_i, uv, uv_i / 1 \leq i \leq n\}$ be the vertex set and the edge set of the restricted square of bistar $B_{n,n}$. The restricted square graph of $B_{n,n}$ has $2n + 2$ vertices and $4n + 1$ edges.

Define a bijection $f : V \cup E \rightarrow \{1, 2, 3, ..., 6n + 3\}$ such that

$f(u) = 1$
$f(u_i) = i + 1, 1 \leq i \leq n$
$f(v) = 2n + 2$
$f(v_i) = n + i + 1, 1 \leq i \leq n$
$f(uu_i) = 4n - i + 3, 1 \leq i \leq n$
$f(uv_i) = 3n - i + 3, 1 \leq i \leq n$
$f(uv) = 6n + 3$
$f(vv_i) = 5n - i + 3, 1 \leq i \leq n$
$f(vu_i) = 6n - i + 3, 1 \leq i \leq n$

Using these labelings, there exist two magic constants for each edge $xy \in E$, $[(f(x) + f(y))(\text{mod} \ q) + f(xy)]$ yields any one of the magic constants $k_1 = 4n + 5$ and $k_2 = 8n + 6$. Therefore, the restricted square of bistar $B_{n,n}$ admits an edge bimagic harmonious labeling for all $n$. □
Example 2. Bimagic harmonious labeling of restricted square of bistar \( B_{5,5} \) is given in figure 3.

![Figure 3: Restricted square of bistar \( B_{5,5} \) with \( k_1 = 25 \) and \( k_2 = 46 \).](image)

Theorem 2.3. Duplication of apex vertex of \( B_{n,n} \) admits an edge bimagic harmonious labeling for all \( n \).

Proof. Let \( V = \{ u, v, u_i, v_i, v'/1 \leq i \leq n \} \) and \( E = \{ uu_i, vv_i, uv, uv', vv'/1 \leq i \leq n \} \) be the vertex set and the edge set of the duplication of apex vertex of \( B_{n,n} \). The duplication of apex vertex of \( B_{n,n} \) has \( 2n + 3 \) vertices and \( 3n + 2 \) edges.

Define a bijection \( f : V \cup E \to \{ 1, 2, 3, ..., 5n + 5 \} \) such that

\[
\begin{align*}
  f(u) &= 1 \\
  f(u_i) &= i + 1, 1 \leq i \leq n \\
  f(v) &= n + 2 \\
  f(v_i) &= n + i + 2, 1 \leq i \leq n \\
  f(v') &= 2n + 3 \\
  f(uu_i) &= 5n - i + 3, 1 \leq i \leq n \\
  f(uv) &= 4n + 2 \\
  f(uv') &= 3n + 1 \\
  f(uv') &= 5n + 5, \text{ for } n = 2 \\
  f(vv_i) &= 3n - i + 1, 1 \leq i \leq n - 3
\end{align*}
\]
Using these labelings, there exist two magic constants for each edge \( xy \in E \), 
\[ [(f(x) + f(y))(\text{mod} \ q) + f(xy)] \] yields any one of the magic constants \( k_1 = 5n + 5 \) and \( k_2 = 4n + 5 \). Therefore, duplication of apex vertex of \( B_{n,n} \) admits an edge bimagic harmonious labeling for all \( n \).

Example 3. Bimagic harmonious labeling of duplication of apex vertex of bistar \( B_{5,5} \) is given in figure 4.

![Figure 4: Duplication of apex vertex of bistar \( B_{5,5} \) with \( k_1 = 30 \) and \( k_2 = 25 \).](image)

3. CONCLUSION

In this paper, we proved that the \( \langle B_{m,n} : 2 \rangle \), restricted square graph of \( B_{n,n} \) and duplication of apex vertex of \( B_{n,n} \) are edge bimagic harmonious graphs.

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