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APPLICATION OF BIPOLAR FUZZY ROUGH SETS

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ABSTRACT. The aim this paper is to introduce weighted geometric aggregation operator (WGAO)using bipolar fuzzy rough set (BFRS). A multi-criteria decision making method (MCDM) based on bipolar fuzzy rough set is developed. Further, an example is given to explain this method.

1. INTRODUCTION

Pawlak [4, 5] proposed the rough set theory. Dubois et al. [2, 3] introduced the concept of fuzzy rough sets. Zhang [7] developed bipolar fuzzy set theory. Thillaigovindan et al. [6] dealt with MCDM problems on $IFSSRT$. This paper deals with a score function based on $BFRS.$ A MCDM mothod based on BFR set is developed. Further, an example is given to explain this method.

2. $BFRS$ $WGAO$ AND $BFRS$ SCORE FUNCTION

This section deals with $WGAO$ based on $BFRS$. A score function based on $BFRS$ is defined. $BFRS$ is defined in [1].

Definition 2.1. *The entire data set of $BFRS$ is represented in the form of a $m \times k$ matrix.*

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$$BFRM = \begin{array}{c} \\ BFR(A_1) \\ BFR(A_2) \\ \vdots \\ BFR(A_m) \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_k \end{array} \\ \begin{pmatrix} (\underline{M}_{11}, \overline{M}_{11}) & (\underline{M}_{12}, \overline{M}_{12} & \cdots & (\underline{M}_{1k}, \overline{M}_{1k} \\ (\underline{M}_{21}, \overline{M}_{21}) & (\underline{M}_{22}, \overline{M}_{22}) & \cdots & (\underline{M}_{2k}, \overline{M}_{2k}) \\ \vdots & \vdots & \vdots & \vdots \\ (\underline{M}_{m1}, \overline{M}_{m1}) & (\underline{M}_{m2}, \overline{M}_{m2}) & \cdots & (\underline{M}_{mk}, \overline{M}_{mk}) \end{pmatrix} \end{array}$$

where $\underline{M}_{11} = (\underline{\mu}^n_{11}, \underline{\mu}^p_{11})$ *and* $\overline{M}_{11} = (\overline{\mu}^n_{11}, \overline{\mu}^p_{11})$.

Definition 2.2. *Let A be a $BFRS$ over $U = \{x_1, x_2, ..., x_m\}$ and $C = \{c_1, c_2, ..., c_k\}$ be the set of criteria. A WGAO of $BFRS(U)$ is defined as*

$$\xi_i = \Bigg(\Bigg(\frac{2\prod_{j=1}^{k}(-p_{ij})^{wt_j}}{\prod_{j=1}^{k}(-p_{ij})^{wt_j} - \prod_{j=1}^{k}(2-p_{ij})^{wt_j}}, \frac{2\prod_{j=1}^{k}(q_{ij})^{wt_j}}{\prod_{j=1}^{k}(q_{ij})^{wt_j} + \prod_{j=1}^{k}(2-q_{ij})^{wt_j}}\Bigg),$$

$$\Bigg(\frac{\prod_{j=1}^{k}(1+r_{ij})^{wt_j} - \prod_{j=1}^{k}(1-r_{ij})^{wt_j}}{\prod_{j=1}^{k}(1+r_{ij})^{wt_j} + \prod_{j=1}^{k}(1-r_{ij})^{wt_j}}, \frac{\prod_{j=1}^{k}(1+s_{ij})^{wt_j} - \prod_{j=1}^{k}(1-s_{ij})^{wt_j}}{\prod_{j=1}^{k}(1+s_{ij})^{wt_j} + \prod_{j=1}^{k}(1-s_{ij})^{wt_j}}\Bigg)\Bigg)$$

where

$$\begin{aligned}(p_{ij}, q_{ij}) &= (\mu_{BF\underline{R}^n(A_i)}(x_i), \mu_{BF\underline{R}^p(A_i)}(x_i)), (r_{ij}, s_{ij}) \\ &= (\mu_{BF\overline{R}^n(A_i)}(x_i), \mu_{BF\overline{R}^p(A_i)}(x_i))),\end{aligned}$$

wt_j *is weight of criteria* $c_j \ni wt_j \in [0,1]$, $j = 1, 2, \cdots, k$ *and* $\sum_{j=1}^{k} wt_j = 1$.

Definition 2.3. *The BFR degree is defined as*

$$\lambda_{ij} = 1 - |\mu_{BF\underline{R}^n(A)}(x) - \mu_{BF\overline{R}^n(A)}(x) + \mu_{BF\underline{R}^p(A)}(x) - \mu_{BF\overline{R}^p(A)}(x)|.$$

Definition 2.4. *The $BFRS$ is* $E_j = \frac{1}{m}\sum_{i=1}^{m} \lambda_{ij}$.

Definition 2.5. *The weight* E_j *is,*

$$wt_j = \frac{1 - E_j}{\sum_{j=1}^{k}(1 - E_j)}, j = 1, 2, ..., k.$$

Thus we obtain the weight vector $wt = (wt_1, wt_2, ..., wt_k)$ *which satisfying* $\sum_{j=1}^{k} wt_j = 1$.

Definition 2.6. *For a* BFR *set, the score function is*

$$BFRS(\zeta_i) = \left| \frac{\mu^n_{BF\underline{R}}(x) + \mu^p_{BF\underline{R}}(x) + (-1 - \mu^n_{BF\overline{R}}(x)) + (1 - \mu^p_{BF\overline{R}}(x))}{4 + (-1 - \mu^n_{BF\underline{R}}(x) - \mu^n_{BF\overline{R}}(x)) + (1 - \mu^p_{BF\underline{R}}(x) - \mu^p_{BF\overline{R}}(x))} \right|,$$

where $BFRS(\zeta_i) \in [-1, 1]$.

3. Method

Consider a set $U = \{x_1, x_2, ..., x_m\}$ of m alternatives and a set $C = \{c_1, c_2, ..., c_k\}$ of k criteria. Corresponding to criteria c_j, each alternative x_i is considered as a $BFRS$ over U. Weight wt_j is assigned to each criteria. Each alternative $BFR(A_i)$ to reduced to a single $BFRS$ $((\underline{\mu_i}^n, \underline{\mu_i}^p), (\overline{\mu_i}^n, \overline{\mu_i}^p)) = \zeta_i$ on application of the $WGAO$. The score function is used to convert the $BFRS$ of each alternative to $BFRS(\zeta_i)$. On comparing the score function values between $BFR(A_i)$, the largest value is chosen as the best.

3.1. **Procedure:** The procedure for solving the MCDM problems with $BFRSs$ is as follows:

Step 1: Compute $BFRSs$, $BFR(A_i)$ $(i = 1, 2, ..., m)$ and form the $BFRM$.

Step 2: The $BFRSs$ are aggregated to a single value by finding the fuzzy degree λ_{ij}. Using this the $BFRS$ entropy of evaluation index E_j is computed. The weight wt_j corresponding to c_j $(j = 1, 2, ..., k)$ is then calculated using E_j values.

Step 3: The WGA value ζ_i for each alternative $BFR(A_i)$ is calculated using Definition 2.5.

Step 4: Compute the score function value $BFRS(\zeta_i)$ for each ζ_i by Definition 2.6.

Step 5: The alternative are ranked depending on the values of $BFRS(\zeta_i)$. The alternative corresponding to maximum value of $BFRS(\zeta_i)$ is the best.

3.2. **Application.** A wireless communication engineer has to decide on the working of four types of antennas viz. short dipole, dipole, monopole and loop antennas which are represented by bipolar fuzzy rough sets $BFR(A_1), BFR(A_2), BFR(A_3), BFR(A_4)$ respectively. By considering the properties of the antennas as c_1= antenna gain, c_2= aperture, c_3= bandwidth, c_4= polarization and c_5= effective length, the best performing antenna is to be selected under these five criteria.

Step 1: Consider $U = \{x_1, x_2, x_3, x_4, x_5\}$,
$A_1 = \{x_1/(-0.25, 0.34), x_2/(-0.24, 0.45), x_3/(-0.4, 0.55), x_4/(-0.38, 0.6), x_5/(-0.25, 0.62)\}$
and

$$BF\mathbb{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{pmatrix} (-1,1) & (-0.4,0.64) & (-0.4,0.64) & (-0.4,0.64) & (-0.4,0.64) \\ (-0.4,0.64) & (-1,1) & (-0.7,0.74) & (-0.7,0.74) & (-0.7,0.74) \\ (-0.4,0.64) & (-0.7,0.74) & (-1,1) & (-0.8,0.84) & (-0.8,0.84) \\ (-0.4,0.64) & (-0.7,0.74) & (-0.8,0.84) & (-1,1) & (-0.9,0.94) \\ (-0.4,0.64) & (-0.7,0.74) & (-0.8,0.84) & (-0.9,0.94) & (-1,1) \end{pmatrix} \end{array}$$

$BFR(A_1) = \{\{x_1/(-0.4, 0.34), x_2/(-0.3, 0.34), x_3/(-0.3, 0.34), x_4/(-0.3, 0.34), x_5/(-0.3, 0.34)\}, \{x_1/(-0.4, 0.62), x_2/(-0.4, 0.62), x_3/(-0.4, 0.62), x_4/(-0.4, 0.62), x_5/(-0.4, 0.62)\}\}$.

Consider $U = \{x_1, x_2, x_3, x_4, x_5\}$, $A_2 = \{x_1/(-0.11, 0.4), x_2/(-0.21, 0.5), x_3/(-0.28, 0.53), x_4/(-0.4, 0.58), x_5/(-0.45, 0.35)\}$ and

$$BF\mathbb{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{pmatrix} (-1,1) & (-0.48,0.58) & (-0.48,0.58) & (-0.48,0.58) & (-0.48,0.58) \\ (-0.48,0.58) & (-1,1) & (-0.6,0.68) & (-0.6,0.68) & (-0.6,0.68) \\ (-0.48,0.58) & (-0.6,0.68) & (-1,1) & (-0.68,0.75) & (-0.68,0.75) \\ (-0.48,0.58) & (-0.6,0.68) & (-0.68,0.75) & (-1,1) & (-0.72,0.8) \\ (-0.48,0.58) & (-0.6,0.68) & (-0.68,0.75) & (-0.72,0.8) & (-1,1) \end{pmatrix} \end{array}$$

$BFR(A_2) = \{\{x_1/(-0.45, 0.4), x_2/(-0.4, 0.35), x_3/(-0.32, 0.35), x_4/(-0.28, 0.35), x_5/(-0.28, 0.35)\}, \{x_1/(-0.48, 0.58), x_2/(-0.48, 0.58), x_3/(-0.48, 0.58), x_4/(-0.48, 0.58), x_5/(-0.48, 0.58)\}\}$.

Consider $U=\{x_1,x_2,x_3,x_4,x_5\}$, $A_3=\{x_1/(-0.23,0.21),x_2/(-0.15,0.3),$ $x_3/(-0.008,0.27),x_4/(-0.012,0.38),x_5/(-0.16,0.009)\}$ and

$$BF\mathbb{R}=\begin{array}{c} \\ x_1\\x_2\\x_3\\x_4\\x_5\end{array}\begin{array}{c}\begin{array}{ccccc}x_1 & x_2 & x_3 & x_4 & x_5\end{array}\\ \begin{pmatrix}(-1,1)&(-0.35,0.4)&(-0.35,0.4)&(-0.35,0.4)&(-0.35,0.4)\\(-0.35,0.4)&(-1,1)&(-0.55,0.62)&(-0.55,0.62)&(-0.55,0.62)\\(-0.35,0.4)&(-0.55,0.62)&(-1,1)&(-0.65,0.88)&(-0.65,0.88)\\(-0.35,0.4)&(-0.55,0.62)&(-0.65,0.88)&(-1,1)&(-0.78,0.95)\\(-0.35,0.4)&(-0.55,0.62)&(-0.65,0.88)&(-0.78,0.95)&(-1,1)\end{pmatrix}\end{array}$$

$BFR(A_3)=\{\{x_1/(-0.16,0.21),x_2/(-0.23,0.3),x_3/(-0.23,0.12),$
$x_4/(-0.23,0.05),x_5/(-0.23,0.009)\},$
$\{x_1/(-0.35,0.38),x_2/(-0.35,0.38),x_3/(-0.35,0.38),$
$x_4/(-0.35,0.38),x_5/(-0.35,0.38)\}\}.$

Consider $U=\{x_1,x_2,x_3,x_4,x_5\}$, $A_4=\{x_1/(-0.02,0.11),x_2/(-0.09,0.18),$ $x_3/(-0.07,0.2),x_4/(-0.05,0.13),x_5/(-0.006,0.005)\}$ and

$$BF\mathbb{R}=\begin{array}{c} \\ x_1\\x_2\\x_3\\x_4\\x_5\end{array}\begin{array}{c}\begin{array}{ccccc}x_1 & x_2 & x_3 & x_4 & x_5\end{array}\\ \begin{pmatrix}(-1,1)&(-0.11,0.21)&(-0.11,0.21)&(-0.11,0.21)&(-0.11,0.21)\\(-0.11,0.21)&(-1,1)&(-0.23,0.41)&(-0.23,0.41)&(-0.23,0.41)\\(-0.11,0.21)&(-0.23,0.41)&(-1,1)&(-0.36,0.56)&(-0.36,0.56)\\(-0.11,0.21)&(-0.23,0.41)&(-0.36,0.56)&(-1,1)&(-0.49,0.62)\\(-0.11,0.21)&(-0.23,0.41)&(-0.36,0.56)&(-0.49,0.62)&(-1,1)\end{pmatrix}\end{array}$$

$BFR(A_4)=\{\{x_1/(-0.09,0.11),x_2/(-0.07,0.18),x_3/(-0.09,0.2),$
$x_4/(-0.09,0.13),x_5/(-0.09,0.005)\},$
$\{x_1/(-0.11,0.2),x_2/(-0.2,0.2),x_3/(-0.2,0.2),$
$x_4/(-0.2,0.2),x_5/(-0.2,0.2)\}\}.$

Bipolar fuzzy rough decision matrix

$$BFRM=\begin{array}{c}\\ BFR(A_1)\\BFR(A_2)\\BFR(A_3)\\BFR(A_4)\end{array}\begin{pmatrix} & c_1 & c_2 & c_3 \\ & (-0.4,0.34)(-0.4,0.62) & (-0.3,0.34)(-0.4,0.62) & (-0.3,0.34)(-0.4,0.62)\\ & (-0.45,0.4)(-0.48,0.58) & (-0.4,0.35)(-0.48,0.58) & (-0.32,0.35)(-0.48,0.58)\\ & (-0.16,0.21)(-0.35,0.38) & (-0.23,0.3)(-0.35,0.38) & (-0.23,0.12)(-0.35,0.38)\\ & (-0.09,0.11)(-0.11,0.2) & (-0.07,0.18)(-0.2,0.2) & (-0.09,0.2)(-0.2,0.2)\\ & & c_4 & c_5\\ & & (-0.3,0.34)(-0.4,0.62) & (-0.3,0.34)(-0.4,0.62)\\ & & (-0.28,0.35)(-0.48,0.58) & (-0.28,0.35)(-0.48,0.58)\\ & & (-0.23,0.05)(-0.35,0.38) & (-0.23,0.009)(-0.35,0.38)\\ & & (-0.09,0.13)(-0.2,0.2) & (-0.09,0.005)(-0.2,0.2)\end{pmatrix}$$

Step 2: Using Definitions $2.3, 2.4$ and 2.5 the weight wt_j, corresponding criteria c_j are as follows:

$$wt_1 = 0.191, wt_2 = 0.203,\ wt_3 = 0.191\ wt_4 = 176 \text{ and } wt_5 = 239.$$

Step 3: Using Definition 2.2 the values of WGAO calculated for each alternative are,

$$\begin{aligned}
\zeta_1 &= ((-0.46491, 0.34), (-0.20207, 0.62))\\
\zeta_2 &= ((-0.34872, 0.390365), (-0.48, 0.58))\\
\zeta_3 &= ((-0.30394, 0.076682), (-0.35, 0.38))\\
\zeta_4 &= ((-0.13887, 0.068905), (-0.1635, 0.2)).
\end{aligned}$$

Step 4: Using Definition 2.6 the score function values $BFRS(\zeta_i)$ calculated.

$$\begin{aligned}
BFRS(\zeta_1) &= 0.14644,\\
BFRS(\zeta_2) &= 0.01512,\\
BFRS(\zeta_3) &= 0.06129,\\
BFRS(\zeta_4) &= 0.0264.
\end{aligned}$$

Step 5: We conclude that $BFR(A_1) \succ BFR(A_3) \succ BFR(A_4) \succ BFR(A_2)$. Thus the alternative $BFR(A_1)$, namely dipole antenna is the best.

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