ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.4, 1851–1858
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.42 Spec. Issue on NCFCTA-2020

APPLICATION OF TRAPEZOIDAL FUZZY NUMBER IN RADIAL BASIS FUNCTION OF NEURAL NETWORK

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ABSTRACT. This paper deals with the application of trapezoidal fuzzy number in radial basis function of neural network. Based on this concept an algorithm is developed. Further, examples are given to illustrate the proposed algorithm.

1. INTRODUCTION

Al-Amoudi et al. [1] proposed network, consists of a set of radial basis functions which are directly connected to the input vectors. Further the links between hidden and output units are linear. Scheibel et al. [2] described an algorithm which connected the calculation of the centers and variances of Gaussian nodes to improve the response of radial basis function neural network. Fuzzy numbers were introduced by Zadeh [3] to deal with imprecise numerical quantities. Xiao at al. [4] combined the concept of trapezoidal fuzzy number and soft set and applied it for solving fuzzy multi criteria decision making problem. Motivated by these concepts we developed the notion of radial basis function neural network applying trapezoidal fuzzy number.

This paper deals with the application of trapezoidal fuzzy number in radial basis function of neural network. Based on this concept an algorithm is developed and examples are given to illustrate the proposed algorithm.

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2010 *Mathematics Subject Classification.* 92B20.

Key words and phrases. Trapezoidal fuzzy number, Neural network, Gaussian Function, Radial basis function network.

2. Radial Basis Function Network (RBFN)

Definition 2.1. *Let $A = \{a, b, c, d\}$ be the Trapezoidal fuzzy number then is membership value is defined by $\mu_A = \{\frac{a}{10}, \frac{b}{10}, \frac{c}{10}, \frac{d}{10}\}$, where the parameters a, b, c, d scale from 1 to 10.*

Definition 2.2. *Let $R = \{R_1, R_2, ..., R_n\}$ be the set of $'n'$ elements which are trapezoidal fuzzy numbers. These trapezoidal fuzzy numbers represent the weights of the radial basis function network denoted as $R_1 = (r_{11}, r_{12}, r_{13}, r_{14}), ...,$ $R_n = (r_{n1}, r_{n2}, r_{n3}, r_{n4})$. Let m_i be the mean of the trapezoidal fuzzy number R_i, $m_i = \frac{1}{n}\sum\limits_{i=1}^{n} r_{ij},\ j = 1$ to 4. The variance v_i of radial basis function is defined by the following equation $v_i = \frac{1}{n-1}[(x_1 - m_i) + (x_2 - m_i)]\ i = 1, 2, ..., n.$*

Definition 2.3. *The activation function for radial basis function of neural network is a Gaussian function denoted by $\Phi(r_{ij})$ and defined as*

$$\Phi(r_{ij}) = \exp\left(\frac{-(r_{ij} - m_i)^2}{v_i^2}\right),$$

where r_{ij} are the weights of radial basis function network.

Definition 2.4. *The Radial output function used to find the output value of the network is defined as $f(R_i) = \sum\limits_{i=1}^{n} r_{ij}\Phi(r_{ij}),\ j = 1, 2, 3, 4.$*

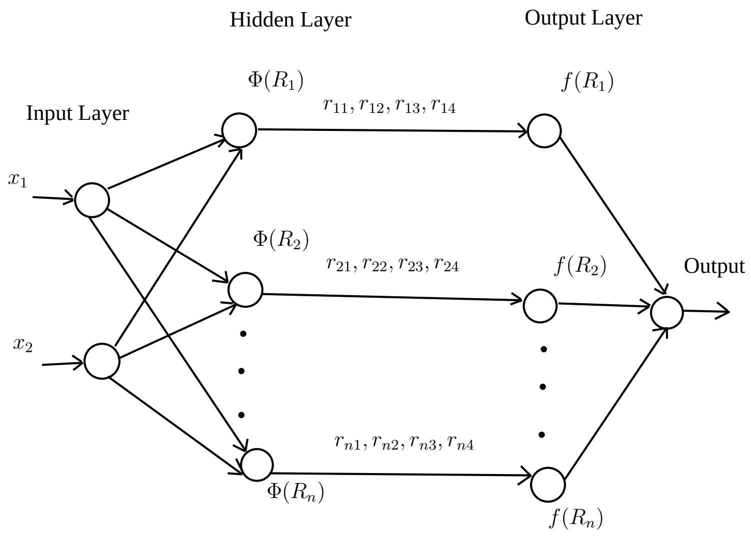

Fig 1: Guassian Radial basis Function Network

3. Algorithm

Step 1: Construct the Trapezoidal fuzzy number.
Step 2: Convert the trapezoidal fuzzy number into its membership function which are weights of radial basis function network.
Step 3: Assume the input value x_i as 0 and 1 which are used for finding variance.
step 4: The hidden output neuron is found by calculating the Gaussian radial basis function, $\Phi(r_{ij}) = \exp(-\frac{(r_{ij}-m_i)^2}{v_i^2})$.
Step 5: The output neuron function is the sum of the product of weights and the Gaussian function $f(R_i) = \sum_{i=1}^{n} r_{ij}\Phi(r_{ij})$.
Step 6: Determine the maximum value.

4. Applications

In this section we give two examples to illustrate the working of the algorithm.

Example 1. *Let* R_1, R_2, R_3, R_4, R_5 *be the yield of five different plants and* $r_{i1}, r_{i2}, r_{i3}, r_{i4}$ *be the four types of fertilizers viz., Nitrogen fertilizer, Phosphate fertilizer, Potassium fertilizer and Organic fertilizer.*
Step 1: *Construct the trapezoidal fuzzy number*

$R1 = (1, 2, 3, 4)$
$R_2 = (0, 1, 3, 5)$
$R_3 = (2, 3, 4, 5)$
$R_4 = (1, 4, 4, 6)$
$R_5 = (0, 5, 5, 7)$

Step 2: *Convert the Trapezoidal fuzzy number into its membership value*

$R_1 = (0.1, 0.2, 0.3, 0.4)$
$R_2 = (0.0, 0.1, 0.3, 0.5)$
$R_3 = (0.2, 0.3, 0.4, 0.5)$
$R_4 = (0.1, 0.4, 0.4, 0.6)$
$R_5 = (0.0, 0.5, 0.5, 0.7)$

Step 3: *Assume the inputs* $x_1 = 0$ *and* $x_2 = 1.$
Step 4: *Calculate the Gaussian function*

$R_1 = (0.1, 0.2, 0.3, 0.4)$
$m_1 = \frac{1}{4} = 0.25$

$v_1 = \frac{1}{3}[(0-0.25)+(1-0.25)] = \frac{1}{3}(0.5) = 0.1667$
$v_1^2 = 0.0278.$
$\Phi(r_{11}) = \exp(-\frac{(r_{11}-m_1)^2}{v_1^2}) = \exp(-\frac{(0.1-0.25)^2}{0.0278}) = \exp(-0.8094) = 0.4451.$
Similarly, $\Phi(r_{12}) = 0.9140$
$\Phi(r_{13}) = 0.9140$
$\Phi(r_{14}) = 0.4451.$
$R_2 = (0.0, 0.1, 0.3, 0.5)$
$m_2 = \frac{0.9}{4} = 0.225.$
$v_2 = \frac{1}{3}[(0-0.225)+(1-0.225)] = \frac{1}{3}(0.55) = 0.1833.$
$v_2^2 = 0.0336.$
$\Phi(r_{21}) = \exp(-\frac{(r_{21}-m_2)^2}{v_2^2}) = \exp(-\frac{(0.0-0.225)^2}{0.0336}) = \exp(-1.5060) = 0.2218.$
Similarly, $\Phi((r_{22}) = 0.6286$
$\Phi(r_{23}) = 0.8465$
$\Phi(r_{24}) = 0.1054.$
$R_3 = (0.2, 0.3, 0.4, 0.5)$
$m_3 = \frac{1.4}{4} = 0.35.$
$v_3 = \frac{1}{3}[(0-0.35)+(1-0.35)] = \frac{1}{3}(0.3) = 0.1$
$v_3^2 = 0.01.$
$\Phi(r_{31}) = \exp(-\frac{(r_{31}-m_3)^2}{v_3^2}) = \exp(-\frac{(0.2-0.35)^2}{0.01}) = \exp(-2.25) = 0.1054.$
Similarly, $\Phi(r_{32}) = 0.7788$
$\Phi(r_{33}) = 0.7788$
$\Phi(r_{34}) = 0.1054.$
$R_4 = (0.1, 0.4, 0.4, 0.6)$
$m_4 = \frac{1.5}{4} = 0.375.$
$v_4 = \frac{1}{3}[(0-0.375)+(1-0.375)] = \frac{1}{3}(0.23) = 0.0833$
$v_4^2 = 0.0069.$
$\Phi(r_{41}) = \exp(-\frac{(r_{41}-m_4)^2}{v_4^2}) = \exp(-\frac{(0.1-0.375)^2}{0.0069}) = \exp(-10.95) = 0.0001.$
Similarly, $\Phi(r_{42}) = 0.9168$
$\Phi(r_{43}) = 0.09168$
$\Phi(r_{44}) = 0.0007.$
$R_5 = (0.0, 0.5, 0.5, 0.7)$
$m_5 = \frac{1.7}{4} = 0.425.$
$v_5 = \frac{1}{3}[(0-0.425)+(1-0.425)] = \frac{1}{3}(0.15) = 0.05$
$v_5^2 = 0.0025.$

$\Phi(r_{51}) = \exp(-\frac{(r_{51}-m_5)^2}{v_5^2}) = \exp(-\frac{(0.0-0.425)^2}{0.0025}) = \exp(-0.7225) = 0.4855.$

Similarly, $\Phi(r_{52}) = 0.1065$

$\Phi(r_{53}) = 0.1065$

$\Phi(r_{54}) = 0.7390.$

Step 5: *Determine the output function*

$f(R_1) = r_{11}\Phi(r_{11}) + r_{12}\Phi(r_{12}) + r_{13}\Phi(r_{13}) + r_{14}\Phi(r_{14})$
$= (0.1)(0.4451) + (0.2)(0.9140) + (0.3)(0.9140) + (0.4)(0.4451)$

$f(R_1) = 0.6795.$

$f(R_2) = r_{21}\Phi(r_{21}) + r_{22}\Phi(r_{22}) + r_{23}\Phi(r_{23}) + r_{24}\Phi(r_{24})$
$= (0.0)(0.2218) + (0.1)(0.6286) + (0.3)(0.8465) + (0.5)(0.1054)$

$f(R_2) = 0.3696.$

$f(R_3) = r_{31}\Phi(r_{31}) + r_{32}\Phi(r_{32}) + r_{33}\Phi(r_{33}) + r_{34}\Phi(r_{34})$
$= (0.2)(0.1054) + (0.3)(0.7788) + (0.4)(0.7788) + (0.5)(0.1054)$

$f(R_3) = 0.6189.$

$f(R_4) = r_{41}\Phi(r_{41}) + r_{42}\Phi(r_{42}) + r_{43}\Phi(r_{43}) + r_{44}\Phi(r_{44})$
$= (0.1)(0.0001) + (0.4)(0.9168) + (0.4)(0.9168) + (0.6)(0.0007)$

$f(R_4) = 0.7338.$

$f(R_5) = r_{51}\Phi(r_{51}) + r_{52}\Phi(r_{52}) + r_{53}\Phi(r_{53}) + r_{54}\Phi(r_{54})$
$= (0.0)(0.4855) + (0.5)(0.1065) + (0.5)(0.1065) + (0.7)(0.7390)$

$f(R_5) = 0.6239.$

Step 6: *Determine the maximum value*

$f(R_1) = 0.6795$

$f(R_2) = 0.3696$

$f(R_3) = 0.6189$

$f(R_4) = 0.7338$

$f(R_5) = 0.6239$

$f(R_4) = 0.7338$ *is the maximum output value.* R_4 *is the best yielding plant.*

Example 2. *Let* R_1, R_2, R_3, R_4 *be the four different types of wireless networks and* r_{i1}*(Multimeter waves),* r_{i2} *(Build in performance),* r_{i3}*(edge to edge analytics),* r_{i4} *(wireless security) be the predictions for the efficiency of wireless networks.*

Step 1: *Construct the trapezoidal fuzzy number*

$R1 = (1.5, 2.6, 2.6, 5.9)$

$R_2 = (0.5.0.6, 0.6, 3)$

$R_3 = (2, 4.3, 4.3, 4.4)$

$R_4 = (3.1, 3.2, 3.2, 5.4)$

Step 2: *Convert the Trapezoidal fuzzy number into its membership value*

$R_1 = (0.15, , 0.26, 0.26, 0.59)$

$R_2 = (0.05, 0.06, 0.21, 0.3)$

$R_3 = (0.2, 0.43, 0.43, 0.44)$

$R_4 = (0.31, 0.32, 0.32, 0.54)$

Step 3: *Assume the inputs $x_1 = 0$ and $x_2 = 1$.*

Step 4: *Calculate the Gaussian function*

$R_1 = (0.15, 0.26, 0.26, 0.59)$

$m_1 = \frac{1.26}{4} = 0.315$

$v_1 = \frac{1}{3}[(0 - 0.315) + (1 - 0.315)] = \frac{1}{3}(0.37) = 0.123.$

$v_1^2 = 0.015$

$\Phi(r_{11}) = \exp(-\frac{(r_{11}-m_1)^2}{v_1^2}) = \exp(-\frac{(0.15-0.315)^2}{0.015}) = \exp(-1.8133) = 0.1632.$

Similarly, $\Phi(r_{12}) = 0.8187$

$\Phi(r_{13}) = 0.8187$

$\Phi(r_{14}) = 0.6039.$

$R_2 = (0.05, 0.06, 0.21, 0.3)$

$m_2 = \frac{0.62}{4} = 0.155$

$v_2 = \frac{1}{3}[(0 - 0.155) + (1 - 0.155)] = \frac{1}{3}(0.69) = 0.23.$

$v_2^2 = 0.0529$

$\Phi(r_{21}) = \exp(-\frac{(r_{21}-m_2)^2}{v_2^2}) = \exp(-\frac{(0.05-0.155)^2}{0.0529}) = \exp(-0.2079) = 1.$

Similarly, $\Phi(r_{22}) = 0.8436$

$\Phi(r_{23}) = 0.9448$

$\Phi(r_{24}) = 0.2157.$

$R_3 = (0.2, 0.43, 0.43, 0.44)$

$m_3 = \frac{1.5}{4} = 0.375$

$v_3 = \frac{1}{3}[(0 - 0.375) + (1 - 0.375)] = \frac{1}{3}(0.25) = 0.0833.$

$v_3^2 = 0.0069$

$\Phi(r_{31}) = \exp(-\frac{(r_{31}-m_3)^2}{v_3^2}) = \exp(-\frac{(0.2-0.375)^2}{0.0069}) = \exp(-4.434) = 0.0119.$

Similarly, $\Phi(r_{32}) = 0.6475$

$\Phi(r_{33}) = 0.6475$

$\Phi(r_{34}) = 0.5441.$

$R_4 = (0.31, 0.32, 0.32, 0.54)$

$m_4 = \frac{1.49}{4} = 0.373$

$v_4 = \frac{1}{3}[(0 - 0.373) + (1 - 0.373)] = \frac{1}{3}(0.254) = 0.0846.$

$v_4^2 = 0.0071$

$\Phi(r_{41}) = \exp(-\frac{(r_{41}-m_4)^2}{v_4^2}) = \exp(-\frac{(0.31-0.373)^2}{0.0071}) = \exp(-0.5492) = 0.5774.$

Similarly, $\Phi(r_{42}) = 0.6741$

$\Phi(r_{43}) = 0.6741$

$\Phi(r_{44}) = 0.0199.$

Step 5: *Determine the output function*

$f(R_1) = r_{11}\Phi(r_{11}) + r_{12}\Phi(r_{12}) + r_{13}\Phi(r_{13}) + r_{14}\Phi(r_{14})$

$= (0.15)(0.1632) + (0.26)(0.8187) + (0.26)(0.8187) + (0.59)(0.6039)$

$f(R_1) = 0.8066.$

$f(R_2) = r_{21}\Phi(r_{21}) + r_{22}\Phi(r_{22}) + r_{23}\Phi(r_{23}) + r_{24}\Phi(r_{24})$

$= (0.05)(1) + (0.06)(0.8436) + (0.21)(0.9448) + (0.3)(0.2157)$

$f(R_2) = 0.3637.$

$f(R_3) = r_{31}\Phi(r_{31}) + r_{32}\Phi(r_{32}) + r_{33}\Phi(r_{33}) + r_{34}\Phi(r_{34})$

$= (0.2)(0.0119) + (0.43)(0.6475) + (0.43)(0.6475) + (0.44)(0.5441)$

$f(R_3) = 0.7985.$

$f(R_4) = r_{41}\Phi(r_{41}) + r_{42}\Phi(r_{42}) + r_{43}\Phi(r_{43}) + r_{44}\Phi(r_{44})$

$= (0.31)(0.5774) + (0.32)(0.6741) + (0.32)(0.6741) + (0.54)(0.0199)$

$f(R_4) = 0.6210.$

Step 6: *Determine the maximum value*

$f(R_1) = 0.8066$

$f(R_2) = 0.3637$

$f(R_3) = 0.7985$

$f(R_4) = 0.6210$

$f(R_1) = 0.8066$

is the maximum output value. R_1 *is the best wireless network.*

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