INTUITIONISTIC INTERVAL VALUED MULTI FUZZY SUBFIELDS OF A FIELD

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Abstract. In this paper, some theorems of intuitionistic interval valued multi fuzzy subfield of a field are defined and noted and also some definitions, results and properties are given.

1. Introduction

The fuzzy set was introduced by L.A.Zadeh [13], A lot of researchers explained on the generalization of the notation of fuzzy set ([3–10]). Some results of intuitionistic multi fuzzy subset was introduced by K.T. Atanassov [1] as a generalization of the notion of fuzzy set. Azriel Rosenfeld [2] was introduced a fuzzy group.

2. Preliminaries

Definition 2.1. Let $X$ be a non-empty set. A fuzzy subset $E$ of $X$ is a function $E : X \rightarrow [0, 1]$.

Definition 2.2. A multi fuzzy subset $E$ of a set $X$ is defined as an object of the form $E = \{(x, E_1(x), E_2(x), E_3(x), ..., E_n(x)) \mid x \in X\}$, where $E_i : X \rightarrow [0, 1]$ for every $i$. $E$ is called multi fuzzy subset of $X$ with dimension $n$. It is denoted as $E = \{E_1, E_2, E_3, ..., E_n\}$.
Definition 2.3. A intuitionistic fuzzy set (IFS) $E$ in $X$ is defined as an object having the form $E = \{(x, \mu_E(x), \nu_E(x))/x \in X\}$, where $\mu_E : X \rightarrow [0, 1]$ and $\nu_E : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and all $x$ in $X$ satisfying $0 \leq \mu_E(x) + \nu_E(x) \leq 1$.

Definition 2.4. An intuitionistic multi fuzzy subset of $A$ of a set $X$ is defined as an object of the form $A(x, \mu_E(x), \nu_E(x), ..., \mu_{E_n}(x), \nu_{E_n}(x))/x \in X$ where $\mu_{E_i}(x) : X \rightarrow [0, 1]$ and $\nu_{E_i}(x) : X \rightarrow [0, 1]$ for every $i$, define the degrees of membership and the degrees of non-membership of the element $x \in X$ respectively and for all $x \in X$ satisfying $0 \leq \mu_{E_i}(x) + \nu_{E_i}(x) \leq 1$, for every $i$. It is noted as $E = (\mu_E, \nu_E)$ where $\mu_E = \{\mu_{E_1}(x), \mu_{E_2}(x), ..., \mu_{E_n}(x)\}$ and $\nu_E = \{\nu_{E_1}(x), \nu_{E_2}(x), ..., \nu_{E_n}(x)\}$.

Definition 2.5. Let $X$ be a non-empty set, An interval valued fuzzy subset $E$ of $X$ is a function $E : X \rightarrow D[0, 1]$, where $D[0, 1]$ is a collection of all subinterval of $[0, 1]$.

Definition 2.6. An interval valued multi fuzzy subset of $E$ of a set $X$ is defined as an object of the form $E = \{(x, E_1(x), E_2(x), E_3(x), ..., E_n(x))/x \in X\}$ where $E_i(x) : X \rightarrow D[0, 1]$ for every $i$, Here $E$ is called an interval valued multi fuzzy subset of $X$ with dimension $n$. This noted as $E = (E_1 E_2 ... E_n)$.

Definition 2.7. An intuitionistic interval valued fuzzy set $E$ in $X$ is an object having the form $E = \{(x, \mu_E(x), \nu_E(x))/x \in X\}$, where $\mu_E : X \rightarrow D[0, 1], \nu_E : X \rightarrow D[0, 1]$ defined the degrees of membership, the degree of non-membership of the elements $x$ in $X$ respectively and for all $x$ in $X$ satisfying $0 \leq \mu_{E(x)} + \nu_{E(x)} \leq 1$.

Definition 2.8. Let $E$ and $F$ be any two intuitionistic interval valued multi fuzzy subset of $X$, we define the following relations and operations

(i) $E \leq F$ iff $\mu_{E_i}(x) \leq \mu_{F_i}(x)$ and $\nu_{E_i}(x) \geq \nu_{F_i}(x)$ for every $x$ in $X$ and for every $i$.

(ii) $E = F$ iff $\mu_{E_i}(x) = \mu_{F_i}(x)$ and $\nu_{E_i}(x) = \nu_{F_i}(x)$ for every $x$ in $X$ and for every $i$.

(iii) $E \cap F$ iff $E \cap F(x) = \{r \min\{\mu_{E_i}(x), \mu_{F_i}(x)\}, r \max\{\nu_{E_i}(x), \nu_{F_i}(x)\}\}$ for every $x$ in $X$ and for every $i$.

(iv) $E \cup F$ iff $E \cup F(x) = \{r \max\{\mu_{E_i}(x), \mu_{F_i}(x)\}, r \min\{\nu_{E_i}(x), \nu_{F_i}(x)\}\}$ for every $x$ in $X$ and for every $i$. 
Definition 2.9. Let \((F, +, \cdot)\) be a field an interval valued multi fuzzy subset \(E\) of \(F\) is said to be a interval valued multi fuzzy subfield of \(F\) if the following conditions are satisfied.

(i) \(E_i(x - y) \geq r \min\{E_i(x), E_i(y)\}\) for every \(x, y\) in \(F\), for every \(i\).

(ii) \(E_i(xy^{-1}) \geq r \min\{E_i(x), E_i(y)\}\) for every \(x, y \neq 0\) in \(F\) for every \(i\), where 0 is the additive identity element of \(F\).

Definition 2.10. Let \((F, +, \cdot)\) be a field. An intuitionistic interval valued multi fuzzy subset \(E\) of \(F\) is said to be an intuitionistic interval valued multi fuzzy subfield of \(F\), if it satisfies the following axioms

(i) \(\mu_{E_i}(x - y) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(y)\}\) for every \(x, y\) in \(F\) for every \(i\).

(ii) \(\mu_{E_i}(xy^{-1}) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(x)\}\) for every \(x, y \neq 0\) in \(F\) for every \(i\).

(iii) \(\nu_{E_i}(x - y) \leq r \min\{\nu_{E_i}(x), \nu_{E_i}(y)\}\) for every \(x, y\) in \(F\) for every \(i\).

(iv) \(\nu_{E_i}(xy^{-1}) \leq r \min\{\nu_{E_i}(x), \nu_{E_i}(y)\}\) for every \(x, y \neq 0\) in \(F\) for every \(i\), where 0 is the additive identity element in \(F\).

3. Some Properties

Note 1. \(0 = [0, 0], \ 1 = [1, 1]\).

Theorem 3.1. If \(E\) is an intuitionistic interval valued multi fuzzy subfield \((F, +, \cdot)\), then \(\mu_{E_i}(-x) = \mu_{E_i}(x)\) for all \(x\) in \(F\) and \(\mu_{E_i}(x^{-1}) = \mu_{E_i}(x)\) for all \(x \neq 0\) in \(F\) and \(\nu_{E_i}(-x) = \nu_{E_i}(x)\) for every \(x\) in \(F\) and \(\nu_{E_i}(x^{-1}) = \nu_{E_i}(x)\) for all \(x \neq 0\) in \(F\), \(\mu_{E_i}(x) \leq \mu_{E_i}(0)\) for every \(x\) in \(F\) and \(\mu_{E_i}(x) \leq \mu_{E_i}(1)\) for every \(x \neq 0\) in \(F\) and \(\nu_{E_i}(x) \geq \nu_{E_i}(0)\) for every \(x\) in \(F\) and \(\nu_{E_i}(x) \geq \nu_{E_i}(1)\) for all \(x \neq 0\) in \(F\), for all \(i\), where 0 and 1 are identify element in \(F\).

Proof. For \(x\) in \(F\) and 0,1 there are identify elements in \(F\). Now \(\mu_{E_i}(x) = \mu_{E_i}(-(-x)) \geq \mu_{E_i}(-x) \geq \mu_{E_i}(x)\). Therefore,

\[\mu_{E_i}(-x) = \mu_{E_i}(x)\]

for every \(x\) in \(F\) and for every \(i\);

\[\mu_{E_i}(x) = \mu_{E_i}(x^{-1})^{-1} \geq \mu_{E_i}(x^{-1}) \geq \mu_{E_i}(x) \cdot \mu_{E_i}(x^{-1}) = \mu_{E_i}(x)\]

for every \(x \neq 0\) in \(F\) and for every \(i\). So,

\[\nu_{E_i}(x) = \nu_{E_i}(-(-x)) \leq \nu_{E_i}(-x) \leq \nu_{E_i}(x)\].
Therefore,
\[ \nu_{E_i}(-x) = \nu_{E_i}(x) \]
for every \( x \) in \( F \) and for every \( i \), and further,
\[ \nu_{E_i}(x) = \nu_{E_i}(x^{-1})^{-1} \leq \nu_{E_i}(x^{-1}) \leq \nu_{E_i}(x) \]
for every \( x \) in \( F \) and for every \( i \),
\[ \mu_{E_i}(0) = \mu_{E_i}(x - x) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(-x)\} = \mu_{E_i}(x). \]
Therefore,
\[ \mu_{E_i}(0) \geq \mu_{E_i}(x) \]
for all \( x \) in \( F \) and for every \( i \).

Now
\[ \mu_{E_i}(1) = \mu_{E_i}(xx^{-1}) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(x^{-1})\} = \mu_{E_i}(x). \]
Therefore,
\[ \mu_{E_i}(1) \geq \mu_{E_i}(x) \]
for every \( x \neq 0 \) in \( F \) and for every \( i \), and
\[ \nu_{E_i}(0) = \nu_{E_i}(x - x) \leq r \max\{\nu_{E_i}(x), \nu_{E_i}(-x)\} = \nu_{E_i}(x). \]
Therefore,
\[ \nu_{E_i}(0) \leq \nu_{E_i}(x) \]
for every \( x \) in \( F \) and for every \( i \),
\[ \nu_{E_i}(1) = \nu_{E_i}(xx^{-1}) \leq r \max\{\nu_{E_i}(x), \nu_{E_i}(x^{-1})\} = \nu_{E_i}(x). \]
Therefore,
\[ \nu_{E_i}(1) \leq \nu_{E_i}(x) \]
for every \( x \neq 0 \) in \( F \) and for every \( i \).

\( \square \)

**Theorem 3.2.** If \( A \) is an intuitionistic yinterval valued multi fuzzy subfield of a field \( (F, +, \cdot) \), then for each \( i \),

(i) \( \mu_{E_i}(x - y) = \mu_{E_i}(0) \) gives \( \mu_{E_i}(x) = \mu_{E_i}(y) \) for each \( x \) and \( y \) in \( F \).
(ii) \( \mu_{E_i}(xy^{-1}) = \mu_{E_i}(1) \) gives \( \mu_{E_i}(x) = \mu_{E_i}(y) \) for every \( x \) and \( y = 0 \) in \( F \).
(iii) \( \nu_{E_i}(x - y) = \nu_{E_i}(0) \) gives \( \nu_{E_i}(x) = \nu_{E_i}(y) \) for every \( x \) and \( y \) in \( F \).
(iv) \( \nu_{E_i}(xy^{-1}) = \nu_{E_i}(1) \) gives \( \nu_{E_i}(x) = \nu_{E_i}(y) \) for all \( x \) and \( y \neq 0 \) in \( F \) where \( 0 \) and \( 1 \) are identity elements in \( F \).
Theorem 3.3. Let $E$ be an intuitionistic interval valued multi fuzzy subset of a field $(F, +, \cdot)$. If for every $i$, $\mu_{E_i}(e) = \mu_{E_i}(e_1) = 1$ and $\nu_{E_i}(e) = \nu_{E_i}(e_1) = 0$ and $\mu_{E_i}(x - y) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(y)\}$ for every $x$ and $y$ in $F$,
$\mu_{E_i}(xy^{-1}) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(y)\}$ for every $x$ and $y \neq e$ in $F$ and
$\nu_{E_i}(x - y) \leq r \max\{\nu_{E_i}(x), \nu_{E_i}(y)\}$ for every $x$ and $y$ in $F$,
$\nu_{E_i}(xy^{-1}) \leq r \max\{\nu_{E_i}(x), \nu_{E_i}(y)\}$ for every $x$ and $y \neq e$ in $F$,
then $E$ is an intuitionistic multi fuzzy subfield of $F$, where $e$ and $e_1$ are identity elements of $F$.

Proof. It is well defined. □

Theorem 3.4. Let $E$ be an intuitionistic multi fuzzy subfield of a field $(F, +, \cdot)$, then $H = \{x/x \in F : \mu_{E_i}(x) = 1, \nu_{E_i}(x) = 0 \text{ for every } i\}$ is either empty or a subfield of $F$.

Proof. If no element satisfies this conditions, then $H$ is empty. If $x$ and $y$ in $H$, then

$$\mu_{E_i}(x - y) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(y)\}$$
$$\geq r \min\{\mu_{E_i}(x), \mu_{E_i}(y)\}$$
$$= r \min(1, 1)$$
$$= 1.$$

Therefore, $\mu_{E_i}(x - y) = 1$ for every $x, y$ in $H$ and for every $i$.

$$\mu_{E_i}(xy^{-1}) \geq r \min\{\mu_{E_i}(x), \mu_{E_i}(y^{-1})\}$$
$$\geq r \min\{\mu_{E_i}(x), \mu_{E_i}(y)\}$$
$$= r \min(1, 1)$$
$$= 1.$$

Therefore, $\mu_{E_i}(xy^{-1}) = 1$ for every $x$ and $y \neq 0$ in $H$ and for every $i$.

$$\nu_{E_i}(x - y) \leq r \max\{\nu_{E_i}(x), \nu_{E_i}(y)\}$$
$$\leq r \max\{\nu_{E_i}(x), \nu_{E_i}(y)\}$$
$$= r \max(0, 0)$$
$$= 0$$

Therefore, $\nu_{E_i}(x - y) = 0$ for every $x$ and $y$ in $H$ and for every $i$. $x - y, xy^{-1} \in H$. 
Therefore, $H$ is a subfield of $F$. Hence $H$ is either empty or a subfield of $F$. \hfill \square

**Theorem 3.5.** Let $E$ be an intuitionistic interval valued multi fuzzy subfield of a field $(F, +, \cdot)$, then for every $i$,

(i) If $\mu_{E_i}(x - y) = 1$ then $\mu_{E_i}(x) = \mu_{E_i}(y)$ for every $x$ and $y \neq e$ in $F$.

(ii) If $\nu_{E_i}(x - y) = 0$ then $\nu_{E_i}(x) = \nu_{E_i}(y)$ for every $x$ and $y$ in $F$ and if $\nu_{E_i}(xy^{-1}) = 0$, then $\nu_{E_i}(x) = \nu_{E_i}(y)$ for every $x$ and $y \neq e_1$ in $F$.

Here $e$ and $e_1$ are identity elements of $F$.

**Theorem 3.6.** If $E$ be a Intuitionistic Interval valued multi fuzzy subfield of a field $(F, +, \cdot)$, then for every $i$:

(i) If $\mu_{E_i}(x - y) = 0$ then either $\mu_{E_i}(x) = 0$ or $\mu_{E_i}(y) = 0$ for every $x, y$ in $F$ and if $\mu_{E_i}(xy^{-1}) = 0$ then either $\mu_{E_i}(x) = 0$ or $\mu_{E_i}(y) = 0$ for every $x$ and $y \neq e$ in $F$.

(ii) If $\nu_{E_i}(x - y) = 1$ then either $\nu_{E_i}(x) = 1$ or $\nu_{E_i}(y) = 1$ for every $x, y$ in $F$ and if $\nu_{E_i}(xy^{-1}) = 1$ then either $\nu_{E_i}(x) = 1$ or $\nu_{E_i}(y) = 1$ for every $x$ and $y \neq e_1$ in $F$, where $e$ and $e_1$ are identity elements of $F$.

**References**


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