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CORE ON FUZZY NEAR SETS

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ABSTRACT. In this paper we present the notion of fuzzy near sets, lower and upper approximation and the boundary region of fuzzy near sets. Reduction on fuzzy near sets is defined. Using reduction, core on fuzzy near sets is calculated and represented diagrammatically.

1. INTRODUCTION

The solution to the problem of approximating sets of perceptual objects results from a generalization of the classification of objects introduced by Pawlak [2]. This generalization lead to the introduction of near set by James F. Peters [3]. Peters considered approximation of sets of perceptual objects that have matching descriptions. Lellis Thivagar et al. [1] applied rough topology in decision making problems.

This paper aims to introduce the notions of fuzzy near set, lower and upper approximation and the boundary region of fuzzy near sets. Reduction on fuzzy near set is defined. Using reduction, core on fuzzy near set is calculated and represented diagrammatically.

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2. Fuzzy Near Sets

Definition 2.1. *Let $X \neq \phi$ and $A \subseteq X$. Let μ be the membership function. A set denoted by $FN_s(A)$ is called a fuzzy near set if at least one pair of objects have the same membership value. i.e., $\exists$ at least one pair $x, x' \in X \ni \mu_{FN_s(A)}(x) = \mu_{FN_s(A)}(x')$.*

Definition 2.2. *Let $X \neq \phi$ and $A \subseteq X$ and $FN_s(A)$ be a fuzzy near set. A class $[x]_{FN_s(A)}$ in a fuzzy near set, is a set of objects having the same membership values*

$$[x]_{FN_s(A)} = \{x \in X : \mu_{FN_s(A)}(x) = \mu_{FN_s(A)}(x')\}.$$

Definition 2.3. *Let $X \neq \phi$ and $A \subseteq X$ and $FN_s(A)$ be a fuzzy near set. The lower and upper approximation of a fuzzy near set and the boundary region $FN_s(A)$ is denoted by $\underline{FN}_s(A)$ and defined as:*

$$\underline{FN}_s(A) = \bigcup_{x:[x]_{FN_s(A)} \subseteq X} [x]_{FN_s(A)},$$
$$\overline{FN}_s(A) = \bigcup_{x:[x]_{FN_s(A)} \bigcap X \neq \phi} [x]_{FN_s(A)},$$
$$B_{FN_s}(A) = \overline{FN}_s(A) - \underline{FN}_s(A).$$

Definition 2.4. *Let X be a non-empty set, $A \subseteq X$ and $FN_s(A)$ be a fuzzy near set. A topology $\tau FN_s(A)$ on X is a collection*

$$\tau_{FN_s(A)} = \{X, \phi, \underline{FN_s(A)}, \overline{FN_s(A)}, B_{FN_s(A)}\}$$

satisfying the following axioms:

(i) *ϕ, $X \in \tau FN_s(A)$.*
(ii) *The $\bigcup$ be any sub collection of $\tau FN_s(A)$ is in $\tau FN_s(A)$.*
(iii) *The $\bigcap$ be sub collection of $\tau FN_s(A)$ is in $\tau FN_s(A)$.*

$\tau FN_s(A)$ is called fuzzy near topology on X.

Example 1. *Let $X = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$, $A = \{\gamma_1, \gamma_2, \gamma_3, \gamma_6\}$ and $FN(A) = \{\gamma_1, 0.4\}, \{\gamma_2, 0.4\}, \{\gamma_3, 0.61\}, \{\gamma_4, .61\}, \{\gamma_5, 0.61\}, \{\gamma_6, 0.83\}\}$, Then*

$$[x]_{FN_s(A)} = \{\{\gamma_1, \gamma_2\}, \{\gamma_3, \gamma_4, \gamma_5\}, \{\gamma_6\}\},$$

the fuzzy near class of U, $\underline{FN_s(A)} = \{\gamma_1, \gamma_2, \gamma_6\}, \overline{FN_s(A)} = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$ and $B_F N_s(A) = \{\gamma_3, \gamma_4, \gamma_5\}$. Therefore the fuzzy near topology

$$\tau_{FN_s(A)} = \{\gamma, \phi, \{\gamma_1, \gamma_2, \gamma_5\}, \gamma_6\}, \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_6\}, \{\gamma_3, \gamma_4, \gamma_6\}\}.$$

Definition 2.5. *If τ_{FN_s} is the fuzzy near topology on X then the set*

$$\mathbb{B} = \{X, \underline{FN_s(A)}, B_{FN_s(A)}\}$$

is the basis for τ_{FN}.

Theorem 2.1. $\mathbb{B}$ *is a basis of $\tau_{FN_s(A)}$.*

Proof.

(i) $\bigcup\limits_{FN_s(A_i)\in\mathbb{B}} FN_s(A_i) = X$

(ii) Let $W = \underline{FN_s(A)}$. Since $X \bigcap \underline{FN_s(A)} = \underline{FN_s(A)}$, $W \subset X \bigcap \underline{FN_s(A)}$ and every x in $X \bigcap \underline{FN_s(A)} \in W$.

If we consider X and $B_{FN_s(A)}$ from $\mathbb{B}$, taking $V = B_{FN_s(A)}$, since

$$X \bigcap B_{FN_s(A)} = B_{FN_s(A)}, V \subset X \bigcap B_{FN_s(A)}$$

and every x in $X \bigcap B_{FN_s(A)} \in V$. $\underline{FN_s(A)} \bigcap B_{FN_s(A)} = \phi$. Thus, $\mathbb{B}$ is a basis for τ_{FN}. □

Definition 2.6. *Let $X \neq \phi$ and FN_s be fuzzy near sets, τ_{FN_s} be the fuzzy near topology on U and $\mathbb{B}$ be the basis for τ_{FN_s}. A subset $SFN_s(A)$ of $FN_s(A)$, is called core viz. $CFN_s(A)$ of $FN_s(A)$ if $\mathbb{B}_{SFN_s(A)} \neq \mathbb{B}_{(FN_s(A))-(FN_s(A_i))}$ for every $x \in SFN_s(A)$. i.e., a core of $FN_s(A)$ is a subset of fuzzy near sets such that its element cannot be removed without much effect on the classification of fuzzy near sets.*

3. Applications of core on fuzzy near sets

Example 2. Let $X = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7, \delta_8\}$ represent eight different types of tomatoes affected by the disease bacterial stem and fruit canker. Let the five symptoms namely,

* Cracks develop in streaks and form cankers
* Slimy bacterial oooze through the cracks
* White blister like spots in the margins of leaves
* Leaflets on one side of rachis show withering initially
* Vascular discolouration in split open stems

FIGURE 1. Bacterial stem and Fruit canker of tomato

TABLE 1. Five fuzzy near sets

X	$FN_s(A_1)$	$FN_s(A_2)$	$FN_s(A_3)$	$FN_s(A_4)$	$FN_s(A_5)$	Decision
δ_1	0.2	0.4	0.4	0.7	0.4	1
δ_2	0.7	0.5	0.35	0.7	0.8	0
δ_3	0.7	0.5	0.35	0.7	0.8	1
δ_4	0.1	0.5	0.52	0.19	1	0
δ_5	0.2	0.4	0.4	0.4	0.3	0
δ_6	0.2	0.5	0.52	0.19	0.3	1
δ_7	0.1	0.4	0.52	0.19	1	0
δ_8	0.1	0.4	0.52	0.19	1	1

be represented by the fuzzy near sets $FN_s(A_1), FN_s(A_2), FN_s(A_3), FN_s(A_4)$ and $FN_s(A_5)$ respectively.

Case 1: Consider, $X = \{\delta_1, \delta_3, \delta_6, \delta_8\}$ where the decision class corresponds to 1. The partitions corresponding to relation of all the fuzzy near sets

$FN_s(A_i)$ where $i = 1$ to 5, is given by

$$\begin{aligned}
[\delta]FN_s(A) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4\},\{\delta_5\},\{\delta_6\},\{\delta_7\},\{\delta_8\}\}\\
\underline{FN_s(A)} &= \{\delta_1,\delta_6,\delta_8\}\\
\overline{FN_s(A)} &= \{\delta_1,\delta_2,\delta_3,\delta_6,\delta_8\}\\
B_{FN_s(A)} &= \{\delta_2,\delta_3\}\\
\tau_{FN_s(A)} &= \{\delta,\phi,\{\delta_1,\delta_6,\delta_8\},\{\delta_1,\delta_2,\delta_3,\delta_6,\delta_8\},\{\delta_2,\delta_3\}\}\\
\mathbb{B}_{FN_s(A)} &= \{\delta,\{\delta_1,\delta_6,\delta_8\},\{\delta_2,\delta_3\}\}.
\end{aligned}$$

Excluding the fuzzy near set $FN_s(A_1)$, the partitions corresponding to the remaining fuzzy near sets are

$$\begin{aligned}
[\delta](FN_s(A)-FN_s(A_1)) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4,\delta_6\},\{\delta_5\},\{\delta_7\},\{\delta_8\},\}\\
\underline{(FN_s(A)-FN_s(A_1))} &= \{\delta_1,\delta_8\}\\
\overline{(FN_s(A)-FN_s(A_1))} &= \{\delta_1,\delta_2,\delta_3,\delta_4,\delta_6,\delta_8\}\\
B_{(FN_s(A)-FN_s(A_1))} &= \{\delta_2,\delta_3,\delta_4,\delta_6\}\\
\tau_{(FN_s(A)-FN_s(A_1))} &= \{\delta,\phi,\{\delta_1,\delta_8\},\{\delta_1,\delta_2,\delta_3,\delta_4,\delta_6,\delta_8\},\{\delta_2,\delta_3,\delta_4,\delta_6\}\}\\
\mathbb{B}_{(FN_s(A)-FN_s(A_1))} &= \{\delta,\{\delta_1,\delta_8\},\{\delta_2,\delta_3,\delta_4,\delta_6\}\} \neq \mathbb{B}_{FN_s(A)}.
\end{aligned}$$

Excluding the fuzzy near set $FN_s(A_2)$, the partitions corresponding to the remaining fuzzy near sets are

$$\begin{aligned}
[\delta](FN_s(A)-FN_s(A_2)) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4,\delta_8\},\{\delta_5\},\{\delta_6\},\{\delta_7\}\}\\
\underline{(FN_s(A)-FN_s(A_2))} &= \{\delta_1,\delta_6\}\\
\overline{(FN_s(A)-FN_s(A_2))} &= \{\delta_1,\delta_2,\delta_3,\delta_4,\delta_6,\delta_8\}\\
B_{(FN_s(A)-FN_s(A_2))} &= \{\delta_2,\delta_3,\delta_4,\delta_8\}\\
\tau_{(FN_s(A)-FN_s(A_2))} &= \{\delta,\phi,\{\delta_1,\delta_6\},\{\delta_2,\delta_3,\delta_4,\delta_8\},\{\delta_1,\delta_2,\delta_3,\delta_4,\delta_6,\delta_8\}\}\\
\mathbb{B}_{(FN_s(A)-FN_s(A_2))} &= \{\delta,\{\delta_1,\delta_6\},\{\delta_2,\delta_3,\delta_4,\delta_8\}\} \neq \mathbb{B}_{FN_s(A)}.
\end{aligned}$$

Excluding the fuzzy near sets $FN_s(A_3)$ and $FN_s(A_4)$the partitions corresponding to each of the remaining fuzzy near sets are equal.

$$[\delta](FN_s(A)-FN_s(A_3)) = \{\{\delta_1\},\{\delta_4\},\{\delta_5\},\{\delta_6\},\{\delta_7\},\{\delta_8\},\{\delta_2,\delta_3\}\}$$

$$[\delta](FN_s(A)-FN_s(A_4)) = \{\{\delta_1\},\{\delta_4\},\{\delta_5\},\{\delta_6\},\{\delta_7\},\{\delta_8\},\{\delta_2,\delta_3\}\}.$$

Excluding the fuzzy near set $FN_s(A5)$, the partitions corresponding to the remaining fuzzy near sets are

$$\begin{aligned}
[\delta](FN_s(A)-FN_s(A_5)) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4\},\{\delta_5\},\{\delta_6\},\{\delta_7,\delta_8\}\}\\
\underline{FN_s(A)-FN_s(A_5)} &= \{\delta_1,\delta_6\}\\
\overline{FN_s(A)-FN_s(A_5)} &= \{\delta_1,\delta_2,\delta_3,\delta_6,\delta_7,\delta_8\}\\
B_{FN_s(A)-FN_s(A_5)} &= \{\delta_2,\delta_3,\delta_7,\delta_8\}
\end{aligned}$$

$$\begin{aligned}
\tau_{FN_s(A)-FN_s(A_5)} &= \{\delta,\phi,\{\delta_1,\delta_6\},\{\delta_1,\delta_2,\delta_3,\delta_6,\delta_7,\delta_8\},\{\delta_2,\delta_3,\delta_7,\delta_8\}\}\\
\mathbb{B}_{FN_s(A)-FN_s(A_5)} &= \{t,\{t_1,t_6\},\{t_2,t_3,t_7,t_8\}\} \neq \mathbb{B}_{FN_s(A)}.
\end{aligned}$$

If $M = \{FN_s(A_1), FN_s(A_2), FN_s(A_5)\}$, then the basis for the fuzzy near set topology corresponding to M is

$$\mathbb{B}_M = \{\delta, \{\{\delta_1,\delta_6,\delta_8\},\{\delta_2,\delta_3\}\}.$$

Also $\mathbb{B}_M \neq \mathbb{B}_{FN_s(A)-FN_s(A_i)}$. Therefore core,

$$\mathbf{CFN_s(A)} = \{\mathbf{FN_s(A_1)}, \mathbf{FN_s(A_2)}, \mathbf{FN_s(A_5)}\}.$$

Case 2: Consider, $X = \{\delta_2,\delta_4,\delta_5,\delta_7\}$ where the decision class corresponds to 0. the partitions corresponding to relation of all the fuzzy near sets $FN_s(A_i)$ where i=1 to 5 are

$$\begin{aligned}
[\delta]FN_s(A) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4\},\{\delta_5\},\{\delta_6\},\{\delta_7\},\{\delta_8\}\}\\
\underline{FN_s(A)} &= \{\delta_4,\delta_5,\delta_7\}\\
\overline{FN_s(A)} &= \{\delta_2,\delta_3,\delta_4,\delta_5.\delta_7\}\\
B_{FN_s(A)} &= \{\delta_2,\delta_3\}\\
\tau_{FN_s(A)} &= \{t,\phi,\{\delta_4,\delta_5,\delta_7\},\{\delta_2,\delta_3,\delta_4,\delta_5,\delta_7\},\{\delta_2,\delta_3\}\}\\
\mathbb{B}_{FN_s(A)} &= \{\delta,\{\delta_4,\delta_5,\delta_7\},\{\delta_2,\delta_3\}\}.
\end{aligned}$$

Excluding the fuzzy near set $FN_s(A_1)$, the partitions corresponding to the remaining fuzzy near sets are

$$\begin{aligned}
[\delta](FN_s(A)-FN_s(A_1)) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4,\delta_6\},\{\delta_5\},\{\delta_7\},\{\delta_8\},\}\\
\underline{(FN_s(A)-FN_s(A_1))} &= \{\delta_5,\delta_7\}\\
\overline{(FN_s(A)-FN_s(A_1))} &= \{\delta_2,\delta_3,\delta_4,\delta_5,\delta_6,\delta_7\}\\
B_{(FN_s(A)-FN(A_1))} &= \{\delta_2,\delta_3,\delta_4,\delta_6\}\\
\tau_{(FN_s(A)-FN_s(A_1))} &= \{\delta,\phi,\{\delta_5,\delta_7\},\{\delta_2,\delta_3,\delta_4,\delta_5,\delta_6,\delta_7\},\{\delta_2,\delta_3,\delta_4,\delta_6\}\}\\
\mathbb{B}_{(FN_s(A)-FN_s(A_1))} &= \{\delta,\{\delta_5,\delta_7\},\{\delta_2,\delta_3,\delta_4,\delta_6\}\} \neq \mathbb{B}_{FN_s(A)}.
\end{aligned}$$

Excluding the fuzzy near set $FN_s(A_2)$, the partitions corresponding to the remaining sets are

$$\begin{aligned}
[\delta](FN_s(A)-FN_s(A_2)) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4,\delta_8\},\{\delta_5\},\{\delta_6\},\{\delta_7\}\}\\
\underline{(FN_s(A)-FN_s(A_2))} &= \{\delta_5,\delta_7\}\\
\overline{(FN_s(A)-FN_s(A_2))} &= \{\delta_2,\delta_3,\delta_4,\delta_5,\delta_7,\delta_8\}\\
B_{(FN_s(A)-FN_s(A_2))} &= \{\delta_2,\delta_3,\delta_4,\delta_8\}\\
\tau_{(FN_s(A)-FN_s(A_2))} &= \{\delta,\phi,\{\delta_1,\delta_6\},\{\delta_2,\delta_3,\delta_4,\delta_8\},\{\delta_1,\delta_2,\delta_3,\delta_4,\delta_6,\delta_8\}\}\\
\mathbb{B}_{(FN_s(A)-FN_s(A_2))} &= \{\delta,\{\delta_5,\delta_7\},\{\delta_2,\delta_3,\delta_4,\delta_8\}\} \neq \mathbb{B}_{FN_s(A)}.
\end{aligned}$$

Excluding the fuzzy near set $FN_s(A_3)$ and $FN_s(A_4)$ the partitions corresponding to each of remaining fuzzy near sets are equal.

$$[\delta](FN_s(A)-FN_s(A_3)) = \{\{\delta_1\},\{\delta_2,\delta_3\},\{t_7\},\{\delta_4\},\{\delta_5\},\{t_8\},\{\delta_6\}\}$$

$$[\delta](FN_s(A)-FN_s(A_4)) = \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4\},\{\delta_5\},\{\delta_6\},\{\delta_7\},\{\delta_8\}\}.$$

Excluding the fuzzy near set $FN_s(A_5)$, the partitions corresponding to the remaining sets are

$$\begin{aligned}
[\delta](FN_s(A)-FN_s(A_5)) &= \{\{\delta_1\},\{\delta_2,\delta_3\},\{\delta_4\},\{\delta_5\},\{\delta_6\},\{\delta_7,\delta_8\}\}\\
\underline{FN_s(A)-FN_s(A_5)} &= \{\delta_4,\delta_5\}\\
\overline{FN_s(A)-FN_s(A_5)} &= \{\delta_2,\delta_3,\delta_4,\delta_5,\delta_7,\delta_8\}\\
B_{FN_s(A)-FN_s(A_5)} &= \{\delta_2,\delta_3,\delta_7,\delta_8\}\\
\tau_{FN_s(A)-FN_s(A_5)} &= \{\delta,\phi,\{\delta_4,\delta_5\},\{\delta_2,\delta_3,\delta_4,\delta_5,\delta_7,\delta_8\},\{\delta_2,\delta_3,\delta_7,\delta_8\}\}\\
\mathbb{B}_{FN_s(A)-FN_s(A_5)} &= \{\delta,\{\delta_4,\delta_5\},\{\delta_2,\delta_3,\delta_7,\delta_8\}\} \neq \mathbb{B}_{FN_s(A)}.
\end{aligned}$$

If $M = \{FN_s(A_1), FN_s(A_2), FN_s(A_5)\}$, then the basis for the fuzzy near set topology corresponding to M is given by $\mathbb{B}_M = \{\delta, \{\{\delta_4, \delta_5, \delta_7\}, \{\delta_2, \delta 3\}\}$. Also $\mathbb{B}_M \neq \mathbb{B}_{FN_s(A)-FN_s(A_i)}$. Therefore core,

$$\mathbf{CFN_s(A) = \{FN_s(A_1), FN_s(A_2), FN_s(A_5)\}}.$$

In both cases we find that the fuzzy near sets $FN_s(A_1), FN_s(A_2), FN_s(A_5)$ form the core showing that Cracks develop in streaks and form cankers, Slimy bacterial oooze through the cracks, Vascular discolouration in split open stems are the three symptoms necessary to decide that the tomato plant is infected by the disease bacterial stem and fruit canker.

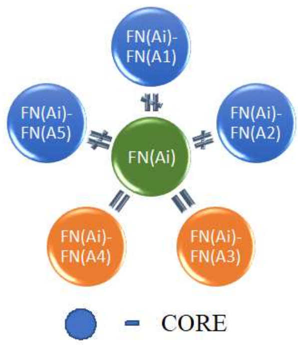

FIGURE 2

Example 3. Let $X = \{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6\}$ represent six types of paddy. Let $FN_s(A_1)$, $FN_s(A_2)$, $FN_s(A_3)$ and $FN_s(A_4)$ represent four qualities of rice. viz., bran removed rice, germ removed rice, polished rice and brown rice, respectively.

Case 1. Consider, $X = \{\eta_1, \eta_2, \eta_3\}$ the partitions corresponding to $FN_s(A)$ are

$$\begin{aligned}
[\eta]FN_s(A) &= \{\{\eta_1, \eta_2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\} \\
\underline{FN_s(A)} &= \{\eta_1, \eta_2, \eta_3\} \\
\overline{FN_s(A)} &= \{\eta_1, \eta_2, \eta_3\} \\
B_{FN_s(A)} &= \{\phi\} \\
\tau_{FN_s(A)} &= \{\eta, \phi, \{\eta_1, \eta_2, \eta_3\}, \{\eta_1, \eta_2, \eta_3\}\} \\
\mathbb{B}_{FN_s(A)} &= \{\eta, \{\eta_1, \eta_2, \eta_3\}\}.
\end{aligned}$$

TABLE 2. Four fuzzy near sets

X	$FN_s(A_1)$	$FN_s(A_2)$	$FN_s(A_3)$	$FN_s(A_4)$	Decision
η_1	0.5	0.5	0.6	1	1
η_2	0.5	0.5	0.6	1	1
η_3	0.7	0.7	0.8	0	1
η_4	0.9	0.5	0.8	0	0
η_5	0.9	0.5	0.4	0.2	0
η_6	0.7	0.7	0.8	0.2	0

Excluding the fuzzy near sets $FN_s(A_1), FN_s(A_2)$ and $FN_s(A_3)$ the partitions corresponding to each of the remaining fuzzy near sets are equal.

$$[\eta](FN_s(A) - FN_s(A_1)) = \{\{\eta_1, \eta_2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\}.$$

$$[\eta](FN_s(A) - FN_s(A_2)) = \{\{\eta_1, \eta_2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\}.$$

$$[\eta](FN_s(A) - FN_s(A_3)) = \{\{\eta_1, \eta_2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\}.$$

Excluding the fuzzy near set $FN_s(A_4)$, the partitions corresponding to the remaining fuzzy near sets are

$$\begin{aligned}
[\eta](FN_s(A) - FN_s(A_4)) &= \{\{\eta_1, \eta_2\}, \{\eta_3, \eta_6\}, \{\eta_4\}, \{\eta_5\}\} \\
\underline{FN_s(A) - FN_s(A_4)} &= \{\eta_1, \eta_2\} \\
\overline{FN_s(A) - FN_s(A_4)} &= \{\eta_1, \eta_2, \eta_3, \eta_6\} \\
B_{FN_s(A)-FN_s(A_4)} &= \{\eta_3, \eta_6\} \\
\tau_{FN_s(A)-FN_s(A_4)} &= \{\eta, \phi, \{\eta_1, \eta_2\}, \{\eta_1, \eta_2, \eta_3, \eta_6\}, \{\eta_3, \eta_6\}\} \\
\mathbb{B}_{FN_s(A)-FN_s(A_4)} &= \{\eta, \{\eta_1, \eta_6, \eta_8\}, \{\eta_2, \eta_3\}\} \neq \mathbb{B}_{FN_s(A)-FN(A_4)}.
\end{aligned}$$

If $M = \{FN_s(A_4)\}$, then the basis for the fuzzy near set topology corresponding to M is $\mathbb{B}_M = \{\eta, \{\{\eta_1, \eta_2\}, \{\eta_3, \eta_4\}\}$. Also $\mathbb{B}_M \neq \mathbb{B}_{FN_s(A)-FN_s(A_i)}$.
therefore core,

$$\mathbf{CFN_s(A)} = \{\mathbf{FN_s(A_4)}\}.$$

FIGURE 3. Qualities of Rice

Case 2. Consider, $X = \{\eta_4, \eta_5, \eta_6\}$.

$$\begin{aligned}
[\eta]FN_s(A) &= \{\{\eta_1, \eta_2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\} \\
\underline{FN_s(A)} &= \{\eta_4, \eta_5, \eta_6\} \\
\overline{FN_s(A)} &= \{\eta_4, \eta_5, \eta_6\} \\
B_{FN_s(A)} &= \{\phi\} \\
\tau_{FN_s(A)} &= \{P, \phi, \{\eta_4, \eta_5, \eta_6\}\} \\
\mathbb{B}_{FN_s(A)} &= \{P, \{\eta_4, \eta_5, \eta_6\}\} = \mathbb{B}_{FN_s(A)}.
\end{aligned}$$

Excluding the fuzzy near sets $FN_s(A_1), FN_s(A_2)$ and $FN_s(A_3)$ the partitions corresponding to each of the remaining fuzzy near sets are equal:

$$[\eta](FN_s(A) - FN_s(A_1)) = \{\{\eta_1, \eta_2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\}$$

$$[\eta](FN_s(A) - FN_s(A_2)) = \{\{\eta_1, \eta 2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\}$$

$$[\eta](FN_s(A) - FN_s(A_3)) = \{\{\eta_1, \eta_2\}, \{\eta_3\}, \{\eta_4\}, \{\eta_5\}, \{\eta_6\}\}.$$

Excluding the fuzzy near set $FN_s(A_4)$, the partitions corresponding to the remaining fuzzy near sets are

$$\begin{aligned}
[\eta](FN_s(A) - FN_s(A_4)) &= \{\{\eta_1, \eta_2\}, \{\eta_3, \eta_4\}, \{\eta_5\}\} \\
\underline{FN_s(A) - FN_s(A_4)} &= \{\eta_1, \eta_2\} \\
\overline{FN_s(A) - FN_s(A_1)} &= \{\eta_1, \eta_2, \eta_3, \eta_6\} \\
B_{FN_s(A)-FN_s(A_1)} &= \{\eta_3, \eta_6\} \\
\tau_{FN_s(A)-FN_s(A1)} &= \{\eta, \phi, \{\eta_1, \eta_2\}, \{\eta_1, \eta_2, \eta_3\}, \eta_6\}, \{\eta_3, \eta_6\}\} \\
\mathbb{B}_{FN_s(A)-FN_s(A_1)} &= \{\eta, \{\eta_1, \eta_2\}, \{\eta_3, \eta_6\}\}.
\end{aligned}$$

Hence $\mathbb{B}_{FN_s(A)} \neq \mathbb{B}_{FN_s(A)-FN_s(A_4)}$.

If $M = \{FN_s(A_4)\}$, then the basis for the fuzzy near set topology corresponding to M is $\mathbb{B}_M = \{\eta, \{\{\eta_5, \eta_6\}, \{\eta_3, \eta_4\}\}$. Also $\mathbb{B}_M \neq \mathbb{B}_{FN_s(A)-FN_s(A_i)}$. Therefore core,

$$\mathbf{CFN_s(A)} = \{\mathbf{FN_s(A_4)}\}.$$

In both cases we find that the fuzzy near set $FN_s(A_4)$ forms the core showing that brown rice is the best quality of paddy.

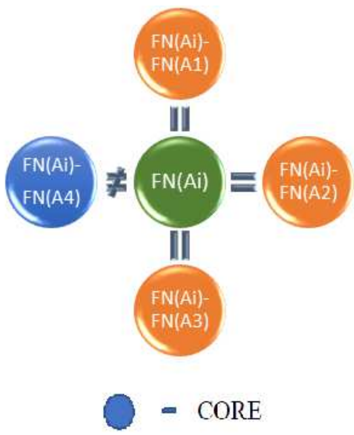

FIGURE 4

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