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STRONGLY VERTEX MULTIPLICATIVE MAGIC GRAPHS

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ABSTRACT. A graph $G = (V;\ E)$ with p vertices and q edges is claimed to be strongly vertex multiplicative magic if the vertices of G can be marked from $\{1,\ 2,\ldots,k\}$ in which no two neighboring vertices gets same label such that labels give raise to the edges obtained by the product of the labels of its end vertices are same. Right now, we show that the existence and non existence for some families of graphs.

1. INTRODUCTION

Graph labellings, where the vertices are doled out qualities subject to specific conditions, have frequently been inspired by useful issues, however they are likewise of enthusiasm for their possess right [1]. A gigantic group of writing has developed around the subject, particularly over the most recent thirty years or thereabouts, and even to make reference to the assortment of issues that have been examined would take us too far aeld here. Most intriguing graph labelling issues have some ingredients, a set of numbers $\mathbb{S}$ from which vertex labels are picked and a condition that these values must fulfil [2, 3].

Sustainable two of the foremost facinating labelling problems are gracefulness and harmoniousness. Graceful labellings were presented under the pertense of β-valuations by Rosa [7], and quite a bit of their unique interigue lay in their association with disintegration of complete graphs, specifically, into trees.

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(See Bloom [4] for a conversation of this point.) In a graceful labelling of a graph with q edges, the names are picked as different values from $\{0, 1, \ldots, q\}$ each edge is given the supreme worth of the names on its vertices, and the prerequisite is that all edge labels be unique.

Harmonious labellings were presented by Graham and Sloane [6] and have associations with blunder adjusting codes'. In a harmonious labelling, the vertices have distinct values from $\{1, 2, \ldots, q\}$ an edge is given the sum modulo q of the marks on its vertices, and, once more, all edge must be distinct. Gallian [5] has composed a broad review, refreshed occasionally, in which results on numerous varieties of these two kinds of labeling are complied.

Right now, consider a labelling that has a lot the same flavoras graceful, harmonious labellings in its simplicity of definition. In any case, it utilizes products more or less than sums. The property, which we call 'strongly vertex multiplicative magic'.

Finally, we show that existence and non existence of strongly vertex multiplicative magic graphs.

2. Strongly vertex multiplicative magic graphs

Theorem 2.1. *The path P_n is strongly vertex multipliicative magic of any non-negative number $n \geq 2$.*

Proof. Define $f : \mathbb{V}(P_n) \to \{1; 2\}$ by

$$f(v_i) = \begin{cases} 1 & \text{if, } i \text{ is odd number} \\ 2 & \text{if, } i \text{ is even number} \\ & \end{cases}$$

and $f(v_i v_{i+1}) = 2, i = 1; 2; 3, \ldots, n-1$. Here all the edge values are same. Hence P_n is strongly vertex multiplicative magic for all non-negative number $n \geq 2$. □

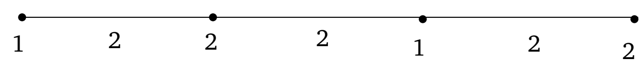

FIGURE 1. Strongly vertex multiplicative magic labeling of P_4

Corollary 2.1. *The graph mP_n is strongly vertex multipliicative magic for any two non-negative numbers m and $n \geq 2$.*

Theorem 2.2. *The cycle C_{2n} is strongly vertex multipliicative magic for any positive integer n.*

Proof. Define $f : V(C_{2n}) \to \{1; 2\}$ by

$$f(v_i) = \begin{cases} 1 & \text{if } i \textit{ is odd number} \\ 2 & \text{if } i \textit{ is even number} \end{cases},$$

and the edge labels are $f(v_i v_{i+1}) = 2$, $\forall\ i = 1, 2, 3, \ldots, 2n - 1$ and $f(v_2 v_1) = 2$. Here all the edge labels are same. Hence C_{2n} is strongly vertex multiplicative magic for any positive integer n. □

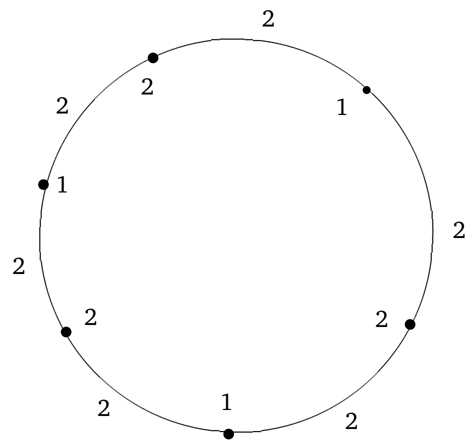

FIGURE 2. Strongly vertex multiplicative magic labeling of C_6

Corollary 2.2. *The graph mC_{2n} is strongly vertex multipliicative magic for any two non-negative numbers m and n.*

Corollary 2.3. *The graph C_{2n+1} is not storngly vertex multiplicative magic for any non-negative numbers n.*

Corollary 2.4. *The graph mC_{2n+1} is not strongly vertex multiplicative magic for any two non-negative numbers m and n.*

Theorem 2.3. *The complete graph K_n is not strongly vertex multiplicative magic for any positive integer n.*

Proof. Suppose K_n is strongly vertex multiplicative magic forall $n \geq 1$. Since all the vertices are adjacent, all the vertex labels are distinct. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$. Since K_n is strongly vertex multiplicative magic, then there exist a labeling $\mho : \mathbb{V}(K_n) \to \{1, 2, \ldots, n\}$ thus $f(v_i) = i \; \forall i = 1; 2; \ldots; n$. Without loss of generality, we assume $f(v_j v_{j+1}) = f(v_l v_{l+1}) = k$ for some $j,\ l \in \{1, 2, \ldots, n\}$ where k is the magic constant.

$$\begin{aligned} \text{Now, } f(v_j v_{j+1}) &= f(v_{j+1} v_{j+2}) = k \\ j(j+1) &= (j+1)(j+2) = k \qquad (2.1) \\ j(j+1) &= k \\ j+1 &= \frac{k}{j} \qquad (2.2) \\ \text{From (2.1), } (j+1)(j+2) &= k \\ (j+2) &= \frac{k}{j+1} = j \text{ , } by(2.2) \\ j+2 &= j \end{aligned}$$

This implies $0 = 2$, which is contradiction. Hence K_n is not strongly vertex multiplicative magic for any positive integer n. □

Corollary 2.5. *The graph mK_n is not strongly vertex multipliicative magic for any two positive integers m and n.*

Theorem 2.4. *The star-graph $K_{1,\,n}$ is strongly vertex multiplicative magic for any positive integer n.*

Proof. Define $\mho : \mathbb{V}(K_{1,n}) \to \{1; 2\}$ by

$$\begin{aligned} f(v_1) &= 1 \\ f(v_i) &= 2,\ i = 2, 3, \ldots, n+1 \\ f(v_1 v_i) &= 2,\ i = 2, 3, \ldots, n+1 \end{aligned}$$

Now, we can easily check that edge labels are same. Hence $K_{1,\,n}$ is strongly vertex multiplicative magic for any positive integer n. □

Corollary 2.6. *The graph $mK_{1\,n}$ is strongly vertex multiplicative magic for any two non-negative number m and n.*

Theorem 2.5. *Every Wheel graph is not strongly vertex multiplicative magic.*

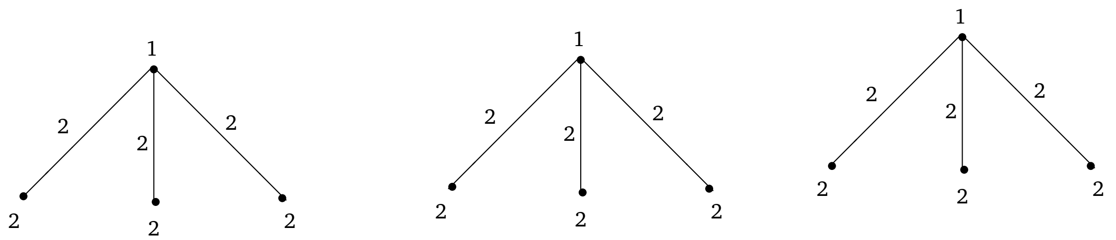

FIGURE 3. Strongly vetex multiplicative magic labeling of $3K_{1,\,3}$

Proof. Suppose the wheel W_4 is strongly vertex multiplicative magic.

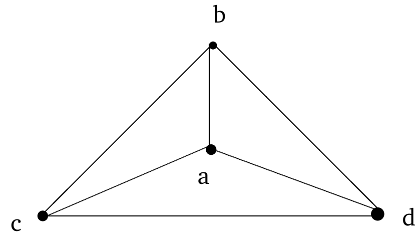

FIGURE 4. A wheel graph w_4

Assume a, b, c, d are the vertex labels and $a \neq b \neq c \neq d$. By the definition, $ab = ac = ad = bc = bd = cd = k$, where k is the magic constant.

$$\begin{aligned} \text{Now}, ab &= k \\ b &= \frac{k}{a} \\ \text{Moreover}, bd &= k \\ d &= \frac{k}{b} = a \\ a &= d \end{aligned}$$

Which is a contradiction to our aim. Hence W_4 is not strongly vertex multiplicative magic. In general, W_n is not strongly vertex multiplicative magic. □

Corollary 2.7. *mW_n is not strongly vertex multiplicative magic for any two non-negative numbers m and n.*

Theorem 2.6. *Every tree is strongly vertex multiplicative magic.*

Proof. Let $\mathbb{T}$ be a tree and let $\gtrapprox$ be any vertex of $\mathbb{T}$. Implant $\mathbb{T}$ in the plane with v as root, and label the vertices in progression utilizing a broadness research. To see that this labeling is strongly vertex multiplicative magic, let 'd' and 'e' be two edges and expect that the ends of 'd' are labelled 'i' and 'j' with $i < j$ and those of 'e' are 'i' and 'j' such that the adjacent edges 'd' and 'e' receives the label ij. Repeat the above process, we get the following labeling as shown in the figure. we can easily check that all edges receives same label. Hence the

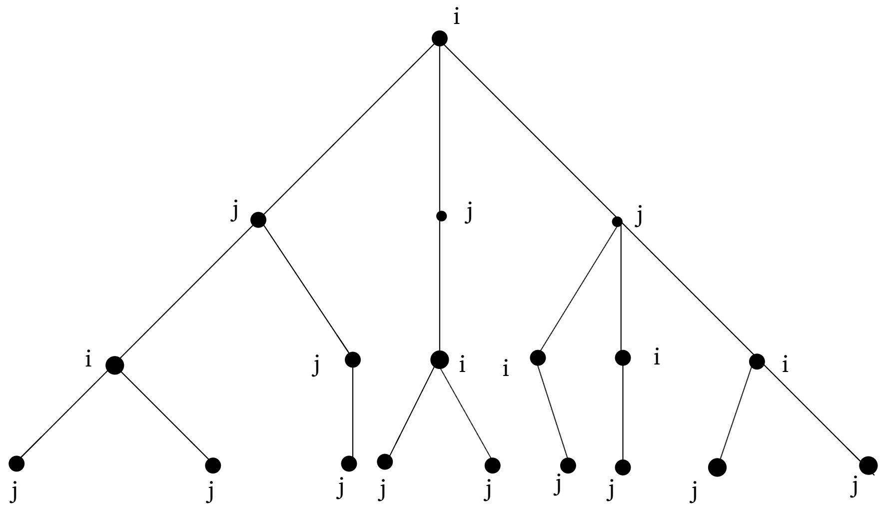

FIGURE 5. Strongly vertex multiplicative labeling of a tree

tree T is strongly vertex multiplicative magic. □

3. ACKNOWLEDGEMENTS

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