REGULAR SEMICLOSED SETS ON NEUTROSOPHIC CRISP TOPOLOGICAL SPACES

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**Abstract.** In this paper, we introduce another idea of neutrosophic crisp generalised sets called neutrosophic crisp regular semi closed sets and examined their central properties in neutrosophic crisp topological spaces. We additionally present neutrosophic crisp regular semi closure and neutrosophic crisp regular semi interior and concentrate a portion of their major properties.

1. Introduction and Preliminaries

In 1965, Zadeh [10] had introduced a fuzzy set as a degree of membership. In 1986, Atanassove [1] proposed the degree of non-membership to fuzzy sets. In addition to this Smarandache [9] added the degree of indeterminacy in 1998. In [7], Salama and Smarandache introduced the following notions, we select one type alone in each case, as more than two types [3]. Let a NCS (neutrosophic crisp set) \(L = (L_1, L_2, L_3)\) of a \(X \neq \phi\), where \(L_1, L_2, L_3 \subseteq X\), \(\phi_N = (\phi, \phi, X)\), \(X_N = (X, X, \phi)\). We will denote the set of all NCSs in \(X\) as \(NCS(X)\).

Let \(L = (L_1, L_2, L_3)\), \(M = (M_1, M_2, M_3) \in NCS(X)\) and \((L_j)_{j \in J} \subset NCS(X)\), where \(L_j = (L_{j,1}, L_{j,2}, L_{j,3})\). Then

(i) \(L \subseteq M\), if \(L_1 \subseteq M_1, L_2 \subseteq M_2, L_3 \supseteq M_3\).

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2010 Mathematics Subject Classification. 54A05, 54A10.

*Key words and phrases.* neutrosophic crisp regular closed (resp. open) sets, neutrosophic crisp regular semi closed (resp. open) sets and neutrosophic crisp regular semi closure (resp. interior).
Let \( L, M, C \in NCS(X) \) and \((L_j)_{j \in J} \subset NCS(X)\). Then

(i) \( \phi_N \subset L \subset X_N \).
(ii) if \( L \subset M \) and \( M \subset C \), then \( L \subset C \).
(iii) \( L \cap M \subset L \) and \( L \cap M \subset M \).
(iv) \( L \cap M = (L_1 \cap M_1, L_2 \cap M_2, L_3 \cap M_3) \).
(v) \( L \cup M = (\cup L_j, \cup L_j, \cup L_j, \cup L_j) \).
(vi) \( \cap L_j = (\cap L_j, \cap L_j, \cap L_j, \cap L_j) \).
(vii) \( \cup L_j = (\cup L_j, \cup L_j, \cup L_j, \cup L_j) \).

Moreover, Salama et al. [5, 7, 8] applied the concept of neutrosophic crisp sets to
concept of \( NCT \) (neutrosophic crisp topology), \( NCT \) (neutrosophic crisp topological space), \( NCS \) (neutrosophic crisp closed set), \( NCT \) (neutrosophic crisp open set), \( NCcl(L) \) (neutrosophic crisp closure of \( L \)) and neutrosophic crisp interior of \( L \). A neutrosophic crisp subset \( L \) of a \( NCTS(X, \Gamma) \) is said to be neutrosophic crisp pre (resp. semi, \( \alpha \) and \( \beta \)) open set \[6\] (briefly, \( NCPos \) (resp. \( NCos, NC\alphaos \) and \( NC\betaos \)) if \( L \subset NCint(NCcl(L)) \) (resp. \( L \subseteq NCcl(NCint(L)) \) and \( L \subseteq NCcl(NCint(NCcl(L))) \)).
The complement of a \( NCP_{os} \) (resp. \( NCS_{os}, NCA_{os} \) and \( NCB_{os} \)) is called a neutrosophic crisp preclosed (resp. semi, \( \alpha \) and \( \beta \)) closed set (briefly, \( NCP_{cs} \) (resp. \( NCS_{cs}, NCA_{cs} \) and \( NCB_{cs} \))) in \( (X, \Gamma) \). The family of all \( NCP_{os} \) (resp. \( NCP_{cs}, NCS_{os}, NCS_{cs}, NCA_{os}, NCA_{cs}, NCB_{os} \) and \( NCB_{cs} \)) \( X \) is denoted by \( NCP_{OS}(X) \) (resp. \( NCP_{CS}(X), NCS_{OS}(X), NCS_{CS}(X), NCA_{OS}(X), NCA_{CS}(X), NCB_{OS}(X) \) and \( NCB_{CS}(X) \)). [6] Let \( L \) be a \( NCS \) of \( NCTS(X, \Gamma) \). Then, the neutrosophic crisp pre (resp. semi, \( \alpha \) and \( \beta \)) interior of \( L \) is the union of all \( NCP_{os} \) (resp. \( NCS_{os}, NC\alpha_{os} \) and \( NC\beta_{os} \)) contained in \( L \) and is denoted by \( NCP_{int}(L) \) (respectively \( NCS_{int}(L), NC\alpha_{int}(L) \) and \( NC\beta_{int}(L) \)). The undefined notions from [7] and cited therein. In general topology Cameron [2] defined a regular semi open sets and Di Maio and Noiri [4] defined semi regular open sets.

2. Neutrosophic crisp regular semi closed sets

**Definition 2.1.** A \( NCS \), \( L \) of a \( NCTS \) \( (X, \Gamma) \) is called a neutrosophic crisp (i) regular open (resp. closed) set (briefly, \( NC_{ros} \) (resp. \( NC_{rcs} \))) if \( L = NC_{int}(NC_{cl}(L)) \) (resp. \( L = NC_{cl}(NC_{int}(L)) \)). (ii) regular semi closed (resp. open) sets (briefly, \( NC_{rS_{cs}} \) (resp. \( NC_{rS_{os}} \))) if \( \exists NC_{res} \) (resp. \( NC_{res} \)) \( H \) in \( X \ni NC_{int}(H) \subseteq L \subseteq H \) (resp. \( H \subseteq L \subseteq NC_{cl}(H) \)).

\( NC_{rS_{CS}}(X) \) (resp. \( NC_{rS_{OS}}(X) \)) denotes the family of all \( NC_{rS_{cs}} \) (resp. \( NC_{rS_{os}} \)) of \( X \)

**Proposition 2.1.** If a \( NCS \), \( L \) is a \( NC_{ros} \) (resp. \( NC_{rS_{os}} \)) then \( L^{c} \) is \( NC_{res} \) (resp. \( NC_{rS_{res}} \)).

**Proposition 2.2.** In a \( NCTS \) \( (X, \Gamma) \), the following hold:

(i) Every \( NC_{ros} \) is a \( NC_{os} \) (resp. \( NC_{rS_{os}} \)).
(ii) Every \( NC_{rS_{os}} \) is a \( NC_{os} \).
(iii) Every \( NC_{os} \) is a \( NC_{rS_{os}} \) (resp. \( NC_{os} \)).
(iv) Every \( NC_{os} \) is a \( NC_{os} \) (resp. \( NC_{pos} \)).
(v) Every \( NC_{pos} \) is a \( NC_{os} \).
(vi) Every \( NC_{os} \) is a \( NC_{os} \).
But not conversely.

**Definition 2.2.** A neutrosophic crisp subset $L$ of a NCTS $(X, \Gamma)$ is called a neutrosophic crisp semi regular open sets (briefly, NCSros) if it is both NCSo and NCSc or equivalently, $L = NC\text{Int}(NC\text{Cl}(L))$. The family of all NCSro (resp. NCSO) of $X$ is denoted by NCSrO($X$) (resp. NCSO($X$)).

**Theorem 2.1.** For any NCS, $L$ of a NCTS $(X, \Gamma)$. (a) (i) $L \in$ NCSrO($X$). (ii) $L = NC\text{Int}(NC\text{Cl}(L))$ (iii) There exist a NCRos $H$ of $X \ni H \subseteq L \subseteq NC\text{Cl}(H)$. are equivalent. (b) (i) $L \in$ NCSrC($X$). (ii) $L = NC\text{Cl}(NC\text{Int}(L))$ (iii) There exist a NCrCS $H$ of $X \ni NC\text{Int}(H) \subseteq L \subseteq H$. are equivalent.

From this discussion, we have,

**Figure 1**

\[ NC\text{ros} \rightarrow NC\text{ros} \rightarrow NC\text{pos} \]
\[ \downarrow \quad \quad \downarrow \quad \quad \downarrow \]
\[ NC\text{rSos} \rightarrow NC\text{Sos} \rightarrow NC\text{bos} \]

3. **Neutrosophic Crisp Regular Semi Closure (resp. Interior)**

**Definition 3.1.** The intersection (resp. union) of all NCrSos (resp. NCrSos) in a NCTS $(X, \Gamma)$ containing (resp. contained in) $L$ is called neutrosophic crisp regular semi closure of $L$ (resp. neutrosophic crisp regular semi interior of $L$) (briefly, NCrSel($L$) (resp. NCrSInt($L$))), $NC\text{rSel}(L) = \cap \{M : L \subseteq M, M \text{ is a } NC\text{rSos}\}$ (resp. $NC\text{rSInt}(L) = \cup \{M : B \subseteq L, M \text{ is a } NC\text{rSos}\}$).

**Proposition 3.1.** Let $L$ be any neutrosophic crisp set in a NCTS $(X, \Gamma)$, the following properties are true:

(i) $NC\text{rSel}(L) = L$ iff $L$ is a NCrSos.
(ii) $NC\text{rSInt}(L) = L$ iff $L$ is a NCrSos.
(iii) $NC\text{rSel}(L)$ is the smallest NCrSos containing $L$.
(iv) $NC\text{rSInt}(L)$ is the largest NCrSos contained in $L$. 
Theorem 3.1. Let $L$ and $M$ be two neutrosophic crisp set in a NCTS $(X, \Gamma)$, the following properties hold:

(i) $NC\text{rScl}(\phi_N) = \phi_N, NC\text{rScl}(X_N) = X_N$.

(ii) $L \subseteq NC\text{rScl}(L)$.

(iii) $L \subseteq M \Rightarrow NC\text{rScl}(L) \subseteq NC\text{rScl}(M)$.

(iv) $NC\text{rScl}(L \cap M) \subseteq NC\text{rScl}(L) \cap NC\text{rScl}(M)$.

(v) $NC\text{rScl}(L) \cup NC\text{rScl}(M) \subseteq NC\text{rScl}(L \cup M)$.

(vi) $NC\text{rScl}(NC\text{rScl}(L)) = NC\text{rScl}(L)$.

(vii) $NC\text{rSint}(\phi_N) = \phi_N, NC\text{rSint}(X_N) = X_N$.

(viii) $NC\text{rSint}(L) \subseteq L$.

(ix) $L \subseteq M \Rightarrow NC\text{rSint}(L) \subseteq NC\text{rSint}(M)$.

(x) $NC\text{rSint}(L \cap M) \subseteq NC\text{rSint}(L) \cap NC\text{rSint}(M)$.

(xi) $NC\text{rSint}(L) \cup NC\text{rSint}(M) \subseteq NC\text{rSint}(L \cup M)$.

(xii) $NC\text{rSint}(NC\text{rSint}(L)) = NC\text{rSint}(L)$.

Proposition 3.2. For any NCS, $L$ of a NCTS $(X, \Gamma)$, then:

(i) $NC\text{rint}(L) \subseteq NC\text{int}(L) \subseteq NC\text{rSint}(L) \subseteq NC\beta\text{rint}(L) \subseteq \quad\quad\quad\quad L \subseteq NC\beta\text{cl}(L) \subseteq NC\text{Scl}(L) \subseteq NC\text{rScl}(L) \subseteq NC\text{cl}(L) \subseteq NC\text{rcl}(L)$.

(ii) $NC\text{int}(L) \subseteq NC\text{oint}(L) \subseteq NC\text{Sint}(L) \subseteq NC\text{Scl}(L) \subseteq NC\text{cl}(L) \subseteq NC\text{calc}(L)$.

(iii) $NC\text{oint}(L) \subseteq NC\text{Pint}(L) \subseteq NC\beta\text{int}(L) \subseteq NC\beta\text{cl}(L) \subseteq NC\text{Pcl}(L) \subseteq NC\text{calc}(L)$.

Theorem 3.2. If a $NC\text{rSos} L$ is such that $L \subseteq M \subseteq NC\text{cl}(L)$, then $M$ is also a $NC\text{rSos}$.

Corollary 3.1. If a $NC\text{rScs} L$ is such that $NC\text{int}(L) \subseteq M \subseteq L$, then $M$ is also a $NC\text{rScs}$.

Theorem 3.3. A NCS $L \in NC\text{rSO}(X)$ iff for every neutrosophic crisp point $p \in L$, $\exists$ a NCS $M \in NC\text{rSO}(X)$ such that $p \in M \subseteq L$.

Proposition 3.3. If $L \in NC\text{rSO}(X)$, then $NC\text{rScl}(L) \subseteq NC\text{rSO}(X)$.

Proposition 3.4. If $L$ is $NC\text{rSos} in X$, then $L^c$ is $NC\text{rScs}$.
Proposition 3.5. In a NCTS \((X, \Gamma)\), the \(\text{NC}\text{cres}\), \(\text{NCros}\) and \(\text{NCrclos}\) are \(\text{NCrSos}\).

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