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LAGUERRE WAVELET TRANSFORM OF GENERALIZED FUNCTIONS IN $K'\{M_P\}$ SPACES

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ABSTRACT. In this paper we have obtained bounded results for Laguerre Wavelet Transform using convolution Theory of $K'\{M_p\}$ Spaces.Also inversion formula for Lagurre Wavelet Transform of Generalized Functions is obtained.

1. INTRODUCTION

The wavelet transform $(W_a f)(b,a)$ of an element $f \in L^2(\mathbb{R})$ is defined by,

$$W_a(f)(b,a) = \int_{-\infty}^{\infty} f(t)\overline{\Phi_{b,a}(t)}dt \tag{1.1}$$

provided the integral exists. Also note that wavelet $\Phi_{b,a}(f)$ is defined as,

$$\Phi_{b,a}(f) = a^{-1/2}\Phi\left(\frac{t-b}{a}\right).$$

In terms of translation τ_b is defined by,

$$\tau_b\Phi(t) = \Phi(t-b), b \in \mathbb{R}$$

and Dilation D_a is defined by,

$$D_a\Phi(t) = a^{-1/2}\Phi\left(\frac{t}{a}\right) \ a < 0.$$

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We can write

$$\Phi_{b,a}(t) = \tau_b D_a \Phi(t).$$

Therefore (1.1) can be written as,

$$W_a(f)(b,a) = (f \star g_{0,a})(b), \tag{1.2}$$

where $g(t) = \overline{\Phi(-t)}$.

In view of (1.2) the wavelet transform $W_a(b,a)$ can be considered as the convolution on f and $g_{0,a}$.The convolution product play an important roles among the various mordern function compositions.It is also one of the very powerful tool for symbolic calculation.Fourier series,approximation theory and in the solution of boundary value problems.

A. K. Shukala [1] has defined Leguerre Transform.

$$F_n(x,y) = \int_0^\infty \int_0^\infty e^{-(x+y)} x^\alpha y^\beta K_n^{(\alpha,\beta)} f(x,y)\, dx\, dy, \tag{1.3}$$

where $K_n^{(\alpha,\beta)} = L_n^\alpha(n) L_n^\beta(y)$ and $L_n^\alpha(n)$ in the Laguerre polynomial of degree and order $\alpha > -1$ given by,

$$L_n^\alpha(x) = \frac{x^n e^x}{n!} \left(\frac{d}{dn}\right)^n x^{x+\alpha} e^{-x}.$$

Therefore equivalent definitions of (1.3) becomes,

$$F_n(\alpha,\beta) = \int_0^\infty e^{-(x+y)} x^\alpha y^\beta L_n^\alpha(x) L_x^\beta(y) f(x,y)\, dx\, dy. \tag{1.4}$$

Inverse of (1.4) is given as,

$$f(x,y) = \sum_{n=0}^{\infty} (\delta_n)^{-1} K_n^{(\alpha,\beta)} F_n(x,y),$$

where, $\delta_n = \frac{\Gamma(x+\alpha+1)\Gamma(x+\beta+1)}{(n!)^2}$. Note that $d \wedge (x) = e^{-x} x^\alpha dx$.

In [9–12] T. G. Thange has studied Wavelet Transform and Laguerre Wavelet Transform. M.S Choudhury and T.G.Thange [2] has introduced Laguerre Wavelet Transform for functions of two variables with inversion formula as follows.

Laguerre Wavelet transform of functions of two variables is defined as

$$(L_\Psi f)(a, b_1 b_2) = \left\langle f(x,y), \Psi^a_{b_1 b_2}(x,y) \right\rangle_\wedge$$
$$= \int_0^\infty \int_0^\infty f(x,y)\Psi^a_{b_1 b_2}(x,y) d\wedge x \, d\wedge y$$
$$= \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty f(x,y)\Psi\overline{(as, at)} d(b_1, b_2, x, y, s, t) d\wedge s d(t) \, d(x) \, d(y). \tag{1.5}$$

Provided in convergent and it is convergent by [2]. Here,Laguerre wavelet is given [2] as:

$$\Psi^a_{b_1 b_2}(t_1 t_2) = \tau_{b_1 b_2}\Psi(t_1 t_2) = \tau_{b_1 b_2}\Psi(at_1 at_2),$$
$$= \int_0^\infty \int_0^\infty \Psi(as, at) d(b_1, b_2, t_1, t_2, \, ds \, dt), \tag{1.6}$$

and Laguerre waveleiven transform and dilation is given in [2].

$L_{p,n}[0,\infty) \times [0,\infty)$ $1 {\leq} p {\leq} \infty$, is the space of there real measurable functions f on$[0,\infty) \times [0,\infty)$, for which,

$$\|f\|_{p,n} = \left(\int_0^\infty \int_0^\infty |f(x,y)|^p \, dx \, dy \right)^{\frac{1}{2}}.$$

Inverse of (1.5) is given as in [2].

For $f \in L_{2,\wedge}$, Ψ be a basic wavelet which defines Laguerre wavelet transform of functions of few variables by (1.5). Let $q(a) >$ be weight function such that

$$Q(n) = \int_0^\infty q(a) |\hat{\Psi}(a,b)|^2 \wedge (a) > 0.$$

Set $\hat{\Psi}^{\star a}_{b_1 b_2} = \dfrac{\hat{\Psi}_{b_1 b_2}(n)}{Q(n)}$. Then

$$f(x,y) = \int_0^\infty \int_0^\infty \int_0^\infty q(a) \left(L_\Psi f\right)(a, b_1 b_2) \Psi^{\wedge \star a}_{b_1 b_2}(x,y) \, da \, db_1 \, db_2.$$

2. Definition and Notation revelent to $K\{M_p\}$ space

The $K\{M_p\}$ space is introduced by Gel'fand and Shilov [4]. $\{M_p\}$ is a sequence of real valued functions define over $\mathbb{R}^n$ such that

$$1 \leq M_1(x) \leq M_2(X) \leq M_3(x) \leq ... \; for \; all \; x \in \mathbb{R}^n,$$

$K\{M_p\}$ is the space of infinitely differentiable functions Φ on $\mathbb{R}^n$ such that

$$\|\Phi\|_p = Sup \, \{M_p(x) \; / D^\alpha \Phi(x) : x \in \mathbb{R}^n, |\alpha| \leq p\} \; for \; all \; p \geq 1, \tag{2.1}$$

$K\{M_p\}$ is a vector space with respect to the norm $\{|| \; ||\}_{p=1}^{\infty}$ defined by (2.1). Under this topology $K\{M_p\}$ is a Frechet space [4]. In $K\{M_p\}$ space M_p satisfies some additional conditions given by [3].

(1) For all $p \in \mathbb{N}$ there exists $p' > p$ such that for every $\epsilon > 0$ there exists $T > 0$ with the property $M_p(x) M_p^{-1}(x) = M_p(x) < \epsilon$ for $|x| > T$.
(2) The function M_p are Quasi-monotonic in each co-ordinate that is $|x_j| \leq |x_j^n|$, then $M_p(x_1, ...x_j...x_n) \leq C_p M_p(x_1, ...x_j...x_n)$ for each fixed points $(x_1, ...x_j, ...x_n)$.
(3) Each M_p is symmetric that is $M_p(x) = M_p(-x)$ and for each p there is $p' > p$ and $C_{p'} > 0$ such that $M_p(x+y) \leq C_{p'} M_{p'}(x) M_{p'}(y)$ for all $x, y \in \mathbb{R}^n$.The dual of $K\{M_p\}$ is denoted by $K' M_p$.The Schwarz space ζ', Gel'fand-shilor space $(S_\alpha, A)'$ and $(w_m, A)'$ are special case of $K'\{M_p\}$ and $K'\{M_p\} \subset D'$.An infinitely differentiable function Ψ on $\mathbb{R}^n$ is said to be a multiplier on $K\{M_p\}$ if (i) $\Psi\phi \in K\{M_p\}$ for each $\phi \in K\{M_p\}$ and the map $\phi \to \Psi\phi$ is continuous from $K\{M_p\}$ into itself. The vector space of all multipliers on $K\{M_p\}$ is denoted by $\Theta_M[K\{M_p\}]$ [5].

3. Convolution in $K\{M_p\}$ space

Let us recall some result related to convolution in $K\{M_p\}$ spaces form [3]. Convolution in $K\{M_p\}$ space is studied in detail by author [5–9].

Translation in $K\{M_p\}$:

If $\Psi \in K\{M_p\}, b \in \mathbb{R}^n$ and $a > 0$,then translation of Ψ by b is denoted by $\tau_b \, \Psi(x)$ That is $\tau_b \Psi(x) = \Psi(x-b)$.

(1) If $T \in \epsilon'(\mathbb{R}^n)$ and the function $\Psi(\zeta) = \left\langle T_n, \Psi(\xi - b) \right\rangle$ belongs to $K\{M_p\}$. Furthermore,if $\{\Psi_j\}$ converges to zero in $K\{M_p\}$.

(2) Also recalling $(T \star \Phi)(\xi) = \left\langle T_n, \Psi(\xi - b) \right\rangle$. And by above theorem $T \star \Psi \in K\{M_p\}$.

(3) We also assume that $K\{M_p\}$ satisfies additional conditional that for any two $p, r(p \leq r)$ there exists $s \geq p$ such that

$$M_p(x)M_r(x) \leq C_{pr}M_{s'}(x) \; for \; x \in \mathbb{R}^n. \tag{3.1}$$

(4) If $\Psi \in K\{M_p\}, U \in K'\{M_p\}$ and (3.1) holds then $U \star \Psi \in \Theta_M[K\{M_p\}]$.

(5) If $U \in K'\{M_p\}$ then there exist a positive integer p and bounded measurable function f_α where $\alpha \in \mathbb{N}^n, |\alpha| \leq P$ such that

$$U = \sum_{|\alpha| \leq P} D^\alpha(M_p f_\alpha). \tag{3.2}$$

(6) If $U \in K'\{M_p\}$ and then $U \star T \in K'\{M_p\}$.

4. Generalized Laguerre Wavelet Transform for $K'\{M_p\}$ space.

By [3] $D \subset K\{M_p\}$. Also D is dense in $K\{M_p\}$ space.So every element of $K'\{M_p\}$ can be identified with distribution. We recall following result from [3] which is useful for defining Laguerre Wavelet Transform for $K'\{M_p\}$ spaces.

Theorem 4.1. *If $\{M_p\}$ satisfy (1),(2) and (3) conditions.Then for $U \in D'$ following are equivalent:*

(1) $U \in K'\{M_p\}$;

(2) *We use above result i.e For $U \in K'\{M_p\} \exists$ a positive integer p and bounded measurable $f_\alpha(\alpha \in \mathbb{N}, |\alpha| \leq p)$ such that,*

$$U = \sum_{|\alpha| \leq p} D^\alpha(M_p f_\alpha).$$

Using above theorem and definition we define generalized Laguerre wavelet transform for $U_{(t_1,t_2)} \in K\{M_p\}$ by,

$$\begin{aligned}(LWT)U(a, b_1, b_2) &= \left\langle U_{(t_1,t_2)}, \Psi^{\star\wedge a}_{b_1 b_2}(t_1, t_2) \right\rangle. \\ &= \left\langle \sum_{|\alpha| \leq p} D^\alpha(M_p(t_1, t_2)) f_\alpha(t_1, t_2), \Psi^{\star\wedge a}_{b_1, b_2}(t_1, t_2) \right\rangle,\end{aligned}$$

where $\Psi^{\star\wedge a}_{b_1,b_2}$ is given by (1.6).

Definition 4.1. *($W_a\{M_p\}$ space) The Space of all C^∞. Functions such that,*

$$\underset{t\in\mathbb{R}^n, a\in\mathbb{R}_+}{Sup}\left|\frac{D^\beta[U\star h_{a,0}](b)}{M_{p'}(a)M_{p'}(b)}\right| < \infty \; for \; p' > p.$$

Following result can be proved for LWT of $U \in K'\{M_p\}$ as in [3].

Theorem 4.2. *For $U \in K'\{M_p\}$ and $\Psi \in K\{M_p\}$, then $LWT(U) \in W_a\{M_p\}$. Now we give inversion formula for the generalized Laguerre wavelet transform to generalized functions in $K\{M_p\}$ space.*

Theorem 4.3. *Let Laguerre wavelet transform (LWT) (a, b_1, b_2) of $U \in K'\{M_p\}$. Then*

$$\lim_{R\to\infty}\left\langle\int_0^\infty\int_0^\infty\int_0^\infty (L_\Psi f)(a,b_1,b_2)\Psi^{\wedge\star a}_{b_1,b_2}(x,y)\,da\,db_1\,db_2\;,\;\Phi(x,y)\right\rangle = \left\langle U,\Phi\right\rangle,$$

for each $\Phi \in L_{p,n}[0,\infty)\times[0,\infty)$ space [2].

Proof. By structure formula for f and function g_{1v} and g_{2v} defined in [2] we have,

$$W_{U_V}(a,b_1,b_2) = \int_0^\infty\int_0^\infty g_{1v}(t_1,t_2)M_{p'}(t_1,t_2)D_1^{k+1}\Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2)dt_1dt_2$$
$$+\int_{-\infty}^\infty\int_{-\infty}^\infty g_{2v}(t_1,t_2)D_1^k\Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2)dt_1\;dt_2.$$

Our aim is to derive the inversion formula,

$$J = \int_0^\infty\int_0^\infty\int_0^\infty (L_\Psi f)_v(a,b_1,b_2)\Psi^{\wedge\star a}_{b_1,b_2}(x,y)\,da\,db_1\,db_2 = U_v.$$

Interchanging convergence in the weak topology of D', i.e,

$$J = \left\langle\int_0^\infty\int_0^\infty\int_0^\infty (L_\Psi f)_v(a,b_1,b_2)\Psi^{\wedge\star a}_{b_1,b_2}(x,y)\,da\,d_1\,d_2\;,\;\Phi(x,y)\right\rangle$$
$$= \left\langle U_v,\Phi\right\rangle \;\forall\;\Phi\in D.$$

Now using structure formula, we have

$$J = \Big\langle \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty g_{1v}(t_1,t_2) M_{p'}(t_1 t_2) D^{k+1}_{(t_1,t_2)} \overline{\Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2)}$$

$$\Psi^{\wedge\star a}_{b_1,b_2}(x,y)\, dt_1\, dt_2\, da\, db_1\, db_2$$

$$+ \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty g_{2v}(t_1,t_2) M_{q'}(t_1,t_2) D^{k}_{(t_1,t_2)} \overline{\Psi^{\wedge\star a}_{b_1,b_2}(x,y)}$$

$$\Psi^{\wedge\star a}_{b_1,b_2}(x,y)\, dt_1\, dt_2\, da\, db_1 db_2, \Phi(x,y) \Big\rangle$$

$$J = \Big\langle \int_0^\infty \int_0^\infty [\int_0^\infty \{\int_0^\infty \int_0^\infty g_{1v}(t_1,t_2) M_{p'}(t_1,t_2)(-1)^{k+1} D^{k+1}_{b_1,b_2} \overline{\Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2)}$$

$$\Psi^{\wedge\star a}_{b_1,b_2}(x,y) dt_1 dt_2\} da] db_1 db_2, \Phi(x,y) \Big\rangle$$

$$+ \Big\langle \int_0^\infty \int_0^\infty [\int_0^\infty \int_0^\infty \{\int_0^\infty g_{2v}(t_1,t_2) M_{q'}(t_1,t_2)(-1)^{k} D^{k}_{b_1,b_2}$$

$$\overline{\Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2)} \Psi^{\wedge\star a}_{b_1,b_2}(x,y) dt_1\ dt_2\}\ da\ db_1\ ] db_2\ ,\ \Phi(x,y) \Big\rangle,$$

as $D_{(t_1,t_2)} \Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2) = -D^{\wedge\star a}_{b_1,b_2}(t_1,t_2)$. Therefore,

$$J = \Big\langle \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty g_{1v}(t_1.t_2) M_{p'}(t_1,t_2) \overline{\Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2)} D^{k+1}_{b_1,b_2}$$

$$\Psi^{\wedge\star a}_{b_1,b_2}\, dt_1\, dt_2\, da\, db_1\, dt_2\ ,\ \Phi(x,y) \Big\rangle$$

$$+ \Big\langle g_{2v}(t_1,t_2) M_{q'}(t_1,t_2) \overline{\Psi^{\wedge\star a}_{b_1,b_2}} D^{k}_{b_1,b_2} \Psi^{\wedge\star a}_{b_1,b_2}(x,y)\, dt_1\, dt_2\, da\, db_1\, db_2\ ,\ \Phi(x,y) \Big\rangle$$

By integrating by parts with respect to b,

$$J = \Big\langle \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty g_{1v}(t_1,t_2) M_{p'}(t_1,t_2) \overline{\Psi^{\wedge\star a}_{b_1,b_2}(t_1,t_2)} (-1)^{k+1}$$

$$D^{k+1}_{(x,y)} \Psi^{\wedge\star a}_{b_1,b_2}(x,y)\, dt_1\, dt_2\, da\, db_1\, db_2\ ,\ \Phi(x,y) \Big\rangle,$$

as $D_{b_1,b_2} = -D_{(x,y)}\Psi^{\star\wedge a}_{b,a}(x,y)$. Hence by distributional differentiation

$$J = \Big\langle \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty g_{1v}(t_1,t_2)M_{p'}(t_1,t_2)\overline{\Psi^{\star\wedge a}_{b_1,b_2}(t_1,t_2)}$$

$$\Psi^{\star\wedge a}_{b_1,b_2}(x,y)\,dt_1\,dt_2\,da\,db_1\,db_2\ ,\ D^k_{(x,y)}\Phi(x,y)\Big\rangle$$

$$+\Big\langle \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty g_{2v}(t_1,t_2)M_{q'}(t_1,t_2)\overline{\Psi^{\star\wedge a}_{b_1b_2}(t_1,t_2)}$$

$$\Psi^{\star\wedge a}_{b_1,b_2}(x,y)\,dt_1\,dt_2\,da\,db_1\,db_2\ ,\ D^k_{(x,y)}\Phi(x,y)\Big\rangle.$$

The integrands $D^{k+1}_{(x,y)}\Phi(X,y)\overline{\Psi^{\star\wedge a}_{b_1,b_2}(t_1,t_2)}g_{1v}(t_1,t_2)M_{p'}(t_1,t_2)$ and $D^k_{(x,y)}(x,y)\Psi_{b_1,b_2}$ $(t_1,t_2)g_{2v}(t_1,t_2)$ $M_{q'}(t_1,t_2)$ are absolutely integrable with respect to x,y and t_1,t_2. By Fubini's theorem [3], (3.2) gives.

$$J = \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty D^{k+1}_{(x,y)}\Phi(x,y)\overline{\Psi^{\star\wedge a}_{b_1,b_2}(t_1,t_2)}g_{1v}(t_1,t_2)$$

$$M_{p'}(t_1,t_2)\,dx\,dy\,dt_1\,dt_2\,da\,db_1\,db_2$$

$$+\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty D^k_{(x,y)}\Phi^{\star\wedge a}_{b_1,b_2}\overline{\Psi^{\star\wedge a}_{b_1,b_2}(t_1,t_2)}g_{2v}(t_1,t_2)M_{q'}(t_1,t_2)\,dx\,dy\,dt_1\,dt_2$$

$$J = \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \overline{[\overline{W_\Psi\{D^{k+1}_{(x,y)}\Phi(x,y)\}}(b_1,b_2,b_3)\Psi^{\star\wedge a}_{b_1,b_2}\,da\,db_1\,db_2]}$$

$$\times\ g_{1v}(t_1,t_2)M_{p'}(t_1,t_2)\,dt_1\,dt_2$$

$$+\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \overline{[\overline{W_\Psi\{D^k_{(x,y)}\Phi(x,y)\}}(b_1,b_2,b_3)\Psi^{\star\wedge a}_{b_1,b_2}\,da\,db_1\,db_2]}$$

$$\times\ g_{2v}(t_1,t_2)M_{q'}(t_1,t_2)\,dt_1\,dt_2.$$

By Fubini's theorem [3],

$$J = \int_0^\infty \int_0^\infty \overline{D^{k+1}_{(t_1,t_2)} \Phi(t_1,t_2)} g_{1v}(t_1,t_2) M_{p'}(t_1,t_2) dt_1 dt_2$$
$$+ \int_0^\infty \int_0^\infty \overline{D^{k}_{(t_1,t_2)} \Phi(t_1,t_2)} g_{2v}(t_1,t_2) M_{q'}\, dt_1\, dt_2.$$

By inversion formula, we get,

$$J = \left\langle g_{1v}(t_1,t_2), M_{p'}(t_1,t_2) D^{k+1}_{t_1,t_2} \Phi(t_1,t_2) \right\rangle + \left\langle g_{2v}(t_1,t_2), M_{q'}(t_1,t_2) D^{k}_{t_1,t_2} \Phi(t_1,t_2) \right\rangle$$

$$J = \left\langle U_v\ ,\ \Phi \right\rangle \longrightarrow \left\langle U, \Phi \right\rangle \quad \text{as} \quad v \longrightarrow \infty.$$

Hence the proof. □

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