FUZZY SOFT CS-CLOSED SPACES IN FUZZY SOFT TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we present a fuzzy soft Cs-closed spaces in fuzzy soft Topological Spaces. A few properties and portrayals of this space are discussed. Fuzzy soft regular semi open set is introduced with example.

1. INTRODUCTION AND PRELIMINARIES

Zadeh [12], presented the fuzzy set in 1965 to solve the many real life problems and in 1999, Molodtsov [5] established the concept of soft sets as a sufficient mathematical tool for dealing the problems with uncertainties. Many researchers have applied the concept of fuzzy sets and soft sets separately. Later Maji [2] have initiated the generalized concept of fuzzy soft sets which combines the fuzzy and soft sets. The fuzzy soft topological structure was introduced by Tanay [10] in 2011. Based on this work, some authors studied the concept of fuzzy soft topological spaces [5,8].

In 2012, Zahran [13] introduced the concept of fuzzy Cs-closed and some of the characterizations in fuzzy topological spaces were studied. Let $U$ be an initial Universe & $E$ be a set of parameters, $P(U)$ denote the power set of $U$ and $A$ be a nonempty subset of $E$. In 2001 fuzzy soft set (briefly, $fSs$) [2], fuzzy soft topology (briefly, $fSt$) [9], fuzzy soft neighborhood [10], fuzzy soft closure (resp. interior) $F_A$ [6], fuzzy soft semi open (briefly, $fSso$), fuzzy soft semi closed (briefly, $fSsc$) set [8], fuzzy soft semi closure (resp. interior) of
F_A and will be denoted by Scl(F_A) (resp. Sint(F_A)), fuzzy soft regular open (resp. closed) set [4], fSo cover [7], fuzzy soft compact [7], fuzzy soft nearly C-compact [3], fuzzy soft filter [1] were given.

**Definition 1.1.** [11] Let F_A & G_B be two fSs's over (U, E). The following operations are defined as:
- **Subset:** F_A ⊂ G_B, if F_A(1e) ⊆ G_B(1e), ∀ 1e ∈ E.
- **Equal:** F_A = G_B, if F_A(1e) ⊆ G_B(1e) & G_B(1e) ⊆ F_A(1e).
- **Union:** H_{A∪B} = F_A ∪ G_B where H_{A∪B}(1e) = F_A(1e) ∪ G_B(1e) ∀ 1e ∈ E.
- **Intersection:** H_{A∩B} = F_A ∩ G_B where H_{A∩B}(1e) = F_A(1e) ∩ G_B(1e) ∀ 1e ∈ E.

**Definition 1.2.** [11] The fSs F_A over (U, E) is called a fuzzy soft point in (U, E) denoted by 1e(F_A), if for the element 1e ∈ F_A, F(1e) ≠ 0 and F(1e) = 0 ∀ 1e ∉ F_A.

**Definition 1.3.** [8] Let (U_1, E_1, τ_1) and (U_2, E_2, τ_2) be two fSts's. A fuzzy soft function f_{up} : (U_1, E_1, τ_1) → (U_2, E_2, τ_2) is said to be fuzzy
(i) soft semi-continuous (resp. fSsCts) if f_{up}^{-1}(g_B) is a fSc set in (U_1, E_1, τ_1) ∀ fuzzy soft closed set g_B in (U_2, E_2, τ_2).
(ii) fSsCts if ∀ fSs g_B in (U_2, E_2, τ_2), Scl(f_{up}^{-1}(g_B)) ⊆ f_{up}^{-1}(cl(g_B)).
(iii) soft semi-irresolute (resp. fSsIrr) if the inverse image of each fSc set is fSsC (U_1, E_1, τ_1).
(iv) fSsIrr if ∀ fSs g_B in (U_2, E_2, τ_2), Scl(f_{up}^{-1}(g_B)) ⊆ f_{up}^{-1}(Scl(cl(g_B))).

2. Fuzzy soft Cs-closed space

**Definition 2.1.** Let (U, E, τ) be fSts. Then (U, E, τ) is said to be a fuzzy soft Cs-closed (resp. fSCsc) if given a fSc set F_A on (U, E) and ∀ fSsSo cover ψ of F_A ∃ a finite subfamily \{F_{A_i} : i = 1, 2, 3, \ldots, n\} of ψ ⊇ F_A ⊆ \bigcup_{i=1}^{n} Scl(F_{A_i}).

**Remark 2.1.** It is clear that fSCsc implies fuzzy soft nearly C-compactness.

**Definition 2.2.** A fSs F_A in a fSts (U, E, τ) is called a fuzzy soft regular semi open (briefly, fSrso) set iff ∃ a fuzzy regular open set G_B ⊇ G_B ⊆ F_A ⊆ cl(G_B).

**Example 1.** Let X = \{a_1, b_1, c_1\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\} ⊆ E. Let us consider the following fSs's over (X, E):
- F_{A_1} = \{F(e_1) = \{a_1/0.5, b_1/0.3, c_1/0.3\}, F(e_2) = \{a_1/0.3, b_1/0.3, c_1/0.3\}\},
- F_{A_2} = \{F(e_1) = \{a_1/1, b_1/0, c_1/0.5\}, F(e_2) = \{a_1/0.5, b_1/0.3, c_1/1\}\},
Let us consider the fSt τ = \{0_E, 1_E, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}\} over \( (X, E) \). Now,

\[
F_{A_1} = \{F^c(e_1) = \{a_1/0.5, b_1/0.7, c_1/0.7\}, F^c(e_2) = \{a_1/0.7, b_1/0.7, c_1/0.7\}\},
\]

\[
F_{A_2} = \{F^c(e_1) = \{a_1/0, b_1/1, c_1/0.5\}, F^c(e_2) = \{a_1/0.5, b_1/0.7, c_1/0\}\},
\]

\[
F_{A_3} = \{F^c(e_1) = \{a_1/0.5, b_1/1, c_1/0.7\}, F^c(e_2) = \{a_1/0.7, b_1/0.7, c_1/0.7\}\},
\]

\[
F_{A_4} = \{F^c(e_1) = \{a_1/0, b_1/0.5, c_1/0.5\}, F^c(e_2) = \{a_1/0.5, b_1/0.5, c_1/0\}\}.
\]

Clearly, \( F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4} \) are fuzzy soft closed sets.

Obviously \( F_{A_i}^c \) is fuzzy soft regular open set. Consider the fSs \( G_A = \{G(e_1) = \{a_1/0.5, b_1/0.4, c_1/0.3\}, G(e_2) = \{a_1/0.3, b_1/0.5, c_1/0.5\}\} \) in \( \tau \). Since \( F_{A_i} \subseteq G_A \subseteq \text{cl}(F_{A_i}) \). Hence \( G_A \) is fSrso set in \( \tau \).

**Lemma 2.1.** For a fSs \( F_A \) in a fSts \( (U, E, \tau) \). The following

(i) Every fSrso set is fSso and fSsc.

(ii) Scl\((F_A)\) is fSrso \( \forall \) fSso set \( F_A \) in \( (U, E) \) are hold.

**Theorem 2.1.** In a fSts \( (U, E, \tau) \) the following conditions are equivalent:

(i) \( U \) is fSCsc.

(ii) \( \forall \) fSsc set \( F_A \) on \( (U, E) \) and \( \forall \) fSrso cover \( \psi \) of \( F_A \) \( \exists \) a finite subfamily \( \{F_{A_i} : i = 1, 2, 3, \ldots, n\} \) of \( \psi \) \( \ni \) \( F_A \subseteq \bigcup_{i=1}^{n} F_{A_i} \).

(iii) \( \forall \) fSsc set \( F_A \) on \( (U, E) \) and \( \forall \) family \( \xi = \{G_{A_\alpha}\}_{\alpha \in \Delta} \) of non-empty fSsc sets \( \ni \) \( \bigcap \xi \cap F_A = \varnothing \) \( \exists \) a finite subfamily \( \{G_{A_i} : i = 1, 2, 3, \ldots, n\} \) of \( \xi \) \( \ni \) \( \bigcap_{i=1}^{n} \text{Sint}(G_{A_i}) \cap F_A = \varnothing \).

(iv) \( \forall \) fSsc set \( F_A \) on \( (U, E) \) and \( \forall \) family \( \xi = \{G_{A_\alpha}\}_{\alpha \in \Delta} \) of fSsc sets, if \( \forall \) finite subfamily \( \{G_{A_i} : i = 1, 2, 3, \ldots, n\} \) of \( \xi \) we have \( \bigcap_{i=1}^{n} \text{Sint}(G_{A_i}) \cap F_A = \varnothing \) then \( \bigcap \xi \cap F_A \neq \varnothing \).

**Proof.**

(i) \( \Rightarrow \) (ii): Suppose (i) holds. Let \( F_A \) be fSsc set and \( \psi \) of be a fSrso cover. Then by Lemma 2.1 (i), \( \psi \) is a fSso cover of \( F_A \). There exist a finite subfamily \( \{F_{A_i} : i = 1, 2, 3, \ldots, n\} \) of \( \psi \ni F_A \subseteq \bigcup_{i=1}^{n} \text{Scl}(F_{A_i}). \) By Lemma 2.1 (ii), Scl\((F_{A_i})\) is regular semi open and Scl\((F_{A_i})\) is fSsc, since \( F_{A_i} \) is fSsc. Hence \( F_A \subseteq \bigcup_{i=1}^{n} \text{Scl}(F_{A_i}) \subseteq \bigcup_{i=1}^{n} F_{A_i}. \)

(ii) \( \Rightarrow \) (i): Suppose (ii) holds. Let \( F_A \) be fSsc set and \( \psi = \{F_{A_i} : i = 1, 2, 3, \ldots, n\} \) be a fSso cover of \( F_A \). Then \( \xi = \{\text{Scl}(F_{A_i})\} \) is a fSrso cover
of $F_A$. By (ii) there exist a finite subfamily \{\text{Scl}(F_{A_i}) : i = 1, 2, 3, \cdots, n\} \ni F_A \subseteq \bigcup_{i=1}^{n} \text{Scl}(F_{A_i})$.

(ii) $\Rightarrow$ (iii): Let $\xi = \{G_{A_0}\}_{\alpha \in \Delta}$ be a family of \text{fSsc} sets of \text{fSs} $(U, E, \tau) \ni \nexists \emptyset \cap F_A = \phi \forall \text{\text{fSsc}}$ sets of $(U, E, \tau)$. Then $\xi = \{\text{Scl}(G_{A_0})\}_{\alpha \in \Delta}$ is a \text{fS} \text{sc} cover of $F_A$. Thus there exist a finite subfamily $\text{Scl}(F_A) = \{\text{Scl}(G_{A_0}), i = 1, 2, 3, \cdots, n\}$ of $\xi \ni F_A \subseteq \bigcup_{i=1}^{n} \text{Scl}(F_{A_i})$.

Now for each $i$, we have $\text{int}(G_{A_i}) = \text{Sint}(F_{A_i}) = \text{Sint}(E-F_{A_i}) = E-\text{Scl}(E-(E-F_{A_i}))$.

So $\bigcap_{i=1}^{n} \text{Sint}(G_{A_i}) = E-\bigcup_{i=1}^{n} \text{Scl}(F_{A_i}) \subseteq E-F_A$. By (i). i.e., $\bigcap_{i=1}^{n} \text{Sint}(G_{A_i}) \cap F_A = \phi$.

(iii) $\Rightarrow$ (ii): Let $\psi = \{F_{A_i} : i = 1, 2, 3, \cdots, n\}$ be a \text{fS} \text{so} cover of the \text{fSsc} set $F_A$ of \text{fSs} $(U, E, \tau)$. Since $F_A \subseteq \bigcup_{\alpha \in \Delta} F_{A_\alpha}$. We will show that $\bigcap_{i=1}^{n} F_{A_i} \cap F_A = \phi$. Since, $F_{A_i}$ is a family of soft semi closed sets, then by (iii), $\bigcup_{\alpha \in \Delta} F_{A_\alpha} \cap F_A = \phi$, there exist a finite subfamily $F_{A_i}$, such that $\bigcap_{i=1}^{n} \text{Sint}(F_{A_i}) \cap F_A = \phi$. Thus $F_A \subseteq \bigcup_{i=1}^{n} (E-\text{Sint}(E-F_{A_i}))$. Now for each $i$, $\text{Sint}(E-F_{A_i}) = E-\text{Scl}(E-(E-F_{A_i}))$. So $F_A \subseteq \bigcup_{i=1}^{n} \text{Scl}(F_{A_i})$. Since $F_{A_i}$ are \text{fSsc} sets, Hence $F_A \subseteq \bigcup_{i=1}^{n} F_{A_i}$.

(iii) $\Rightarrow$ (iv): Let $F_A$ be \text{fSsc} set and $\xi = \{G_{A_0}\}_{\alpha \in \Delta}$ be a family of \text{fSsc} sets, if for each finite subfamily $\{G_{A_i} : i = 1, 2, 3, \cdots, n\} \ni \xi, \bigcap_{i=1}^{n} \text{Sint}(G_{A_i}) \cap F_A = \phi$.

Suppose that $\nexists G_{A_i} \cap F_A = \phi$. Then by (ii) $\exists$ a finite subfamily $\{G_{A_i} : i = 1, 2, 3, \cdots, n\} \ni \exists \bigcap_{i=1}^{n} \text{Sint}(G_{A_i}) \cap F_A = \phi$, which is a contradiction. Hence $\nexists G_{A_i} \cap F_A = \phi$.

(iv) $\Rightarrow$ (iii): Obvious. \hfill $\Box$

**Theorem 2.2.** Every \text{fSsc} subset of a \text{fSCsc} space $(U, E, \tau)$ is \text{Cs-closed}.

**Theorem 2.3.** In a \text{fSs} $(U, E, \tau)$ the following statements are equivalent:

(i) $U$ is \text{fSsc}.

(ii) If $F_A$ is a proper \text{fSsc} set & $\phi$ is a family of \text{fSsc} sets of $(U, E, \tau) \ni F_A \subseteq (E-\bigcap_{i=1}^{n} F_{A_i})$ then there exist a finite subfamily of $\phi$ say $F_{A_1}, F_{A_2}, \cdots, F_{A_n}$ $\ni F_A \subseteq (E-\bigcap_{i=1}^{n} \text{Sint}(F_{A_i}))$.

**Definition 2.3.** Let $(U, E, \tau)$ be \text{fSs}. A fuzzy soft filter in $U$ is said to be semi adherence convergent if every \text{fS} \text{so} neighborhood of the adherence set of $\xi$ contains an element of $\xi$ where the adherence set is defined by $\bigcap \{\text{Scl}(F_A) : F_A \in \xi\}$. 
Theorem 2.4. If \((U, E, \tau)\) is fSCsc then every fSso filter is semi adherence convergent.

Theorem 2.5. Let \(f_{up} : (U_1, E_1, \tau_1) \to (U_2, E_2, \tau_2)\) be a fuzzy soft function from a fSts \((U_1, E_1, \tau_1)\) to a fSts \((U_2, E_2, \tau_2)\). Then the image of a fSCsc space under a fuzzy soft irresolute function is fSCsc space.

Proof. Let \(f_{up} : (U_1, E_1, \tau_1) \to (U_2, E_2, \tau_2)\) be a fuzzy soft irresolute function from fSCsc space \(U_1\) onto \(U_2\) and let \(F_A\) be a proper fSsc set in \(U_2\). Let \(\psi = \{F_{A_\alpha}\}_{\alpha \in \Delta}\) be a fSso cover of \(F_A\) in \(U_2\). Since \(f_{up}\) is fuzzy soft irresolute, then \(f_{up}^{-1}(F_A)\) is a fSc set in \(U_1\) & \(\{f_{up}^{-1}(F_{A_\alpha})\}_{\alpha \in \Delta}\) is a fSso cover of \(f_{up}^{-1}(F_A)\) in \(U_1\). Since \(U_1\) is fSCsc, then there is a finite subfamily \(\{f_{up}^{-1}(F_{A_\alpha}), i = 1, 2, \ldots, n\}\) such that \(f_{up}^{-1}(F_{A_\alpha}) \subseteq \bigcup_{i=1}^{n} \text{Scl}(f_{up}^{-1}(F_{A_\alpha})) \subseteq \bigcup_{i=1}^{n} f_{up}^{-1}(\text{Scl}(F_{A_\alpha}))\) by Definition 1.3 (iv) and hence \(F_A \subseteq \bigcup_{i=1}^{n} \text{Scl}(F_{A_\alpha})\). Thus \(U_2\) is fSCsc Space. \(\square\)

References


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