ANTI Q-FUZZY BI-IDEALS IN NEAR-SUBTRACTION SEMIGROUPS

V. MAHALAKSHMI¹, M. MUTHU MEENAL, K. MUMTHA, AND J. SIVA RANJINI

ABSTRACT. Our primary focus is to examine the notion of anti Q-fuzzy bi-ideals in near-subtraction semigroups. This is a continuation/furtherance of our earlier study regarding Q-fuzzy bi-ideals in near-subtraction semigroups. In this paper, we have attempted to define the notation of anti Q-fuzzy bi-ideals and investigated their properties in near-subtraction semigroups.

1. INTRODUCTION

The Concepts of fuzzy sets and fuzzy subsets, fuzzy logic finds roots in seminal work of L. A. Zadeh [4] in 1965. Fuzzy Logic and fuzzification is a transformative development in set theory, having been a ring in many latest scientific applications. Ideal of subtraction semigroup is thoroughly examined by K. H. Kim et.al. [3]. Anti Q-fuzzy bi-ideals of near-rings are researched and characterized by A. Balavickhnswari, V. Mahalakshmi [2]. Also the notation of Q-fuzzy bi-ideals of near-subtraction semigroups are researched and characterized by P. Annamalai Selvi and et.al [1].

Our present study is inspired by the above study and we have examined the concept of anti Q-fuzzy bi-ideals in near-subtraction semigroups and its characteristics.

¹corresponding author

2010 Mathematics Subject Classification. 08A72.

Key words and phrases. ideal, bi-ideal, Q-fuzzy bi-ideal, anti Q-fuzzy bi-ideal.
2. Preliminaries

In this section, we collect all basic concepts of near-subtraction semigroups, which are used in this paper. Throughout this paper, by a near subtraction semigroup, we mean only a zero-symmetric right near-subtraction semigroup.

**Definition 2.1.** A family of Q-fuzzy sets \( \{ \mu_i / i \in \Omega \} \) in \( X \), the union of \( \{ \mu_i / i \in \Omega \} \) is defined by, \( \cup_{i \in \Omega} \mu_i(x, q) = \sup \{ \mu_i(x, q) / i \in \Omega \}, \forall x \in X, q \in Q \) and the intersection of \( \{ \mu_i / i \in \Omega \} \) is defined by, \( \cap_{i \in \Omega} (x, q) = \inf \{ \mu_i(x, q) / i \in \Omega \}, \forall x \in X, q \in Q \).

**Definition 2.2.** Let \( f : X \rightarrow X' \). Let \( \mu \) and \( \lambda \) be a Q-fuzzy sets of \( X \) and \( X' \) respectively. Then \( f(\mu) \), the image of \( \mu \) under \( f \), is a subset of \( X' \) defined by:

\[
f(\mu)(b, q) = \begin{cases} 
(a, q) \in f^{-1}\inf_{b,q} \mu(a, q) & \text{if } f^{-1}(b, q) \neq \emptyset \\
0 & \text{otherwise}
\end{cases}
\]

and pre-image of \( \lambda \) under \( f \) is a Q-fuzzy subset of \( X \), defined by \( f^{-1}(\lambda(x, q)) = \lambda(f(x, q)) \), for all \( x \in X, q \in Q \) and \( f^{-1}(y, q) = \{(x, q)/x \in X, q \in Q, f(x, q) = (y, q)\} \) and also referred the notations of \( (\mu \cap \lambda), (\mu - \lambda), (\mu \lambda) \& (\mu * \lambda) \) in [1].

In this paper, \( f_I \) is the characteristic function of the subsets \( I \) of \( X \), and the characteristic function of \( X \times Q \) is denoted by \( \chi : X \times Q \rightarrow [0, 1] \) and it is mapping each element of \( X \times Q \) to 1.

**Definition 2.3.** For any Q-fuzzy set \( \mu \) in \( X \) and \( t \in [0, 1] \), we define \( L(\mu; t) = \{ (x, q)/x \in X, q \in Q, \mu(x, q) \leq t \} \), which is called a lower \( t \)-level cut of \( \mu \).

**Definition 2.4.** A mapping \( f : X \rightarrow X' \) is called a homomorphism if \( f(x - y) = f(x) - f(y) \) and \( f(xy) = f(x)f(y) \), for all \( x, y \in X \).

**Definition 2.5.** A mapping \( f : X \rightarrow X' \) is called an anti-homomorphism if \( f(x - y) = f(y) - f(x) \) and \( f(xy) = f(y)f(x) \) for all \( x, y \in X \).

**Definition 2.6.** A mapping \( \mu : X \times Q \rightarrow [0, 1] \), where \( X \) is an arbitrary non-empty set is called Q-fuzzy set in \( X \).

**Definition 2.7.** A Q-fuzzy subset \( \mu \) is called Q-fuzzy ideal of \( X \) if \( \forall x, y \in X \) and \( q \in Q \) it hold:

\[(i) \ \mu(x - y, q) \geq \min \{ \mu(x, q), \mu(y, q) \}\]
\(\mu(x - y, q) \geq \mu(x, q), \mu(y, q)\)

\(\mu(xy, q) \geq \mu(x, q)\).

**Definition 2.8.** \(\) A Q-fuzzy set \(\mu\) in \(X\) is a a Q-fuzzy bi-ideal of \(X\) if for \(\forall x, y, z \in X\) and \(q \in Q\) it hold:

(i) \(\mu(x - y, q) \geq \mu(x, q), \mu(y, q)\)

(ii) \(\mu(xy, q) \geq \mu(x, q)\).

**Definition 2.9.** A Q-fuzzy subset \(\mu\) is called an anti Q-fuzzy ideal of \(X\) if it satisfies:

(i) \(\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}\)

(ii) \(\mu(x, q) \leq \min\{\mu(x, q), \mu(z, q)\}\).

**3. Anti Q-fuzzy bi-ideal in near-subtraction semigroups**

**Definition 3.1.** A Q-fuzzy set \(\mu\) in \(X\) is said to be an anti Q-fuzzy bi-ideal of \(X\) if for all \(x, y, z \in X\) and \(q \in Q\) it hold:

(i) \(\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}\)

(ii) \(\mu(xy, q) \leq \max\{\mu(x, q), \mu(z, q)\}\)

**Example 1.** Let \(X = \{0, a, b, c\}\) with "−" and "•" defined as

\[
\begin{array}{cccc}
- & 0 & a & b & c \\
0 & 0 & 0 & 0 & 0 \\
a & a & 0 & a & a \\
b & b & b & 0 & b \\
c & c & c & c & 0 \\
\end{array}
\quad
\begin{array}{cccc}
• & 0 & a & b & c \\
0 & 0 & 0 & 0 & 0 \\
a & 0 & b & 0 & b \\
b & 0 & 0 & 0 & 0 \\
c & 0 & b & 0 & b \\
\end{array}
\]

Define an anti Q-fuzzy subset \(\mu : X \times Q \rightarrow [0, 1]\) by \(\mu(0, q) = 0.5, \mu(a, q) = 0.6, \mu(b, q) = 0.7, \mu(c, q) = 0.8\). It is easy to verify that \(\mu\) is an anti Q-fuzzy bi-ideal of \(X\).

**Theorem 3.1.** Let \(\{\mu_i / i \in \Omega\}\) be a family of an anti Q-fuzzy bi-ideal of \(X\), then \(\cup_{i \in \Omega} \mu_i\) is also an anti Q-fuzzy bi-ideal of \(X\), where \(\Omega\) is any index set.
Proof. Let \( \{ \mu_i / i \in \Omega \} \) be a family of an anti \( Q \)-fuzzy bi-ideals of \( X \). Let \( x, y, z \in X \), \( q \in Q \) and \( \mu = \bigcup_{i \in \Omega} \mu_i \). Then

\[
\mu(x, q) = \bigcup_{i \in \Omega} \mu_i(x, q) = \sup_{i \in \Omega} \mu_i(x, q)
\]

\[
\mu(x - y, q) = \sup_{i \in \Omega} \mu_i(x - y, q) \leq \sup_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(y, q)\}
\]

\[
= \max\{\sup_{i \in \Omega} \mu_i(x, q), \sup_{i \in \Omega} \mu_i(y, q)\}
\]

\[
= \max\{\bigcup_{i \in \Omega} \mu_i(x, q), \bigcup_{i \in \Omega} \mu_i(y, q)\}
\]

\[
= \max\{\mu(x, q), \mu(y, q)\}.
\]

Therefore, \( \mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\} \). Thus \( \mu \) is an anti \( Q \)-fuzzy subalgebra of \( X \).

\[
\mu(xyz, q) = \sup_{i \in \Omega} \mu_i(xyz, q)
\]

\[
\leq \sup_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(z, q)\}
\]

\[
= \max\{\sup_{i \in \Omega} \mu_i(x, q), \sup_{i \in \Omega} \mu_i(z, q)\}
\]

\[
= \max\{\bigcup_{i \in \Omega} \mu_i(x, q), \bigcup_{i \in \Omega} \mu_i(z, q)\}
\]

\[
= \max\{\mu(x, q), \mu(z, q)\}.
\]

Therefore, \( \mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\} \). Hence \( \mu = \bigcup_{i \in \Omega} \mu_i \) is an anti \( Q \)-fuzzy bi-ideal of \( X \), where \( \Omega \) is any index set. \( \square \)

Theorem 3.2. Let \( f : X \to X' \) be an epimorphism of \( X \). If \( \lambda \) is an anti \( Q \)-fuzzy bi-ideal of \( X' \), then \( f^{-1}(\lambda) \) is an anti \( Q \)-fuzzy bi-ideal in \( X \).

Proof. Let \( \lambda \) be an anti \( Q \)-fuzzy bi-ideal of \( X' \). For \( x, y, z \in X \) and \( q \in Q \),

\[
f^{-1}(\lambda)(x - y, q) = \lambda(f(x - y, q))
\]

\[
= \lambda(f(x, q) - f(y, q))
\]

\[
\leq \max\{\lambda f(x, q), \lambda f(y, q)\}.
\]
Therefore \( f^{-1}(\lambda)(x - y, q) \leq \max\{f^{-1}(\lambda)(x, q), f^{-1}(\lambda)(y, q)\} \).

\[
f^{-1}(\lambda)(xyz, q) = \lambda(f(xyz, q)) = \lambda\{f(x, q)f(y, q)f(z, q)\} \leq \max\{\lambda f(x, q), \lambda f(z, q)\}.
\]

Therefore, \( f^{-1}(\lambda)(xyz, q) \leq \max\{f^{-1}(\lambda)(x, q), f^{-1}(\lambda)(z, q)\} \). Hence \( f^{-1}(\lambda) \) is an anti Q-fuzzy bi-ideal in \( X \).

\[\square\]

**Theorem 3.3.** Let \( f : X \to X' \) be an epimorphism of \( X \). If \( \mu \) is an anti Q-fuzzy bi-ideal in \( X \), then \( f(\mu) \) is an anti Q-fuzzy bi-ideal in \( X' \).

**Proof.**

(i) Let \( \mu \) is an anti Q-fuzzy bi-ideal in \( X \) and \( y_1, y_2, y_3 \in X' \) and \( q \in Q \). Then we have:

\[
(x, q) \in f^{-1}(\inf_{y_1-y_3} (x, q)) \subseteq (x_1, q) \in f^{-1}(y_1, q)
\]

\[
(x_2, q) \in f^{-1}(\inf_{y_2} (x_1 - x_2, q))
\]

\[
f(\mu)(y_1 - y_2, q) = (x, q) \in f^{-1}(\inf_{y_1-y_2} (x, q)) \leq (x_1, q) \in f^{-1}(y_1, q)
\]

\[
(x_2, q) \in f^{-1}(\inf_{y_2} \mu(x_1 - x_2, q)) \leq (x_1, q) \in f^{-1}(y_1, q)
\]

\[
(x_2, q) \in f^{-1}(\inf_{y_2} \max\{\mu(x_1, q), \mu(x_2, q)\}) = \max\{x_1, q) \in f^{-1}(\inf_{y_1} \mu(x_1, q))
\]

\[
(x_2, q) \in f^{-1}(\inf_{y_2} \mu(x_2, q))
\]

Therefore, \( f(\mu)(y_1 - y_2, q) \leq \max\{f(\mu)(y_1, q), f(\mu)(y_2, q)\} \). Thus \( f(\mu) \) is an anti Q-fuzzy subalgebra in \( X' \).

(ii) Let \( y_1, y_2, y_3 \in X' \) and \( q \in Q \). Then we have:

\[
f(\mu)(y_1y_2y_3, q) = (x, q) \in f^{-1}(\inf_{y_1y_2y_3} (x, q)) \leq (x_1, q) \in f^{-1}(y_1, q)
\]

\[
(x_3, q) \in f^{-1}(\inf_{y_3} \mu(x_1x_2x_3, q)) \leq (x_1, q) \in f^{-1}(y_1, q)
\]

\[
(x_3, q) \in f^{-1}(\inf_{y_3} \max\{\mu(x_1, q), \mu(x_3, q)\}) = \max\{x_1, q) \in f^{-1}(\inf_{y_1} \mu(x_1, q))
\]

\[
(x_3, q) \in f^{-1}(\inf_{y_3} \mu(x_3, q))
\]

Therefore, \( f(\mu)(y_1y_2y_3, q) \leq \max\{f(\mu)(y_1, q), f(\mu)(y_3, q)\} \). Hence \( f(\mu) \) is an anti Q-fuzzy bi-ideal in \( X' \).
Theorem 3.4. Let \( \mu \) be an anti Q-fuzzy subalgebra of \( X \), then \( \mu \) is an anti Q-fuzzy bi-ideal of \( X \) iff \( \mu \) is a bi-ideal of \( X \).

Proof. Assume that \( \mu \) is an anti Q-fuzzy bi-ideal of \( X \). To prove: \( \mu \) is a bi-ideal of \( X \), let \( \mu(x,y) \). Now, \( \mu \) is a bi-ideal of \( X \).

\[
(\mu X \mu)(x', q) = \inf_{x' = xy} \max\{\mu X(x, q), \mu(y, q)\}
\]
\[
= \inf_{x' = xy} \max\{\inf_{x' = x_1 x_2} \max\{\mu(x_1, q), X(x_2, q), \mu(y, q)\}\}
\]
\[
= \inf_{x' = xy} \max\{\inf_{x' = x_1 x_2} \max\{\mu(x, q), 1, \mu(y, q)\}\}
\]
\[
= \inf_{x' = x_1 x_2} \max\{\mu(x, q), \mu(y, q)\}
\]
\[
\geq \inf_{x' = x_1 x_2} \mu(x_1 x_2 y, q).
\]

Therefore, \( (\mu X \mu)(x', q) \geq \mu(x', q) \). Hence \( \mu X \mu \supseteq \mu \).

Conversely, assume that \( \mu X \mu \supseteq \mu \). To prove: \( \mu \) is a bi-ideal of \( X \), let \( x, y, z \in X \) and \( q \in Q \). Now,

\[
\mu(xyz, q) \leq \mu X \mu(xyz, q)
\]
\[
= \inf_{xyz = ab} \max\{\mu X(a, q), \mu(b, q)\}
\]
\[
= \max\{(\mu X)(xy, q), \mu(z, q)\}
\]
\[
= \max\{\mu(x, q), X(y, q), \mu(z, q)\}
\]
\[
= \max\{\mu(x, q), 1, \mu(z, q)\}
\]
\[
= \max\{\mu(x, q), \mu(z, q)\}.
\]

Therefore, \( \mu(xyz, q) \leq \mu(x, q), \mu(z, q)\). Hence \( \mu \) is a bi-ideal of \( X \).

\]

Theorem 3.5. Let \( X \) and \( X' \) be two near-subtraction semigroups. Let a mapping \( f : X \rightarrow X' \) be a homomorphism. If \( \mu \) is an anti Q-fuzzy bi-ideal of \( X' \) and \( L(\mu; t) \) is a bi-ideal of \( X' \), then \( L(f^{-1}(\mu); t) \) is a bi-ideal of \( X \).
Proof. Let $f : X \rightarrow X'$ be a homomorphism, $\mu$ is an anti Q-fuzzy bi-ideal of $X'$ and $L(\mu; t)$ is a bi-ideal of $X'$. Let $x, y \in L(f^{-1}(\mu; t)$ and $q \in Q$. Then we have:

\[
f^{-1}(\mu)(x, q) \leq t
\]

\[
f^{-1}(\mu)(y, q) \leq t \Rightarrow \mu(f(x, q)) \leq t
\]

\[
\mu(f(y, q)) \leq t.
\]

Now,

\[
f^{-1}(\mu)(x - y, q) = \mu(f(x - y, q))
\]

\[
= \mu(f(x, q) - f(y, q))
\]

\[
\leq \max\{\mu(f(x, q)), \mu(f(y, q))\}
\]

\[
= \max\{t, t\} = t.
\]

Therefore, $f^{-1}(\mu)(x - y, q) \leq t$. We get $x - y \in L(f^{-1}(\mu); t)$. Hence $L(f^{-1}(\mu); t)$ is a subalgebra of $X$.

Let $x, z \in L(f^{-1}(\mu); t)$ and $y \in X$. Then we have:

\[
f^{-1}(\mu)(x, q) \leq t
\]

\[
f^{-1}(\mu)(z, q) \leq t \Rightarrow \mu(f(x, q)) \leq t
\]

\[
\mu(f(z, q)) \leq t.
\]

Now,

\[
f^{-1}(\mu)(xyz, q) = \mu(f(xyz, q))
\]

\[
= \mu(f(x, q)f(y, q)f(z, q))
\]

\[
\leq \max\{\mu(f(x, q)), \mu(f(z, q))\}
\]

\[
= \max\{t, t\} = t.
\]

Therefore, $f^{-1}(\mu)(xyz, q) \leq t$. We get $xyz \in L(f^{-1}(\mu); t)$. Hence $L(f^{-1}(\mu); t)$ is a bi-ideal of $X$.

\[
\square
\]

**Theorem 3.6.** Let $\mu$ be a Q-fuzzy set of $X$. Then $\mu$ is an anti Q-fuzzy bi-ideal of $X$ iff $\mu^c$ is a Q-fuzzy bi-ideal of $X$. 

\[
\square
\]
Proof. Assume that $\mu$ is an anti Q-fuzzy bi-ideal of $X$. To prove that $\mu^c$ is a Q-fuzzy bi-ideal of $X$, let $x, y, z \in X$ and $q \in Q$. Now,

$$
\mu^c(x - y, q) = 1 - \mu(x - y, q) \\
\geq 1 - \max\{\mu(x, q), \mu(y, q)\} \\
= \min\{1 - \mu(x, q), 1 - \mu(y, q)\}.
$$

Therefore, $\mu^c(x - y, q) \geq \min\{\mu^c(x, q), \mu^c(y, q)\}$.

Now,

$$
\mu^c(xyz, q) = 1 - \mu(xyz, q) \\
\geq 1 - \max\{\mu(x, q), \mu(z, q)\} \\
= \min\{1 - \mu(x, q), 1 - \mu(z, q)\} \\
= \min\{\mu^c(x, q), \mu^c(z, q)\}.
$$

Therefore, $\mu^c(xyz, q) \geq \min\{\mu^c(x, q), \mu^c(z, q)\}$. Hence $\mu^c$ is a Q-fuzzy bi-ideal of $X$.

Conversely, assume that $\mu^c$ is a Q-fuzzy bi-ideal of $X$. To prove: $\mu$ is an anti Q-fuzzy bi-ideal of $X$, let $x, y, z \in X$ and $q \in Q$. Now,

$$
\mu(x - y, q) = 1 - \mu^c(x - y, q) \\
\leq 1 - \min\{\mu^c(x, q), \mu^c(y, q)\} \\
= \max\{1 - \mu^c(x, q), 1 - \mu^c(y, q)\}.
$$

Therefore $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$. Now,

$$
\mu(xyz, q) = 1 - \mu^c(xyz, q) \\
\leq 1 - \min\{\mu^c(x, q), \mu^c(z, q)\} \\
= \max\{1 - \mu^c(x, q), 1 - \mu^c(z, q)\}.
$$

Therefore $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$. Hence $\mu$ is an anti Q-fuzzy bi-ideal of $X$. $\square$

**Theorem 3.7.** Let $\mu$ be a Q-fuzzy set of $X$. Then $\mu$ is an anti Q-fuzzy bi-ideal of $X$ iff the lower level cut $L(\mu; t)$ of $X$ is a bi-ideal of $X$ for each $t \in [\mu(0), 1]$. 
Proof. Let $\mu$ is an anti $Q$-fuzzy bi-ideal of $X$. Let $x, y \in L(\mu; t)$ and $q \in Q$. Then $\mu(x, q) \leq t$ and $\mu(y, q) \leq t$. Now,

$$\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\} = \max\{t, t\} = t.$$ 

Therefore, $\mu(x - y, q) \leq t$, we get $x - y \in L(\mu; t)$. Hence $L(\mu; t)$ is a subalgebra of $X$.

Let $x, z \in L(\mu; t)$ and $y \in X$, $q \in Q$. Then $\mu(x, q) \leq t$ and $\mu(z, q) \leq t$. Now,

$$\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\} = \max\{t, t\} = t.$$ 

Therefore, $\mu(xyz, q) \leq t$. We get $xyz \in L(\mu; t)$. Hence $L(\mu; t)$ is a bi-ideal of $X$.

Conversely, assume that $L(\mu; t)$ is a bi-ideal of $X$. To prove that $\mu$ is an anti $Q$-fuzzy bi-ideal of $X$, suppose $\mu$ not is an anti $Q$-fuzzy bi-ideal of $X$. Let $x, y \in X$ and $q \in Q$ $\mu(x - y, q) > \max\{\mu(x, q), \mu(y, q)\}$. Choose $t$ such that $\mu(x - y, q) > t > \max\{\mu(x, q), \mu(y, q)\}$. Then we get $x, y \in L(\mu; t)$, but $x - y \notin L(\mu; t)$, which is a contradiction. Hence $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$.

Let $x, y, z \in X$ and $q \in Q$ $\mu(xyz, q) > \max\{\mu(x, q), \mu(z, q)\}$. Choose $t$ such that $\mu(xyz, q) > t > \max\{\mu(x, q), \mu(z, q)\}$. Then we get $xyz \in L(\mu; t)$, but $xyz \notin L(\mu; t)$, which is a contradiction. Hence $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$.

Hence $\mu$ is an anti $Q$-fuzzy bi-ideal of $X$. \qed

Theorem 3.8. Let $\{\mu_i / i \in \Omega\}$ be a family of an anti $Q$-fuzzy bi-ideal of a near-subtraction semigroup $X$. Then $\bigcap_{i \in \Omega} \mu_i$ is also an anti $Q$-fuzzy bi-ideal of $X$, where $\Omega$ is any index set.

Proof. Let $\{\mu_i / i \in \Omega\}$ be a family of an anti $Q$-fuzzy bi-ideals of $X$. Let $x, y, z \in X$ and $q \in Q$ and $\mu = \bigcap_{i \in \Omega} \mu_i$. Then:

$$\mu(x, q) = \bigcap_{i \in \Omega} \mu_i(x, q) = \inf_{i \in \Omega} \mu_i(x, q)$$

$$\mu(x - y, q) = \inf_{i \in \Omega} \mu_i(x - y, q)$$

$$\leq \inf_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(y, q)\}$$

$$= \max\{\inf_{i \in \Omega} \mu_i(x, q), \inf_{i \in \Omega} \mu_i(y, q)\}$$

$$= \max\{\bigcap_{i \in \Omega} \mu_i(x, q), \bigcap_{i \in \Omega} \mu_i(y, q)\}$$

$$= \max\{\mu(x, q), \mu(y, q)\}.$$
Therefore, $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$. Thus $\mu$ is an anti Q-fuzzy subalgebra of $X$.

\[
\begin{align*}
\mu(xyz, q) &= \inf_{i \in \Omega} \mu_i(xyz, q) \\
&\leq \inf_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(z, q)\} \\
&= \max\{\inf_{i \in \Omega} \mu_i(x, q), \inf_{i \in \Omega} \mu_i(z, q)\} \\
&= \max\{\bigcap_{i \in \Omega} \mu_i(x, q), \bigcap_{i \in \Omega} \mu_i(z, q)\} \\
&= \max\{\mu(x, q), \mu(z, q)\}.
\end{align*}
\]

Therefore, $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$. Hence $\mu = \bigcap_{i \in \Omega} \mu_i$ is an anti Q-fuzzy bi-ideal of $X$, where $\Omega$ is any index set.

\[\square\]

References


