CONTRA SUPRA $G^#\alpha$-CONTINUOUS FUNCTION IN SUPRA TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce the concept of contra supra $g^#\alpha$-continuous functions and contra supra $g^#\alpha$-irresolute function. We obtain the basic properties and their relationship with other forms of contra supra continuous functions in supra topological spaces.

1. INTRODUCTION


The purpose of this paper is to introduce the concept of contra supra $g^#\alpha$-continuous functions and contra supra $g^#\alpha$-irresolute and study its basic properties. Also we defined almost contra supra $g^#\alpha$-continuous function, perfectly contra supra $g^#\alpha$-irresolute function and investigated their relationship to other functions in supra topological spaces.

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2. Preliminaries

**Definition 2.1.** [5] A subfamily $\mu$ of $X$ is said to be a supra topology on $X$, if

1. $X, \emptyset \in \mu$
2. If $A_i \in \mu$ for all $i \in J$ then $\bigcup A_i \in \mu$.

The pair $(X, \mu)$ is called supra topological space. The elements of $\mu$ are called supra open sets in $(X, \mu)$ and complement of a supra open set is called a supra closed set.

**Definition 2.2.** [5]

1. The supra closure of a set $A$ is denoted by $cl^\mu(A)$ and is defined as $cl^\mu(A) = \cap \{B : B$ is a supra closed set and $A \subseteq B\}$.
2. The supra interior of a set $A$ is denoted by $int^\mu(A)$ and defined as $int^\mu(A) = \cup \{B : B$ is a supra open set and $A \supseteq B\}$.

**Definition 2.3.** [5] Let $(X, \tau)$ be a topological space and $\mu$ be a supra topology on $X$. We call $\mu$ a supra topology associated with $\tau$ if $\tau \subset \mu$.

**Definition 2.4.** [1] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $\alpha$-open set if $A \subseteq int^\mu(cl^\mu(int^\mu(A)))$. The complement of supra $\alpha$-open set is supra $\alpha$-closed set.

**Definition 2.5.** [6] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $g^*$-closed set if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is supra $g$-open set of $X$. The complement of supra $g$-closed set is supra $g$-open set.

**Definition 2.6.** [6] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called a supra $g^#$-closed set if $cl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha g$-open set of $X$.

**Definition 2.7.** [4] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $g^# \alpha$-closed set if $\alpha cl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is supra $g$-open set of $X$. The complement of supra $g^# \alpha$-closed set is called supra $g^# \alpha$-open set.

**Definition 2.8.** Let $(X, \mu)$ and $(Y, \sigma)$ be two topological spaces and $\tau \subset \mu$. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

1. supra continuous if the inverse image of each open set of $Y$ is a supra open set in $X$ [5].
supra $\alpha$-continuous if the inverse image of each open set of $Y$ is a supra $\alpha$-open set in $X$ [1].

(3) supra $g$-continuous if the inverse image of each closed set of $Y$ is a supra $g$-closed set in $X$ [3].

(4) supra $g^\#$-continuous if the inverse image of each closed set of $Y$ is a supra $g^\#$-closed set in $X$ [6].

**Definition 2.9.** Let $(X, \mu)$ and $(Y, \sigma)$ be two topological spaces and $\tau \subset \mu$. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

(1) supra closed if the image of each closed set of $X$ is a supra closed set in $Y$ [5].

(2) supra $\alpha$-closed if the image of each closed set of $X$ is a supra $\alpha$-closed set in $Y$ [1].

(3) supra $g$-closed if the image of each closed set of $X$ is a supra $g$-closed set in $Y$ [3].

(4) supra $g^\#$-closed if the image of each closed set of $X$ is a supra $g^\#$-closed set in $Y$ [6].

**Definition 2.10.** Let $(X, \mu)$ and $(Y, \sigma)$ be two topological spaces and $\tau \subset \mu$. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

(1) supra irresolute if $f^{-1}(V)$ is supra closed in $X$ for every supra closed set $V$ of $Y$ [5].

(2) supra $\alpha$-irresolute if $f^{-1}(V)$ is supra $\alpha$-closed in $X$ for every supra $\alpha$-closed set $V$ of $Y$ [1].

(3) supra $g$-irresolute if $f^{-1}(V)$ is supra $g^\#$-closed in $X$ for every supra $g^\#$-closed set $V$ of $Y$ [3].

(4) supra $g^\#$-irresolute if $f^{-1}(V)$ is supra $g^\#$-closed in $X$ for every supra $g^\#$-closed set $V$ of $Y$ [6].

**Definition 2.11.** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

(1) Supra $g^\#\alpha$-continuous if $f^{-1}(V)$ is supra $g^\#\alpha$-closed in $(X, \tau)$ for every supra $g^\#\alpha$-closed set $V$ of $(Y, \sigma)$ [4].

(2) Supra $g^\#\alpha$-irresolute if $f^{-1}(V)$ is supra $g^\#\alpha$-closed in $(X, \tau)$ for every supra $g^\#\alpha$-closed set $V$ of $(Y, \sigma)$ [4].

(3) Contra continuous if $f^{-1}(V)$ is closed in $(X, \tau)$ for every open set $V$ of $(Y, \sigma)$ [2].
**Definition 2.12.** [2] A function $f : (X, \tau) \to (Y, \sigma)$ is called contra-continuous functions if $f^{-1}(V)$ is supra-closed in $(X, \tau)$ for every supra open set $V$ of $(Y, \sigma)$.

### 3. Contra Supra $g^\# \alpha$-Continuous Function

**Definition 3.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is called contra supra $g^\# \alpha$-continuous function if $f^{-1}(V)$ is supra $g^\# \alpha$-closed in $(X, \tau)$ for every supra open set $V$ of $(Y, \sigma)$.

**Theorem 3.1.** Every contra continuous function is contra supra $g^\# \alpha$-continuous.

**Proof.** Let $f : X \to Y$ be contra continuous. Let $V$ be any supra open in $Y$. Then the inverse image $f^{-1}(V)$ is supra closed in $X$. Since every supra closed is supra $g^\# \alpha$-closed, $f^{-1}(V)$ is supra $g^\# \alpha$-closed in $X$. Therefore $f$ is contra supra $g^\# \alpha$-continuous.

**Remark 3.1.** The converse of the above theorem is not true and it is shown by the following example.

**Example 1.** Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Here $f$ is contra supra $g^\# \alpha$-continuous function and not contra continuous. Since $V = \{a\}$ is supra open set in $(Y, \sigma)$, $f^{-1}(\{a\}) = \{a\}$ is not supra closed in $(X, \tau)$.

**Remark 3.2.** The composition of two contra supra $g^\# \alpha$-continuous mappings need not be contra supra $g^\# \alpha$-continuous. Let us prove the remark by the following example.

**Example 2.** Let $X = Y = \{a, b, c\}$. Let $\tau = \{X, \emptyset, \{b\}, \{a, b\}\}$, $\sigma = \{Y, \emptyset, \{a\}\}$ and $\nu = \{Z, \emptyset, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \nu)$. Define $f(a) = a$, $f(b) = b$, $f(c) = c$ and $g(a) = c$, $g(b) = b$, $g(c) = a$. Both $f$ and $g$ are contra supra $g^\# \alpha$-continuous. Define $g \circ f : (X, \tau) \to (Z, \nu)$. Hence $\{b\}$ is a supra open set of $(Z, \nu)$. Therefore $(g \circ f)^{-1} = (g \circ f)^{-1}(\{b\}) = g^{-1}(f^{-1}(\{b\})) = g^{-1}(\{b\}) = \{b\}$ is not a supra $g^\# \alpha$-closed set of $(X, \tau)$. Hence $(g \circ f)$ is not contra supra $g^\# \alpha$-continuous.

**Theorem 3.2.** If $f : (X, \tau) \to (Y, \sigma)$ is contra supra $g^\# \alpha$-continuous function and $g : (Y, \sigma) \to (Z, \nu)$ is supra continuous function then the composition $(g \circ f)$ is contra supra $g^\# \alpha$-continuous function.
Proof. Let $V$ be supra open set in $Z$. Since $g$ is supra continuous, then $g^{-1}(V)$ is supra open in $Y$. Since $f$ is contra supra $g^\#\alpha$-continuous function, then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra $g^\#\alpha$-closed in $X$. Therefore $(g \circ f)$ is contra supra $g^\#\alpha$-continuous function. \hfill $\Box$

**Theorem 3.3.** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is supra $g^\#\alpha$- irresolute function and $g : (Y, \sigma) \rightarrow (Z, \nu)$ is contra supra $g^\#\alpha$-continuous function then the composition $(g \circ f)$ is contra supra $g^\#\alpha$-continuous function.

Proof. Let $V$ be supra open set in $Z$. Since $g$ is contra supra $g^\#\alpha$-continuous function, then $g^{-1}(V)$ is supra $g^\#\alpha$-closed in $Y$. Since $f$ is supra $g^\#\alpha$- irresolute function, then $f^{-1}(g^{-1}(V))$ is supra $g^\#\alpha$-closed in $X$. Therefore $(g \circ f)$ is contra supra $g^\#\alpha$-continuous function. \hfill $\Box$

**Remark 3.3.** The concept of supra $g^\#\alpha$-continuity and contra supra $g^\#\alpha$-continuity are independent as shown in the following example.

**Example 3.** Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. $f : (X, \tau) \rightarrow (Y, \sigma)$ be the function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Here $f$ is contra supra $g^\#\alpha$-continuous but not supra $g^\#\alpha$-continuous function, since $V = \{b, c\}$ is supra closed set in $Y$ but $f^{-1}(\{b, c\}) = \{a, b\}$ is not supra $g^\#\alpha$-closed set in $X$.

**Theorem 3.4.** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra supra $g^\#\alpha$-continuous function and $X$ supra $g^\#\alpha Tc$ is -space, then $f$ is contra supra continuous.

Proof. Let $V$ be supra open set in $Y$. Since $f$ is contra supra $g^\#\alpha$-continuous function, then $f^{-1}(V)$ is supra $g^\#\alpha$-closed in $X$. Since $X$ is supra $g^\#\alpha Tc$-space, we have every supra $g^\#\alpha$-closed set is supra closed in $X$, then $f^{-1}(V)$ is supra closed in $X$. Therefore $f$ is contra supra continuous function. \hfill $\Box$

**Definition 3.2.** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called almost contra supra $g^\#\alpha$-continuous function if $f^{-1}(V)$ is supra $g^\#\alpha$-closed in $(X, \tau)$ for every supra regular open set $V$ in $(Y, \sigma)$.

**Theorem 3.5.** Every contra supra continuous function is almost contra supra $g^\#\alpha$-continuous function.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a contra supra continuous function. Let $V$ be a supra regular open set in $(Y, \sigma)$. We know that every supra regular open set is
supra open, then $V$ is supra open in $(Y, \sigma)$. Since $f$ is contra supra continuous function, $f^{-1}(V)$ is supra closed in $(X, \tau)$. We know that every supra closed set is supra $g^\# \alpha$-closed, which implies $f^{-1}(V)$ is supra $g^\# \alpha$-closed in $(X, \tau)$. Therefore $f$ is almost contra supra $g^\# \alpha$-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.** Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}$. $f : (X, \tau) \to (Y, \sigma)$ be the identity. Here $f$ is almost contra supra $g^\# \alpha$-continuous but it is not contra supra continuous function, since $V = \{a\}$ is supra open in $Y$ but $f^{-1}(\{a\}) = \{a\}$ is not supra closed set in $X$.

**Theorem 3.6.** Every contra supra $g^\# \alpha$-continuous function is almost contra supra $g^\# \alpha$-continuous function.

**Proof.** Let $f : (X, \tau) \to (Y, \sigma)$ be a contra supra $g^\# \alpha$-continuous function. Let $V$ be a supra regular open set in $(Y, \sigma)$. We know that every supra regular open set is supra open, then $V$ is supra open in $(Y, \sigma)$. Since $f$ is contra supra $g^\# \alpha$-continuous function, $f^{-1}(V)$ is supra $g^\# \alpha$-closed in $(X, \tau)$. Therefore $f$ is almost contra supra $g^\# \alpha$-continuous function. □

The converse of the above theorem need not be true. It is shown by the following example.

**Example 5.** Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}$. $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Here $f$ is almost contra supra $g^\# \alpha$-continuous, but it is not contra supra $g^\# \alpha$-continuous, since $V = \{a\}$ is open in $Y$ but $f^{-1}(\{a\}) = \{a\}$ is not supra $g^\# \alpha$ closed in $X$.

**Definition 3.3.** A Space $(X, \tau)$ is supra $g^\# \alpha$-locally indiscrete if every supra $g^\# \alpha$-open (supra $g^\# \alpha$-closed) set is supra closed (supra open) in $(X, \tau)$.

**Theorem 3.7.** If $f : (X, \tau) \to (Y, \sigma)$ is supra $g^\# \alpha$-continuous function and $X$ is supra $g^\# \alpha$-locally indiscrete, then $f$ is contra supra $g^\# \alpha$-continuous.

**Proof.** Let $V$ be supra open set in $Y$. Since $f$ is supra $g^\# \alpha$-continuous function, then $f^{-1}(V)$ is supra $g^\# \alpha$-open in $X$. Since $X$ is supra $g^\# \alpha$-locally indiscrete, then $f^{-1}(V)$ is supra closed set in $X$. We know that every supra closed set is supra $g^\# \alpha$-closed set. Therefore $f^{-1}(V)$ is supra $g^\# \alpha$-closed set in $X$. Hence $f$ is contra supra $g^\# \alpha$-continuous function. □
Theorem 3.8. Let \( f : (X, \tau) \to (Y, \sigma) \) be a surjective supra \( g^#\alpha \)-irresolute. \( g : (Y, \sigma) \to (Z, \nu) \) is a function such that \( (g \circ f) : (X, \tau) \to (Z, \nu) \) is contra supra \( g^#\alpha \)-continuous function, iff \( g \) is contra supra \( g^#\alpha \)-continuous.

Proof. Suppose \((g \circ f)\) is contra supra \(g^#\alpha\)-continuous. Let \( V \) be a supra closed set in \( Z \), then \((g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))\) is supra \(g^#\alpha\)-open in \((X, \tau)\). Since \( f \) is surjective and supra \(g^#\alpha\)-irresolute, then \( f((g \circ f)^{-1}) = f(f^{-1}(g^{-1}(V))) = g^{-1}(V) \) is supra \(g^#\alpha\)-open in \((Y, \sigma)\). Hence \( g \) is contra supra \(g^#\alpha\)-continuous function.

Conversely, suppose \( g \) is contra supra \(g^#\alpha\)-continuous. Let \( V \) be supra closed in \( Z \), then \(g^{-1}(V)\) is supra \(g^#\alpha\)-open in \( Y \). Since \( f \) is surjective and supra \(g^#\alpha\)-irresolute, then \( f^{-1}(g^{-1}(V)) \) is supra \(g^#\alpha\)-open in \( X \). Hence \((g \circ f)\) is contra supra \(g^#\alpha\)-continuous function. \( \square \)

Theorem 3.9. If \( f : (X, \tau) \to (Y, \sigma) \) is a supra \(g^#\alpha\)-continuous and \( g : (Y, \sigma) \to (Z, \nu) \) is contra supra \(g^#\alpha\)-continuous function and \((Y, \sigma)\) is supra \(g^#\alpha\)-\(T_c\)-space, then \((g \circ f) : (X, \tau) \to (Z, \nu)\) is contra supra \(g^#\alpha\)-continuous function.

Proof. Let \( V \) be any supra open set in \( Z \), then \(g^{-1}(V)\) is supra \(g^#\alpha\)-closed set in \( Y \). Since \( Y \) is supra \(g^#\alpha\)-\(T_c\)-space, \(g^{-1}(V)\) is supra closed set in \( Y \). Since \( f \) is supra \(g^#\alpha\)-continuous, \( f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)\) is supra \(g^#\alpha\)-closed set in \( X \). Hence \((g \circ f)\) is contra supra \(g^#\alpha\)-continuous. \( \square \)

4. Contra Supra \(g^#\alpha\)-Irresolute Function

Definition 4.1. A function \( f : (X, \tau) \to (Y, \sigma) \) is called contra supra \(g^#\alpha\)-irresolute function if \( f^{-1}(V) \) is supra \(g^#\alpha\)-closed in \((X, \tau)\) for every supra \(g^#\alpha\)-open set \( V \) in \((Y, \sigma)\).

Definition 4.2. A function \( f : (X, \tau) \to (Y, \sigma) \) is called perfectly contra supra \(g^#\alpha\)-irresolute function if \( f^{-1}(V) \) supra \(g^#\alpha\)-closed and supra \(g^#\alpha\)-open in \((X, \tau)\) for every supra \(g^#\alpha\)-open set \( V \) in \((Y, \sigma)\).

Theorem 4.1. Every contra supra \(g^#\alpha\)-irresolute function is contra supra \(g^#\alpha\)-continuous.

Proof. Let \( f : (X, \tau) \to (Y, \sigma) \) be a contra supra \(g^#\alpha\)-irresolute function. Let \( V \) be a supra open set in \((Y, \sigma)\). We know that every supra open set is supra
Let $f$ be a supra $g^\#\alpha$-irresolute function. Since $f$ is contra supra $g^\#\alpha$-irresolute function, $f^{-1}(V)$ is supra $g^\#\alpha$-closed in $(X,\tau)$. Therefore $f$ is contra supra $g^\#\alpha$-continuous function. \hfill \Box

The converse of the above theorem need not be true. It is shown by the following example.

**Example 6.** Let $X = Y = \{a,b,c\}$, $\tau = \{X,\emptyset,\{a\}\}$, $\sigma = \{Y,\emptyset,\{a\},\{b\},\{a, b\}\}$. A function $f : (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = c, f(b) = b, f(c) = a$. Here $f$ is contra supra $g^\#\alpha$-continuous but not contra supra $g^\#\alpha$-irresolute. Since $V = \{b, c\}$ is supra $g^\#\alpha$-open set in $(Y,\sigma)$ and $f^{-1}(\{b, c\}) = \{a, b\}$ is not in supra $g^\#\alpha$-closed set in $(X,\tau)$.

**Theorem 4.2.** If $f : (X,\tau) \to (Y,\sigma)$ is a supra $g^\#\alpha$-irresolute and $g : (Y,\sigma) \to (Z,\nu)$ is contra supra $g^\#\alpha$-irresolute function, then $(g \circ f) : (X,\tau) \to (Z,\nu)$ is contra supra $g^\#\alpha$-irresolute function.

**Proof.** Let $V$ be any supra $g^\#\alpha$-open set in $Z$. Since $g$ is contra supra $g^\#\alpha$-irresolute then $g^{-1}(V)$ is supra $g^\#\alpha$-closed set in $Y$. Since $f$ is supra $g^\#\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra $g^\#\alpha$-closed set in $X$. Hence $(g \circ f)$ is contra supra $g^\#\alpha$-irresolute function. \hfill \Box

**Theorem 4.3.** If $f : (X,\tau) \to (Y,\sigma)$ is a contra supra $g^\#\alpha$-irresolute and $g : (Y,\sigma) \to (Z,\nu)$ is supra $g^\#\alpha$-irresolute function, then $(g \circ f) : (X,\tau) \to (Z,\nu)$ is contra supra $g^\#\alpha$-irresolute function.

**Proof.** Let $V$ be any supra $g^\#\alpha$-open set in $Z$. Since $g$ is supra $g^\#\alpha$-irresolute then $g^{-1}(V)$ is supra $g^\#\alpha$-open set in $Y$. Since $f$ is contra supra $g^\#\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra $g^\#\alpha$-closed set in $X$. Hence $(g \circ f)$ is contra supra $g^\#\alpha$-irresolute function. \hfill \Box

**Theorem 4.4.** Every perfectly contra supra $g^\#\alpha$-irresolute is contra supra $g^\#\alpha$-irresolute function.

**Proof.** Let $f : (X,\tau) \to (Y,\sigma)$ be a perfectly contra supra $g^\#\alpha$-irresolute function. Let $V$ be a supra $g^\#\alpha$-open set in $(Y,\sigma)$. Since $f$ is perfectly contra supra $g^\#\alpha$-irresolute function, $f^{-1}(V)$ is supra $g^\#\alpha$-closed and supra $g^\#\alpha$-open in $(X,\tau)$. Therefore $f$ is contra supra $g^\#\alpha$-irresolute function. \hfill \Box
The converse of the above theorem need not be true. It is shown by the following example.

**Example 7.** Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b\}\} \), \( \sigma = \{Y, \emptyset, \{b\}, \{a, b\}\} \) and let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a function defined by \( f(a) = a \), \( f(b) = c \), \( f(c) = b \). Here \( f \) is contra supra \( g^\#\alpha \)-irresolute function but not perfectly contra supra \( g^\#\alpha \)-irresolute function. Since \( V = \{a, c\} \) is supra \( g^\#\alpha \)-open set in \((Y, \sigma)\), \( f^{-1}(\{a, c\}) = \{a, b\} \) is not supra \( g^\#\alpha \)-closed and supra \( g^\#\alpha \)-open set in \((X, \tau)\).

**Theorem 4.5.** Every perfectly contra supra \( g^\#\alpha \)-irresolute is contra supra \( g^\#\alpha \)-irresolute function.

**Proof.** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a perfectly contra supra \( g^\#\alpha \)-irresolute function. Let \( V \) be a supra \( g^\#\alpha \)-open set in \((Y, \sigma)\). Since \( f \) is perfectly contra supra \( g^\#\alpha \)-irresolute function, \( f^{-1}(V) \) is supra \( g^\#\alpha \)-closed and supra \( g^\#\alpha \)-open in \((X, \tau)\). Therefore \( f \) is supra \( g^\#\alpha \)-irresolute function. \( \square \)

The converse of the above theorem need not be true. It is shown by the following example.

**Example 8.** Let \( X = Y = \{a, b, c\} \) and \( \tau = \{X, \emptyset, \{a\}\} \), \( \sigma = \{Y, \emptyset, \{a\}, \{a, b\}\} \), \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a identity function. Here \( f \) is supra \( g^\#\alpha \)-irresolute function but not perfectly contra supra \( g^\#\alpha \)-irresolute function. Since \( V = \{a, c\} \) is supra \( g^\#\alpha \)-open set in \((Y, \sigma)\), \( f^{-1}(\{a, c\}) = \{a, b\} \) is not supra \( g^\#\alpha \)-closed and supra \( g^\#\alpha \)-open set in \((X, \tau)\).

**References**


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