NEUTROSOPHIC WEAKLY $G^*$-CLOSED SETS

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ABSTRACT. In this paper are presented and explored new sort of Neutrosophic closed set which is called Neutrosophic feebly $g^*$-closed sets in NTSs and furthermore talked about properties and portrayal.

1. INTRODUCTION

A. Salama presented NTSs in [11, 12] by utilizing Smarandache’s NSs, [5, 6]. Neutrosophic $g$ closed set presented by R. Dhavasheelan et al. in [3, 4], what’s more, Neutrosophic $g^*$-closed sets introduced by A. Atkinswesley et al. in [2]. Point of this current paper is, to present and research about new sort of Neutrosophic closed set is called Neutrosophic weakly $g^*$-closed sets in Neutrosophic topological spaces and furthermore examined about properties and portrayal.

2. PRELIMINARIES

In this part, we review required results of Neutrosophic.

Definition 2.1. [4] A Neutrosophic set $W_1^*$ is in the form

$$W_1^* = \{ < r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) > : r \in Nu_X \},$$

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where $\mu_{W_1^*}(w)$ denotes membership function, $\sigma_{W_1^*}(w)$ denotes indeterminacy and $\gamma_{W_1^*}(w)$ denotes non-membership function.

**Definition 2.2.** [4] Neutrosophic set is the set

$$W_1^* = \{r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) : r \in Nu_X^*\},$$

on $Nu_X^*$ and $\forall w \in Nu_X^*$. Then complement of $W_1^*$ is

$$W_1^{*C} = \{r, \gamma_{W_1^*}(w), 1 - \sigma_{W_1^*}(w), \mu_{W_1^*}(w) : r \in Nu_X^*\}.$$

**Definition 2.3.** [4] Let $W_1^*$ and $W_2^*$ are two NSs,

$$\forall w \in Nu_X^* W_1^* = \{r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) : r \in Nu_X^*\}$$

$$W_2^* = \{r, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) : r \in Nu_X^*\}.$$

Then $W_1^* \subseteq W_2^* \iff \mu_{W_1^*}(w) \leq \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \leq \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \geq \gamma_{W_2^*}(w)$.

**Definition 2.4.** [4] Let $W_1^*$ and $W_2^*$ be two NSs are

$$W_1^* = \{r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) : r \in Nu_X^*\},$$

$$W_2^* = \{r, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) : r \in Nu_X^*\}.$$

Then $W_1^* \cap W_2^* = \{r, \mu_{W_1^*}(w) \cap \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cap \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cup \gamma_{W_2^*}(w) : r \in Nu_X^*\}$

$$W_1^* \cup W_2^* = \{r, \mu_{W_1^*}(w) \cup \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cup \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cap \gamma_{W_2^*}(w) : r \in Nu_X^*\}.$$

**Definition 2.5.** [10] Let $Nu_X^*$ be non-empty set and $NS_\tau$ be the collection of Neutrosophic subsets of $Nu_X^*$ satisfying the accompanying properties:

1. $0_{Nu}, 1_{Nu} \in NS_\tau$
2. $Nu_{T_1} \cap Nu_{T_2} \in NS_\tau$ for any $Nu_{T_1}, Nu_{T_2} \in Nu_\tau$
3. $\cup Nu_{T_i} \in NS_\tau$ for every $Nu_{T_i} : i \in j \subseteq Nu_\tau$.

Then the space $(Nu_X^*, NS_\tau)$, is called a $NTS(NS - T - S)$. The component of $NS_\tau$ are called $NS-OS$ (Neutrosophic open set) and its complement is $NS-CS$(Neutrosophic closed set)

**Example 1.** Let $Nu_X^* = \{w\}$ and $\forall w \in Nu_X^*$

$$W_1^* = \langle w, \frac{6}{10}, \frac{6}{10}, \frac{5}{10} \rangle, W_2^* = \langle w, \frac{5}{10}, \frac{7}{10}, \frac{9}{10} \rangle$$

$$W_3^* = \langle w, \frac{6}{10}, \frac{7}{10}, \frac{5}{10} \rangle, W_4^* = \langle w, \frac{5}{10}, \frac{6}{10}, \frac{9}{10} \rangle$$

Then collection $NS_\tau = \{0_{Nu}, W_1^*, W_2^*, W_3^*, W_4^*, 1_{Nu}\}$ is called a $NS-T-S$ on $Nu_X^*$.

**Definition 2.6.** [10] Let $(Nu_X^*, Nu_\tau)$ be $NTS$, Then Neutrosophic closure of $W_1^*$ is $Nu-cl(W_1^*) = \cap \{K : K$ is a Neutrosophic closed set in $Nu_X^*$ and $W_1^* \subseteq K\}$. 
Neutrosophic interior of $W^*_1$ is:

$$\text{Nu-int}(W^*_1) = \bigcup \{ G^*_1 : G^*_1 \text{ is a Neutrosophic open set in } Nu^*_X \text{ and } G^*_1 \subseteq W^*_1 \}.$$  

**Definition 2.7.** Let $(Nu^*_X, Nu_{-\tau})$ be a NTS. Then $W^*_1$ is called

1. **Neutrosophic regular Closed set (Neu-RCS)** if $W^*_1 = \text{Neu-Cl(Neu-Int}(W^*_1))$, [1];
2. **Neutrosophic $\alpha$-Closed set (Neu-$\alpha$ CS)** if $\text{Neu-Cl(Neu-Int(Neu-Cl}(W^*_1))) \subseteq W^*_1$, [1];
3. **Neutrosophic semi Closed set (Neu-SCS)** if $\text{Neu-Int(Neu-Cl}(W^*_1)) \subseteq W^*_1$, [7];
4. **Neutrosophic pre Closed set (Neu-PCS)** if $\text{Neu-Cl(Neu-Int}(W^*_1)) \subseteq W^*_1$, [15];

**Definition 2.8.** Let $(Nu^*_X, Nu_{-\tau})$ be a NTS. Then $W^*_1$ is called:

1. **Neutrosophic (regular open) set Neu-ROS** if $W^*_1 = \text{Neu-Int(Neu-Cl}(W^*_1))$, [1];
2. **Neutrosophic ($\alpha$-open) set (Neu-$\alpha$ OS)** if $W^*_1 \subseteq \text{Neu-Int(Neu-Cl}(\text{Neu-Int}(W^*_1)))$, [1];
3. **Neutrosophic (semi open) set (Neu-SOS)** if $W^*_1 \subseteq \text{Neu-Cl(Neu-Int}(W^*_1))$, [7];
4. **Neutrosophic (pre open) set (Neu-POS)** if $W^*_1 \subseteq \text{Neu-Int(Neu-Cl}(W^*_1))$, [15].

**Definition 2.9.** A Neutrosophic set $W^*_1$ of a NTS $(Nu^*_X, Nu_{-\tau})$ is called

1. **Neutrosophic (g-closed)** if $\text{Nu-cl}(W^*_1) \subseteq G^*_1$ whenever $W^*_1 \subseteq G^*_1$ and $G^*_1$ is Neutrosophic open, [3];
2. **Neutrosophic (sg-closed)** if $\text{Nu-(S)Cl}(W^*_1) \subseteq G^*_1$ whenever $W^*_1 \subseteq G^*_1$ and $G^*_1$ is Neutrosophic semi open, [14];
3. **Neutrosophic ($g^*$-closed)** if $\text{Nu-cl}(W^*_1) \subseteq G^*_1$ whenever $W^*_1 \subseteq G^*_1$ and $G^*_1$ is Neutrosophic g-open, [2];
4. **Neutrosophic ($\alpha$g-closed)** if $\text{Nu-(}$-$\alpha$)$\text{cl}$($W^*_1) \subseteq G$ whenever $W^*_1 \subseteq G^*_1$ and $G^*_1$ is Neutrosophic g-open, [8];
5. **Neutrosophic (go-$\alpha$-closed)** if $\text{Nu-(}$-$\alpha$)$\text{cl}(W^*_1) \subseteq G^*_1$ whenever $W^*_1 \subseteq G^*_1$ and $G^*_1$ is Neutrosophic $\alpha$-open, [4];
6. **Neutrosophic (w-closed)** if $\text{Nu-cl}(W^*_1) \subseteq G$ whenever $W^*_1 \subseteq G^*_1$ and $G^*_1$ is Neutrosophic semi open, [13];
(7) Neutrosophic (gP-closed) if \( \text{Nu-(P)Cl}(W_1^*) \subseteq G_1^* \) whenever \( W_1^* \subseteq G_1^* \) and \( G_1^* \) is Neutrosophic open, [9];

(8) Neutrosophic (gs-closed) if \( \text{Nu-(S)Cl}(W_1^*) \subseteq G_1^* \) whenever \( W_1^* \subseteq G_1^* \) and \( G_1^* \) is Neutrosophic open, [14].

The complements of the above mentioned closed set are their respective open sets.

**Definition 2.10.** [3] If \( W_1^* \) is a Neutrosophic set in NTS\( (\text{Nu}^*X, \text{Nu} \tau) \) then

1. \( \text{Nu-(S)Cl}(W_1^*) = \bigcap \{ F_1^*: W_1^* \subseteq F_1^*, F_1^* \text{ is Nu(S)CS} \} \).
2. \( \text{Nu-(P)Cl}(W_1^*) = \bigcap \{ F_1^*: W_1^* \subseteq F_1^*, F_1^* \text{ is Nu(P)CS} \} \).
3. \( \text{Nu-}(\alpha)\text{cl}(W_1^*) = \bigcap \{ F_1^*: W_1^* \subseteq F_1^*, F_1^* \text{ is Nu}(\alpha)\text{CS} \} \).

**Remark 2.1.**

1. Every NuCS is Nu\((g)\)CS.
2. Every Nu\((\alpha)\)CS is Nu\((\alpha g)\)CS.
3. Every Nu\((g)\)CS is Nu\((g\alpha)\)CS.
4. Every Nu\((\alpha g)\)CS is Nu\((g\alpha)\)CS.
5. Every Nu\((w)\)CS is Nu\((g)\)CS.
6. Every Nu\((w)\)CS is Nu\((sg)\)CS.
7. Every Nu\((sg)\)CS is Nu\((gs)\)CS.

**Lemma 2.1.** [7] Let \( W_1^* \) and \( W_2^* \) be any two NSs of a NTS \( (\text{Nu}^*_X, \text{Nu} \tau\tau) \). Then:

- (a) \( W_1^* \) is a NuCS in \( \text{Nu}^*_X \) \( \Leftrightarrow \) \( \text{Nu-cl}(W_1^*) = W_1^* \).
- (b) \( W_1^* \) is a NuOS in \( \text{Nu}^*_X \) \( \Leftrightarrow \) \( \text{Nu-int}(W_1^*) = W_1^* \).
- (c) \( \text{Nu-cl}(W_1^*) = (\text{Nu-int}(W_1^*))^C \).
- (d) \( \text{Nu-int}(W_1^*) = (\text{Nu-cl}(W_1^*))^C \).
- (e) \( W_1^* \subseteq W_2^* \Rightarrow \text{Nu-int}(W_1^*) \subseteq \text{Nu-int}(W_2^*) \).
- (f) \( W_1^* \subseteq W_2^* \Rightarrow \text{Nu-cl}(W_1^*) \subseteq \text{Nu-cl}(W_2^*) \).
- (g) \( \text{Nu-cl}(W_1^* \cup W_2^*) = \text{Nu-cl}(W_1^*) \cup \text{Nu-cl}(W_2^*) \).
- (h) \( \text{Nu-int}(W_1^* \cap W_2^*) = \text{Nu-int}(W_1^*) \cap \text{Nu-int}(W_2^*) \).

3. **Neutrosophic weakly \( g^*-\)closed**

**Definition 3.1.** A Neutrosophic set \( W_1^* \) of a NTS \( (\text{Nu}^*_X, \text{Nu} \tau\tau) \) is called Nu\((w\)\(g^*)\)CS Neutrosophic weakly \( g^*-\)closed if \( \text{Nu-cl}(\text{Nu-int}(W_1^*)) \subseteq G_1^* \) whenever \( W_1^* \subseteq G_1^* \) and \( G_1^* \) is Neutrosophic \( g\)-open in \( \text{Nu}^*_X \).
Theorem 3.1. Every $\text{Nu}(w)CS$ set is $\text{Nu}(w^g)CS$.

Proof. Let $W^*_1$ is $\text{Nu}(w)CS$. Let $W^*_1 \subseteq H^*_1$ and $H^*_1$ $\text{Nu}(S)OS$ in $\text{Nu}^*_X$. Since every $\text{Nu}(S)OS$ is $\text{Nu}(g)OS$ $H^*_1$ is $\text{Nu}(g)OS$. using definition $\text{Nu}(w)CS$ Nu-cl($W^*_1$) $\subseteq H^*_1$. But Nu-cl(Nu-int($W^*_1$)) $\subseteq$ Nu-cl($W^*_1$) $\subseteq H^*_1$. We have Nu-cl(Nu-int($W^*_1$)) $\subseteq H^*_1$ whenever $W^*_1 \subseteq H^*_1$ and $H^*_1$ is $\text{Nu}(g)OS$ in $\text{Nu}^*_X$. Therefore $W^*_1$ is $\text{Nu}(w^g)CS$. □

Remark 3.1. Every $\text{Nu}(w^g)CS$ is not $\text{Nu}(w)CS$ set.

Example 2. Let $\text{Nu}^*_X = \{a, b\}$ and $\text{Nu}_r = \{0_{\text{Nu}}, W^*_1, 1_{\text{Nu}}\}$ is Neutrosophic topology on $\text{Nu}^*_X$, where $W^*_1 = \langle w, (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$. Then $W^*_2 = \langle w, (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{1}{10}) \rangle$ is $\text{Nu}(w^g)CS$ but not $\text{Nu}(w)CS$.

Theorem 3.2. Every $\text{Nu}(g^*)CS$ is $\text{Nu}(w^g)CS$.

Proof. Let $W^*_1$ is $\text{Nu}(g^*)CS$. Let $W^*_1 \subseteq H^*_1$ and $H^*_1$ is $\text{Nu}(g)OS$ in $\text{Nu}^*_X$. using definition $\text{Nu}(g^*)CS$ Nu-cl($W^*_1$) $\subseteq H^*_1$. But Nu-cl(Nu-int($W^*_1$)) $\subseteq$ Nu-cl($W^*_1$) $\subseteq H^*_1$. We have Nu-cl(Nu-int($W^*_1$)) $\subseteq H^*_1$ whenever $W^*_1 \subseteq H^*_1$ and $H^*_1$ is $\text{Nu}(g)OS$ in $\text{Nu}^*_X$. Therefore $W^*_1$ is $\text{Nu}(w^g)CS$. □

Remark 3.2. Every $\text{Nu}(w^g)CS$ is not $\text{Nu}(g^*)CS$.

Example 3. Let $\text{Nu}^*_X = \{w_1, w_2, w_3, w_4\}$ and NSs $W^*_1$, $W^*_2$, $W^*_3$, $W^*_4$ defined as $W^*_1 = \langle w, (\frac{9}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{1}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle$ $W^*_2 = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle$ $W^*_3 = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle$ $W^*_4 = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle$ $\text{Nu}_r = \{0_{\text{Nu}}, W^*_1, W^*_2, W^*_3, W^*_4, 1_{\text{Nu}}\}$ be a NT on $\text{Nu}^*_X$. Then $W^*_1 = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle$ is $\text{Nu}(w^g)CS$ but it is not $\text{Nu}(g^*)CS$.

Theorem 3.3. Every $\text{Nu}(g)CS$ is $\text{Nu}(w^g)CS$.

Proof. Let $W^*_1$ is $\text{Nu}(g)CS$. Let $W^*_1 \subseteq H^*_1$ and $H^*_1$ $\text{Nu}OS$ in $\text{Nu}^*_X$. Since every $\text{Nu}OS$ is $\text{Nu}(g)OS$ $H^*_1$ is $\text{Nu}(g)OS$. Presently using definition $\text{Nu}(g)CS$s Nu-cl($W^*_1$) $\subseteq H^*_1$. But Nu-cl(Nu-int($W^*_1$)) $\subseteq$ Nu-cl($W^*_1$) $\subseteq H^*_1$. We have Nu-cl(Nu-int($W^*_1$)) $\subseteq H^*_1$ whenever $W^*_1 \subseteq H^*_1$ and $H^*_1$ is $\text{Nu}(g)OS$ in $\text{Nu}^*_X$. Therefore $W^*_1$ is $\text{Nu}(w^g)CS$ set. □
Remark 3.3. Every \( \text{Nu(wg')CS} \) is not \( \text{Nu(g)CS} \).

Example 4. Let \( \text{Nu}_X^* = \{w_1, w_2, w_3\} \) and \( \text{NSsW}_1^*, W_2^*, W_3^* \), defined as
\[
W_1^* = \langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle.
\]
\[
W_2^* = \langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle.
\]
\[
W_3^* = \langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle.
\]

Let \( \text{Nu}_r = \{0_{Nu}, W_1^*, W_2^*, W_3^*, 1_{Nu}\} \) be a Neutrosophic topology on \( \text{Nu}_X^* \). Then the Neutrosophic set
\[
W_4^* = \langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle
\]
is \( \text{Nu(wg')CS} \) but it is not \( \text{Nu(g)CS} \).

Theorem 3.4. Every \( \text{Nu(α g)CS} \) is \( \text{Nu(wg')CS} \).

Proof. Let \( \text{W}_1^* \) be \( \text{Nu(α g)CS} \). Let \( \text{W}_1^* \subseteq H_1^* \) and \( H_1^* \) NuOS in \( \text{Nu}_X^* \). Since every NuOS is \( \text{Nu(g)OS} \), \( H_1^* \) is \( \text{Nu(g)OS} \). Presently using definition \( \text{Nu(α g)CS} \), \( \text{Nu(α cl(W}_1^*) \subseteq H_1^* \). But \( \text{Nu(α cl(W}_1^*) \subseteq \text{Nu-cl}(W_1^*) \) therefore \( \text{Nu-cl}(W_1^*) \subseteq W_1^* \).

Now \( \text{Nu-cl(Nu-int(W}_1^*)) \subseteq \text{Nu-cl}(W_1^*) \subseteq H_1^* \). We have \( \text{Nu-cl(Nu-int(W}_1^*)) \subseteq H_1^* \) whenever \( W_1^* \subseteq H_1^* \) and \( H_1^* \) is \( \text{Nu(g)OS} \) in \( \text{Nu}_X^* \). Therefore \( W_1^* \) is \( \text{Nu(wg')CS} \).

Remark 3.4. Every \( \text{Nu(wg')CS} \) is not \( \text{Nu(α g)CS} \).

Example 5. Let \( \text{Nu}_X^* = \{w_1, w_2\} \) and \( \text{NSsW}_1^*, W_2^* \), defined as
\[
W_1^* = \langle w, \left(\frac{7}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle.
\]
\[
W_2^* = \langle w, \left(\frac{5}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle.
\]

Let \( \text{Nu}_r = \{0_{Nu}, W_1^*, W_2^*, 1_{Nu}\} \) be a Neutrosophic topology on \( \text{Nu}_X^* \). Then Neutrosophic set
\[
W_3^* = \langle w, \left(\frac{7}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle
\]
is \( \text{Nu(wg')CS} \) but it is not \( \text{Nu(α g)CS} \).

Theorem 3.5. Every \( \text{Nu(g α)CS} \) is \( \text{Nu(wg')CS} \).

Proof. From theorem 3.4 we get every \( \text{Nu(g α)CS} \) is \( \text{Nu(α g)CS} \).

Theorem 3.6. Every \( \text{Nu(gP)CS} \) is \( \text{Nu(wg')CS} \).

Proof. Let \( W_1^* \) be \( \text{Nu(gP)CS} \). Let \( W_1^* \subseteq H_1^* \) and \( H_1^* \) NuOS in \( \text{Nu}_X^* \). Since every NuOS is \( \text{Nu(g)OS} \), \( H_1^* \) is \( \text{Nu(g)OS} \). Presently using definition \( \text{Nu(gP)CS} \), \( \text{Nu-Pcl}(W_1^*) \subseteq H_1^* \). But \( \text{Nu-Pcl}(W_1^*) \subseteq \text{Nu-cl}(W_1^*) \) therefore \( \text{Nu-cl}(W_1^*) \subseteq W_1^* \).

Now \( \text{Nu-cl(Nu-int(W}_1^*)) \subseteq \text{Nu-cl}(W_1^*) \subseteq H_1^* \). We have \( \text{Nu-cl(Nu-int(W}_1^*)) \subseteq H_1^* \).
whenever \( W_1^* \subseteq H_1^* \) and \( H_1^* \) is \( Nu(g)OS \) in \( Nu_X^* \). Therefore \( W_1^* \) is \( Nu(wg^*)CS \).

**Remark 3.5.** Every \( Nu(wg^*)CS \) is not \( Nu(gP)CS \).

**Example 6.** Let \( Nu_X^* = \{ w_1, w_2 \} \) and \( Nu_{\tau} = \{ 0_{Nu}, W_1^*, 1_{Nu} \} \) be a NTon \( Nu_X^* \), where

\[
W_1^* = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{3}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle.
\]

\[
W_2^* = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{1}{10}) \rangle.
\]

Then \( W_2^* \) is \( Nu(wg^*)CS \) but it is not \( Nu(gP)CS \).

**Corollary 3.1.**

1. Every \( NuCS \) is \( Nu(wg^*)CS \).
2. Every \( Nu(\alpha)CS \) is \( Nu(wg^*)CS \).
3. Every \( Nu(P)CS \) is \( Nu(wg^*)CS \).
4. Every \( Nu(R)CS \) is \( Nu(wg^*)CS \).

**Proof.** Obvious.

**Remark 3.6.** The intersection of two \( Nu(wg^*)CS \) is a NTS \( (Nu_X^*, Nu_{\tau}) \) may not be \( Nu(wg^*)CS \).

**Example 7.** Let \( Nu_X^* = \{ w_1, w_2, w_3, w_4 \} \) and NSs \( W_1^*, W_2^*, W_3^*, W_4^* \) defined as

\[
W_1^* = \langle w, (\frac{9}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle.
\]

\[
W_2^* = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle.
\]

\[
W_3^* = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle.
\]

\[
W_4^* = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{9}{10}, \frac{5}{10}, \frac{1}{10}) \rangle.
\]

\( \tau_{Nu} = \{ 0_{Nu}, W_1^*, W_2^*, W_3^*, W_4^* \} \) is Neutrosophic topology on \( Nu_X^* \). Then

\[
W_1^* = \langle w, (\frac{9}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}) \rangle
\]

\( W_2^* = \langle w, (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{10}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}) \rangle \) are \( Nu(wg^*)CS \) in \((Nu_X^*, Nu_{\tau})\) but \( W_1^* \cap W_2^* \) is not \( Nu(wg^*)CS \).

**Theorem 3.7.** Let \( W_1^* \) is \( Nu(wg^*)CS \) is a NTS \( (Nu_X^*, Nu_{\tau}) \) and \( W_1^* \subseteq W_2^* \subseteq Nu-cl(Nu-int(W_1^*)) \). Then \( W_2^* \) is \( Nu(wg^*)CS \) in \( Nu_X^* \).

**Proof.** Let \( G_1^* \) is \( Nu(g)OS \) in \( Nu_X^* \) such that \( W_2^* \subseteq G_1^* \). Then \( W_1^* \subseteq G_1^* \) and since \( W_1^* \) is \( Nu(wg^*)CS \), \( Nu-cl(Nu-int(W_1^*)) \) \( G_1^* \). Now \( W_2^* \subseteq Nu-cl(Nu-int(W_1^*)) \) \( Nu-cl(Nu-int(W_2^*)) \subseteq Nu-cl(Nu-int(Nu-cl(Nu-int(W_2^*)))) = Nu-cl(Nu-int(W_1^*)), Nu-cl(Nu-int(W_2^*)) \subseteq Nu-cl(Nu-int(W_1^*)) \subseteq G_1^* \). Consequently \( W_2^* \) is \( Nu(wg^*)CS \).
Definition 3.2. A Neutrosophic set $W_1^*$ of a NTS $(N_X^*, N_{u_r})$ is called Nu($g^*$)OS if $W_1^{*c}$ is Nu($w^g$)CS.

Remark 3.7. Every Nu($w$)OS is Nu($w^g$)OS.

Example 8. Let $N_X^* = \{w_1, w_2\}$ and $N_{u_r} = \{0_{N_X}, W_1^*, 1_{N_X}\}$ is Neutrosophic topology on $N_X^*$, where $W_1^* = \langle w, (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$.

Then $W_2^* = \langle w, (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{1}{10}, \frac{5}{10}, \frac{8}{10}) \rangle$ is Nu($w^g$)OS in $(N_X^*, N_{u_r})$ but it is not Nu($w$)OS in $(N_X^*, N_{u_r})$.

Theorem 3.8. A Neutrosophic set $W_1^*$ of a NTS $(N_X^*, N_{u_r})$ Nu($w^g$)OS if $F_1^* \subseteq Nu-cl(Nu-int(W_1^*))$ whenever $F_1^*$ is Nu($g$)CS and $F_1^* \subseteq W_1^*$.

Proof. Follows from Definition 3.1 and Lemma 2.1. □

Theorem 3.9. Let $W_1^*$ is Nu($w^g$)OS of a NTS $(N_X^*, N_{u_r})$ and Nu-cl(Nu-int($W_1^*$)) $\subseteq W_2^* \subseteq W_1^*$. Then $W_2^*$ is Nu($w^g$)OS.

Proof. Suppose $W_1^*$ is a Nu($w^g$)OS in $N_X^*$ and Nu-cl(Nu-int($W_1^*$)) $\subseteq W_2^* \subseteq W_1^*$ $\Rightarrow W_1^{*c} \subseteq W_2^{*c} \subseteq (Nu-cl(Nu-int(W_1^*)))C \subseteq W_1^{*c} \subseteq W_2^{*c} \subseteq Nu-cl(Nu-int(W_1^{*c}))$ by Lemma 2.18 and $W_1^{*c}$ is Nu($w^g$)CS it follows from theorem that $W_2^{*c}$ is Nu($w^g$)CS. Hence $W_2^*$ is Nu($w^g$)OS. □

4. Conclusion

The hypothesis of g-closed sets assumes a significant job when all is said in done topology. Since its initiation numerous powerless and solid types of g-closed sets have been presented by and large topology just as fuzzy topology and Neutrosophic topology. The current paper researched another type of Nu($g$)CSs called Nu($w^g$)CS which has been contrasted and the classes of Neutrosophic closed sets, Nu($P$)CS, Nu($\alpha$)CS, Nu($w$)CS, Nu($gP$)CS, Nu($\alpha$ g)CS, Nu($g\alpha$)CS, Nu($g^*$)CS. A few properties and utilization of Nu($w^g$)CS are examined. Numerous models are given to legitimize the outcome.

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