ANALYSIS AND EXTRACTION OF COMMUTING PATTERNS IN RAILWAY NETWORKS USING VARIOUS MATRIX DECOMPOSITION TECHNIQUES

NALLI VINAYA KUMARI\(^1\) AND ANIL KUMAR

ABSTRACT. In this paper, we will review how to extract commuters' patterns using the matrix decomposition method. Matrix decomposition is a vast data processing technique, where we will use multiple inputs as matrix data to find a relative pattern. This pattern often helps us to determine how often commuters travel between places and also we can find human traffic during a particular time like business hours. These findings will help us figure out solutions for managing crowds to ensure access to the passenger. We can use this kind of data mined pattern extraction to help travelers to maintain social distancing. Using this method, we can achieve social distancing, Safety, Traveler convenience, etc. Also, this pattern will help us to segregate people based on their preferred modes (i.e.) using counters for getting tickets, online booking, unplanned travelers, planned travel, Cash payment users, and Digital money users. Separating the groups into different kinds will help each one to use their preferred modes with hassle-free. In this paper, we are going to review various methodologies to achieve the above result. Different methods will produce independent results with slight relations with each other. Using all of the results, we will decide which method produces accurate output.

1. INTRODUCTION

A matrix decomposition is a method of diminishing a framework into its constituent parts. It is a methodology that can disentangle progressively complex

\(^1\)corresponding author

2010 Mathematics Subject Classification. 78M32, 92B20.

Key words and phrases. Matrix factorization and decomposition, Data Mining, Python tools, Commuters.
framework tasks that can be performed on the decayed grid instead of on the first network. A typical similarity for network decay is the figuring of numbers, for example, calculating 10 into $2 \times 5$. Hence, framework decomposition is additionally called grid factorization. Like calculating genuine qualities, there are numerous approaches to deteriorate a grid, and henceforth there is a scope of various lattice decay methods. As defined is abstract, we will use the following methodology to find the matching results to extract the patterns in railway networks.

- Joint and Individual variation Explained (JIVE).
- Lu Decomposition.
- Cholesky Decomposition.
- Singular Value Decomposition.

We will discuss the above techniques in the next session briefly. Each method has its methodology to drive the results. As per the review, JIVE produces the most accurate results compared to other methods.

**Lu Decomposition.** This process of framing two triangles has different applications, such as the arrangement of a set of terms, which in itself is a critical component of many applications. And for example, The discovery of current at the circuit and the organization of discrete dynamic problems.

**Cholesky decomposition.** Cholesky decomposition and other decomposition strategies are significant, as it is a rare occurrence achievable to explicitly perform framework calculations. Cholesky decomposition, otherwise called Cholesky factorization, is a strategy for breaking down a positive-clear grid.

**JIVE.** Joint and Individual Variation Explained (JIVE), a general variety decomposition for a structured analysis of these datasets, has been clarified. JIVE is the easiest way to test the result. This exploratory approach breaks down a dataset into three terms: a low-position estimate that calculates the collection structure between information types. These low-position approximations capture each data type’s particular structure and recurrent concussions. Examining the individual structure offers an approach for separating conceivably useful data, but no other details. Representing a unique structure likewise takes into consideration progressively exact estimation of what is essential between information types.
The Singular Value Decomposition (SVD) is a ground-breaking computational device. Current calculations for getting such decay of general frameworks have profoundly affected various applications in science and building disciplines. The SVD is usually utilized to arrange unconstrained direct least squares issues, lattice rank estimation, and accepted relationship investigation. In computational science, it is usually applied in areas such as data recovery, seismic reflection tomography, and constant sign handling.

1.1. LU Decomposition. \( A=LU \) where \( A \) is a 2 \times 2 square matrix

\[
A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & \ \\ 0 & u_{22} \end{bmatrix} = LU,
\]

where \( A \) is a 2 \times 2 square matrix

\[
A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = LU,
\]

when \( A \) is a 3 \times 3 square matrix.

LU network decomposition refers to the factorization of two triangular laths, one large triangle structure, and one lower triangular grid with the purpose of offering shown in figure 1. The first grid as a result of these two grids. For instance, the arrangement of a conditional arrangement, which itself forms the fundamental element of various applications, discover current in a circuit and organize discrete, dynamic frames; locate backward of a network and find the decisive factor of a matrix. This technology to build a structure by two triangular grids has specific applications, for example, In essence, the LU strategy for decomposition is convenient to display the problem that can be illuminated in the grid structure at any point. Moving to the grid layout and dealing with triangular grids makes counting simple throughout the time the system is spent.

A square grid \( A \) may be disintegration into two square networks \( L \) and \( U \) to the degree that \( A = LU \), with \( U \) being formed by the Gauss disposal method on \( A_n \), is the upper triangular grid and \( L \) being a lower triangular frame with inclination sections equal to 1. Now, LU decomposition is practically Gaussian, but we only deal with the grid a (rather than an expanded matrix). We will manage a 3\times3 arrangement of conditions for succinctness, yet everything here sums up to the \( n \times n \) case. Note that the numpy decomposition utilizes halfway
turning (grid columns are permuted to utilize the biggest rotate). This is on the grounds that little can prompt numerical flimsiness. Another motivation behind why one should utilize library capacities at whatever point conceivable.

1.2. Cholesky Decomposition.

\[ XT A x = X n i = 1 x n j = 1 A_{ij} x_i x_j = X n i = 1 A_{ii} x_i^2 + 2 X i > j A_{ij} x_i x_j \]

Every positive definite matrix \( A \in \mathbb{C}^{n \times n} \) can be factored as \( A = R H R \) where \( R \) is upper triangular with positive real diagonal elements.

Suppose \( A \) is \( n \times n \) and Hermitian defined as: \( (A_{ij} = \bar{A}_{ji}) x H A x = X n i = 1 x n j = 1 A_{ij} \bar{x}_i x_j = X n i = 1 A_{ii} |x_i|^2 + 2 X i > j (A_{ij} \bar{x}_i \bar{x}_j + A_{ij} x_i \bar{x}_j) = X n i = 1 A_{ii} |x_i|^2 + 2 Re X i > j A_{ij} \bar{x}_i x_j \).

Note that \( x H A x \) is real for all \( x \in \mathbb{C}^n \), a strategy for breaking down a positive-unmistakable lattice. A positive-distinct network is characterized as an asymmetric grid for every single imaginable vector \( x \), \( \bar{x} A x > 0 \). Cholesky decomposition and other decomposition strategies are significant, as it is a rare occurrence achievable to perform framework calculations explicitly. Cholesky decomposition, otherwise called Cholesky factorization, is a strategy for breaking down a positive-clear grid. A positive-distinct lattice is characterized as an asymmetric framework where for every conceivable vector \( x \), \( \bar{x} A x > 0 \). Cholesky decomposition and other decomposition strategies are significant, as it is a rare occurrence doable to perform lattice calculations expressly. A few utilizations of Cholesky decomposition incorporate unraveling frameworks of direct conditions, Monte
Carlo recreation, and Kalman filters. Cholesky decomposition factors a positive-unequivocal grid \( A = LL^T \) where \( L \) is a lower triangular lattice. \( L \) is known as the Cholesky factor of \( A \) and can be deciphered as the square foundation of a positive-clear matrix shown in figure 2.

\[ \text{Figure 2. Cholesky Decomposition Result} \]

1.3. **Singular Value Decomposition.** To get the singular worth decomposition, shown in figure 3, we can exploit the way that for any network \( A_n \), \( ATA \) is symmetric (since \( (ATA)^T = AT(ATA)^T = ATA \)). Symmetric lattices have the pleasant property that their eigenvectors structure an orthonormal premise; this isn’t appallingly difficult to demonstrate, yet for curtness, trust me. (To demonstrate some portion of this hypothesis, start with two eigenvectors \( v_1 \) and \( v_2 \), compose their spot item as a framework increase, and basically tinker with the variable based math and their eigenvalues \( \lambda_1 \) and \( \lambda_2 \) until you can show that the speck item should be zero on the grounds that the eigenvalues are unmistakable.)
Another significant lattice decomposition is particular worth decomposition or SVD. For any \(m \times n\) grid \(A\), we may compose:

\[
A = UDV,
\]

where \(U\) is a unitary \(m\) of the network (symmetric in the actual case), \(D\) is a rectangular \(m\) of inclination to corner parts \(d_1, \ldots, d_m\). \(V\) is a (symmetric) \(n\) unitary grid. SVD is used in the basic analysis of the Moore-Penrose component and in the measurement of the pseudo-conversion.

Choleskey decomposition using np array:

\[
A = \text{np.array}([[8,6,4,1],[1,4,5,1],[8,4,1,1],[1,4,3,6]])
\]

\[
b = \text{np.array([19,11,14,14])}
\]

\[
\text{la.solve}(A,b)
\]

**Output:** array([ 1., 1., 1., 1.])

**Figure 3.** Singular Value Decomposition

1.4. **Joint and Individual variation Explained (JIVE).** Plays JIVE decomposition in a round-up of information gatherings, when a measurement is shared
by information, which returns the weaker networks which capture the joint and individual data structure. This makes two rank options when the rank is dark, a stage check, and a BIC calculation.

Similarly, three plotting capabilities for the disparity assigned to each datum source are remembered for the bundle: a bar-plot showing secondary fluctuation levels from the joint structure and the individual structure, a heatmap showing uncertainty structures, and head part tracks. Joint and Individual Variance (JIVE), a general variety decomposition used for the structured study of data sets. The decline consists of three terms: a low position expectation of a mixture of different knowledge types, a low position estimate for individual structured variations, and a leftover clamor. JIVE analyses the combined variety of information types, decreases the dimension of data and introduces new headings for a joint and structure visual study shown in figure 4. The methodology revised speaks of an improvement in the critical component research and has strong points of concern about popular two-square strategies such as canonical and partial lower squares.

**CONCLUSION**

Based on our review of the decomposition of Lu, Cholesky decomposition, and shared variable values. To replicate the outcome, we suggest using the JIVE technique. Thanks to the versatility, precision, and very less noise in JIVE’s performance. The other approaches have a bit of noise in their output, and the
result is very different. JIVE does not allow information to be obtained, as in realistic analysis of knowledge. For example, smoothing regulatory strategies can enhance practical knowledge techniques. JIVE gages for joint and individual systems are not excluded from exceptions as an Associate Editor points out. Exploratory techniques, such as PCA, are fairly stable and important variants of PCA have been established late. An interesting possible expansion is a strong type of JIVE. In addition, all missing qualities should be applied to the JIVE gauges above. Another possible extension is a approach that specifically demonstrates missing attributes in determining the joint and individual structure.

This examination was provoked by the longing to support local and transportation organizers better comprehend the job that worker railway network plays in coordinating intra-provincial turn of events. We explore the perplexing impact of establishing a suburbanite rail framework on encompassing neighborhood travel conduct and the assembled condition. With proceeded with spread and vehicle prompted social and ecological issues, it is a higher priority than any time in recent memory for organizers to comprehend this relationship. Hence we suggest the JIVE method to analyses the findings in the commuter’s railway pattern. With the help, JIVE methodology, we can overcome problems that occurs in the commuter pattern in the railway network.

For details see [1-10].

**Future Work**

The review shows that the proposed AJIVE overtook different models of matrix decomposition. In our study, we saw that AJIVE functioned well in convergence rate, dependability measures, and order precision. This strategy can be actualized as a community-oriented sifting method of data recovery. In the future, a versatile learning rate based strategy can be utilized with AJIVE to show signs of improvement results and convergence.

**References**


