Abstract. Harmonious graphs introduced by Graham and Sloane [1] and Singh and Varkey [3] presented the odd sequential graphs. The main objective of this paper we have shown that the double $m$ triangular snakes, and alternative double $m$ triangular snakes are odd sequential harmonious graph for every $m$.

1. Introduction

All graphs considered as finite, simple and undirected during this paper. A graph with $p-$ vertices and $q-$ edges are referred to as a $(p, q)$ graph. Indicate the set of vertex and edge symbols $V(G)$ and $E(G)$. An arrangement of whole numbers at the vertices or edges or both according to unique conditions is understood as a graph labeling. Graph labeling presented in late 1960’s. Graph labeling is in many applications like coding theory, X-beam Crystallography, radar, space science, circuit structure, correspondence organizing to, information base administration.

Definition 1.1 ($k-$ Odd sequential harmonious labeling ($k-OSSH$)). A graph $G$ is supposed to be $k-$ odd sequential harmonious labeling if there exist a
one-one function \( h : V(G) \rightarrow \{k - 1, k, k + 1, \ldots, k + 2q - 1\} \) specified the actuated mapping \( h^* : E(G) \rightarrow \{2k - 1, 2k + 1, 2k + 3, \ldots, 2k + 2q - 3\} \) defined by

\[
h^*(uv) = \begin{cases} 
  h(u) + h(v) + 1, & \text{if } h(u) + h(v) \text{ is even} \\
  h(u) + h(v), & \text{if } h(u) + h(v) \text{ is odd}
\end{cases}
\]

are distinct.

**Definition 1.2** \((k-\text{Odd sequential harmonious graph } (k-\text{OSHG}))\). A graph is called \(k-\text{odd sequential harmonious graph}\) if it has \(k-\text{odd sequential harmonious labeling}\).

2. Main Results

**Definition 2.1** \((\text{Double } m \text{ triangular snake } (2mTS_n))\). A double \( m \) triangular snake comprises of \( m \) triangular snakes that have path, in like manner, i.e., a \((2mTS_n)\) is gotten from a path \(v_1, v_2, \ldots, v_n\) by joining \(v_i\) and \(v_{i+1}\) to a different vertex \(u_{ji}\) for \(i = 1, 2, \ldots, n - 1, \ j = 1, 2, \ldots, m\). Specifically \(m = 1\) is named double triangular snake.

**Theorem 2.1.** Double \( m \)-triangular snake is an OSHG for every \(m\).

**Proof.** Let the vertices of \(2m(TS_n)\) is:

\[
\{v_i : 1 \leq i \leq n\} \cup \{u_{ji}^i, w_{ji}^i : 1 \leq i \leq n - 1, 1 \leq j \leq m\}.
\]

Then the edges labels of \(2m(TS_n)\) are:

\[
\{v_iv_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_iw_{ji}^i : 1 \leq i \leq n - 1, 1 \leq j \leq m\} \\
\cup \{v_{i+1}w_{ji}^i : 1 \leq i \leq n - 1, 1 \leq j \leq m\} \cup \{v_{i+1}w_{ji}^i : 1 \leq i \leq n - 1, 1 \leq j \leq m\}
\]

and are denoted as the following figure:

The vertices are first labelled as follows:

Let \(h : V \rightarrow \{k - 1, k, k + 1, k + 2, \ldots, k + 2q - 1\}\) be defined by

\[
\begin{align*}
  h(v_i) &= i + k - 2, 1 \leq i \leq n \\
  h(u_{ji}^i) &= (8j - 2)n + 3i + k - (8j + 1) \ i \leq j \leq m, 1 \leq i \leq n - 1 \\
  h(w_{ji}^i) &= (8j - 6)n + 3i + k - (8j - 3)
\end{align*}
\]
At that point the incited edge labels are:

\[
\begin{align*}
    h^*(v_i v_{i+1}) &= 2i + 2k - 3, \ 1 \leq i \leq n - 1 \\
    h^*(v_i u_j) &= (8j - 2)n + 4i + 2k - (8j + 3) \\
    h^*(v_{i+1} u_j) &= (8j - 2)n + 4i + 2k - (8j + 1) \quad 1 \leq j \leq m, 1 \leq i \leq n - 1 \\
    h^*(v_i w_j) &= (8j - 6)n + 4i + 2k - (8j - 1) \\
    h^*(v_{i+1} w_j) &= (8j - 6)n + 4i + 2k - (8j - 3),
\end{align*}
\]

Obviously, we see that the edge labels are distinct. Along these lines, the graph \((2mTS_n)\) is a \((k – OSHL)\), for every \(m\). Thus the graph \(2m(TS_n)\) is an \(k – OSHG\), for every \(m\). □

Example 1.
Example 2.

Theorem 2.2. Alternate double triangular snake $A(2mTS_n)$ starting with an edge is an $k$– odd sequential harmonious graph for every $m$.

Proof. There are two other cases.
Case(i): $n$– is even.
Let the vertices of $A(2mTS_n)$ is

$$\{v_i; 1 \leq i \leq n\} \cup \{w^i_j, w^i_j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\},$$

and the edges of $A(2mTS_n)$ is

$$\{v_iv_{i+1}; 1 \leq i \leq n-1\}$$

$$\cup \{v_{2i}w^i_j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\}$$

$$\cup \{v_{2i+1}w^i_j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\}$$

$$\cup \{v_{2i}w^i_j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\}$$

$$\cup \{v_{2i-1}w^i_j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\},$$

which are denoted as the following figure.
The vertices are first labelled as follows. Let $h : V \to \{k-1, k, k+1, \ldots, k+2q-1\}$ be defined by

$$h(v_i) = i + k - 2, 1 \leq i \leq n,$$

$$\begin{cases} 
h(u_{ij}) = 4jn + 2i + k - (8j + 1), & 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m, \\
h(w_{ij}) = (4j - 2)n + 2i + k - (8j - 3), & 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m. 
\end{cases}$$

At that point the prompted edge labels are

$$h^*(v_iv_{i+1}) = 2i + 2k - 3, \quad 1 \leq i \leq n-1,$$

$$\begin{cases} 
h^*(v_{2i}w_i) = (4j-2)n + 4i + 2k - (8j - 1), \\
h^*(v_{2i+1}w_i) = (4j-2)n + 4i + 2k - (8j - 3) & 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m \\
h^*(v_{2i}u_i) = 4jn + 4i + 2k - (8j + 3) \\
h^*(v_{2i+1}u_i) = 4jn + 4i + 2k - (8j + 1) 
\end{cases}$$

**Case (ii):** $n$ is odd.

Let the vertices of $A(2mTS_n)$ are

$$\{v_i; 1 \leq i \leq n\} \cup \{w^i_1, w^i_2; 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m\}$$
and the edges of $A(2mTS_n)$ are
\[
\{v_i v_{i+1}; 1 \leq i \leq n\} \cup \{v_i u_i^j; 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m\} \\
\cup \{v_{i+1} u_i^j; 1 \leq j \leq m\} \\
\cup \{v_i w_i^j; 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m\} \\
\cup \{v_{i+1} w_i^j; 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m\}.
\]

which are denoted in figure:

The vertices are first labeled as follows:
\[
h(v_i) = i + k - 2, \quad 1 \leq i \leq n,
\]
\[
\begin{cases} 
  h(u_i^j) = 4jn + 2i + k - (4j + 3), & 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m. \\
  h(w_i^j) = (4j - 2)n + 2i + k - (4j + 1), & 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m.
\end{cases}
\]

The edge labels then are,
\[
h^*(v_i v_{i+1}) = 2i - 1, \quad 1 \leq i \leq n - 1,
\]
\[
\begin{cases} 
  h^*(v_i u_i^j) = 4jn + 4i + 2k - (4j + 5), \\
  h^*(v_{i+1} u_i^j) = 4jn + 4i + 2k - (4j + 3), & 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m. \\
  h^*(v_i w_i^j) = (4j - 2)n + 4i + 2k - (4j + 3), \\
  h^*(v_{i+1} w_i^j) = (4j - 2)n + 4i + 2k - (4j + 1), & 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m.
\end{cases}
\]
In both case, we see that the edge labels are distinct. So, $h$ admits $k$ odd sequential harmonious labeling. Hence the alternating double triangular graph $A(2mTS_n)$ is $k$-OSHG for every $m$. □

Example 3. $n$ is even.

Example 4.
REFERENCES


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