A NEW APPROACH FOR SOLVING FLOW SHOP SCHEDULING PROBLEM WITH TRIANGULAR INTUITIONISTIC FUZZY NUMBER

K. SELVAKUMARI1 AND S. SANTHI

ABSTRACT. Scheduling is one of the most critical decision-making problems widely studied in the operations research domain. This paper deals with finding an optimal solution to the scheduling problem under the Triangular Intuitionistic fuzzy Environment. Here processing times are being taken as Triangular I.F.N., which are further defuzzified into crisp values by the ranking procedure. We formulate a new algorithm to get an optimal sequence, and calculate the total elapsed time is minimum. We discussed the machines’ minimum rental cost under the specified rental Policy and compared the results with Johnson’s Algorithm.

1. INTRODUCTION

Initially, the idea of fuzzy set theory was formulated by Zadeh [9] in 1965. It deals with the information of membership function to handle Imprecise situation. Later, to overcome this uncertain situation, Atanassov started the intuitionistic fuzzy set and formulated the idea of belongings & non-belongings. The purpose of ranking the Intuitionistic fuzzy numbers plays a vital role in achieving an accurate solution for decision-makers. Atanassov (1994) defined various arithmetic operators in intuitionistic fuzzy sets, which are very helpful

1corresponding author

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in applications of different areas of decision making. Further intuitionistic fuzzy optimization problem involves the comparison of fuzzy numbers.

Fuzzy sets are applied when the knowledge about the processing time is incomplete. Ishibuchi & Lee [5] formulated the fuzzy flow shop scheduling problem with fuzzy processing time Shakeela Sathish, K. Ganesan [7] studied 3 stage flow shop scheduling to minimize the rental cost of machines.

In this paper, We consider the four-machine flow-shop scheduling with a triangular intuitionistic fuzzy number as processing time. Here processing times are being taken as Triangular intuitionistic fuzzy number, which is further defuzzified into crisp values by the ranking procedure. We propose a new algorithm proposed to get an optimal sequence and calculate the total elapsed time. Minimizing the rental cost of machines under the specified rental Policy was discussed and compared the results with Johnson’s Algorithm.

2. Preliminaries

Fuzzy Set
Let X be a nonempty set, and define a fuzzy set \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A\} \). In the pair \((x, \mu_{\tilde{A}}(x))\), the first element belongs to the classical set A, the second element \(\mu_{\tilde{A}}(x)\), belong to the interval \([0, 1]\) is called the membership function.

Fuzzy number
\( \tilde{A} \) is a fuzzy set on the real line \( R \), must satisfy the following conditions.

1. \( \mu_{\tilde{A}}(x_0) \) is piecewise continuous.
2. There exist at least one \( x_0 \in R \) with \( \mu_{\tilde{A}}(x_0) = 1 \).
3. \( \tilde{A} \) must be normal and convex.

Triangular Fuzzy Numbers
A fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \) is said to be a triangular fuzzy number if its membership function is given by,

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & \text{for } x < a_1 \\
\frac{(x-a_1)}{(a_2-a_1)} & \text{for } a_1 \leq x \leq a_2 \\
\frac{(a_3-x)}{(a_3-a_2)} & \text{for } a_2 \leq x \leq a_3 \\
0 & \text{for } x > a_3
\end{cases}
\]

where \( a_1 \leq a_2 \leq a_3 \) are real numbers.
Intuitionistic Fuzzy number

An Intuitionistic fuzzy subset $A^I = \{(x_i, \mu_{A^I}(x_i), \gamma_{A^I}(x_i)) / x_i \in X\}$ of the real line $\mathbb{R}$ is named as an intuitionistic fuzzy number if the following holds.

(1) There exist $\theta \in \mathbb{R}$, $\mu_{A^I}(\theta) = 1$ and $\gamma_{A^I}(\theta) = 0$, where $\theta$ is the mean value of $A^I$.

(2) $\mu_{A^I}$ is a continuous mapping from $\mathbb{R}$ to $[0,1]$ for all $x \in \mathbb{R}$, the relation $0 \leq \mu_{A^I}(x) + \gamma_{A^I}(x) \leq 1$ holds. The membership and non-membership function of $A^I$ is of the following form,

$$\mu_{A^I}(x) = \begin{cases} 0, & \text{if } -\alpha < x < \theta - \alpha \\ f_1(x), & \text{if } x \in [\theta - \alpha, \theta] \\ 1, & \text{if } x = \theta \\ g_1(x), & \text{if } x \in [\theta, \theta + \beta] \\ 0, & \text{if } \theta + \beta \leq x < \alpha \end{cases}$$

$$\gamma_{A^I}(x) = \begin{cases} 1, & \text{if } -\alpha < x < \theta - \alpha' \\ f_2(x), & \text{if } x \in [\theta - \alpha', \theta]; 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0, & \text{if } x = \theta \\ g_2(x), & \text{if } x \in [\theta, \theta + \beta']; 0 \leq g_1(x) + g_2(x) \leq 1 \\ 1, & \text{if } \theta + \beta' \leq x \leq \alpha \end{cases}$$

where $f_i(x)$ and $g_i(x)$; $i = 1, 2$ which are strictly increasing and decreasing functions in $[\theta - \alpha, \theta], [\theta - \alpha', \theta], [\theta - \alpha, \theta]$, and $[\theta, \theta + \beta']$ respectively. $\alpha, \beta, \alpha'$ and $\beta'$ are left and right spreads of $\mu_{A^I}(x)$ and $\gamma_{A^I}(x)$. 

Fig. 1. Graphical representation of Triangular fuzzy number
3. Triangular Intuitionistic Fuzzy Number (TIFN)

An intuitionistic fuzzy number $A_T = \{(a_1, a_2, a_3)(a'_1, a'_2, a'_3)\}$ is said to be triangular intuitionistic fuzzy number (TIFN) if its membership and non-membership functions are respectively given by

$$
\mu_{A_T}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \leq x \leq a_2 \\
\frac{a_3-x}{a_3-a_2}, & \text{if } a_2 \leq x \leq a_3, \\
0, & \text{Otherwise}
\end{cases}
$$

$$
\gamma_{A_T}(x) = \begin{cases} 
\frac{a_2-x}{a_2-a'_1}, & \text{if } a'_1 \leq x \leq a_2 \\
\frac{x-a_3}{a'_3-a_2}, & \text{if } a_2 \leq x \leq a'_3, \\
1, & \text{Otherwise}
\end{cases}
$$

Here $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$ and $\mu_{A_T}(x), \gamma_{A_T}(x) \leq 0.5$ for $\mu_{A_T}(x) = \gamma_{A_T}(x)$ for every $x \in \mathbb{R}$.

Arithmetic Operators

Let $A = (a_1, a_2, a_3)(a'_1, a_2, a'_3)$ and $B = (b_1, b_2, b_3)(b'_1, b_2, b'_3)$ be the two TIFN then

1. Addition Let $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)(a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3)$
2. Subtraction Let $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)(a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1)$
4. Ranking of TIFN

The Ranking of Triangular intuitionistic fuzzy number is defined as

\[ R(A) = \frac{1}{3} \left[ \frac{(a' - a'_1)(a_2 - 2a'_3 - 2a'_1) + (a_3 - a_1)(a_1 + a_2 + a_3) + 3(a'_2 - a'_1^2)}{a_3 - a'_1 + a_3 - a_1} \right]. \]

If \( R.A. \leq R.B. \), then \( A \leq B \).

5. Flow Shop Scheduling Problem with Intuitionistic Fuzzy Processing Time

Rental Policy

Policy I All the machines are taken and rent at one time and are returned at one time.

Policy II All the tools are taken on rent at once and come back as and when they are no longer required.

Policy III We required all the machines are taking on rent and returned and no longer needed for processing.

Rental Situation

The problem formulated here is under Policy III; whenever the job under machine I completed, it is returned and deal with the job(appointment) to the next device.

Assumption

(1) The tasks to be processed are independent of each other
(2) Pre-emption of employment are not allowed
(3) An appointment is not available to the next machine until and unless we complete the current processing device.
(4) Machines never breakdown and are available throughout the scheduling process.
(5) We conclude each job must be once when it starts.

Notations

\( f_{ij} \) - Processing time of \( i^{th} \) job on a \( j^{th} \) machine
\( R(S) \) - Total rental cost for the sequence (S)
\( U_k(S_K) \) - Utilisation time of each machine
\( C_m \) - Cost for each rent \((m = 1...4)\)
Problem Formulation
Assume that some jobs \( i(1,2,...n) \) are to be processed on machines \( j(1,2,...m) \) under the specified rental policy.

Let \( f_{ij} \) be the processing time of \( i^{th} \) job on the \( j^{th} \) machine described by the triangular intuitionistic fuzzy number. We aim to find the minimal rental cost
\[
R(S) = \sum_{i=1}^{n} f_{ij} \ast C1 + U_2(S_K) \ast C2 + U_3(S_K) \ast C3 + U_4(S_K) \ast C4
\]

Division Algorithm (Proposed method)
\textbf{Step1}: Defuzzify the triangular intuitionistic fuzzy number into a crisp number
\textbf{Step2}: Choose the highest processing time in each row, and all the entries are divided by it, enter the results in the top right of the table. \textbf{Step3}: Choose the highest processing time in each column, and all the entries are divided by it, enter the results in the bottom left of the table.
\textbf{Step4}: Add both the entries and note the minimum entry in the table.
\textbf{Step5}: Delete the corresponding row of the minimum entry and mark the sequence as \( S_k \) \((k=1...5)\)
\textbf{Step6}: Repeat the procedure until we arrange all the jobs in proper order
\textbf{Step7}: Calculate the minimum total elapsed time and idle time of each Machine.
\textbf{Step8}: Calculate the total rental cost.

6. Numerical Example

Prestigious management school in a city is giving an order to stitch the uniform to all the students studying from sixth to the tenth standard to the X, Y, Z. Shop, the works are to stitch: 1) Boys Pants, 2) Boys Shirts, 3) Girls Pants, 4) Girls Tops, 5) Girls Overcoat. For each job, the owner has to arrange some persons for cutting, stitching, fixing school emblem and ironing.

Therefore he has to pay for each person who is doing the work. He is planning to pay Rs.10, Rs.20, Rs.15, and Rs.5 for each task. Triangular Intuitionistic Fuzzy Number is using for the time taken in each job. Calculate the total amount of time he takes to complete the work and the total amount he spends for each job.
Solution
The given TIFN is defuzzified into crisp value by using the proposed ranking procedure as follows

Choose the highest processing time in each row, and all the entries are divided by it, enter the results in the top right of the table.
Similarly, choose the highest processing time in each column, and all the entries are divided by it, enter the results in the bottom left of the table. Adding the effects of each cell, choose the minimum entry, and delete the corresponding row.

Repeat the procedure until we arrange all the jobs

<table>
<thead>
<tr>
<th>Jobs</th>
<th>C</th>
<th>S</th>
<th>F</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.688</td>
<td>2.106</td>
<td>0.600</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.319</td>
<td>0.571</td>
<td>0.168</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6.762</td>
<td>3.688</td>
<td>2.282</td>
<td>7.022</td>
</tr>
<tr>
<td></td>
<td>0.460</td>
<td>0.640</td>
<td>0.942</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9.375</td>
<td>2.106</td>
<td>2.106</td>
<td>6.044</td>
</tr>
<tr>
<td></td>
<td>0.638</td>
<td>0.571</td>
<td>0.591</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10.762</td>
<td>0.600</td>
<td>0.279</td>
<td>7.458</td>
</tr>
<tr>
<td></td>
<td>0.733</td>
<td>0.163</td>
<td>0.842</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>14.688</td>
<td>2.106</td>
<td>3.563</td>
<td>4.688</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.571</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

The resultant value of the simplification is

<table>
<thead>
<tr>
<th>Jobs</th>
<th>C</th>
<th>S</th>
<th>F</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.257</td>
<td>0.992</td>
<td>0.288</td>
<td>1.670</td>
</tr>
<tr>
<td>2</td>
<td>1.423</td>
<td>1.525</td>
<td>0.965</td>
<td>1.942</td>
</tr>
<tr>
<td>3</td>
<td>1.638</td>
<td>0.796</td>
<td>0.816</td>
<td>1.455</td>
</tr>
<tr>
<td>4</td>
<td>1.733</td>
<td>0.219</td>
<td>1.121</td>
<td>1.695</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.714</td>
<td>1.243</td>
<td>0.948</td>
</tr>
</tbody>
</table>
Among all the entries, the value in the fourth row is minimum. Delete the corresponding row and form the sequence as $S_4$. Proceed to the above algorithm until all the jobs arranged.

Here we get the sequence as $S_4 - S_1 - S_5 - S_2 - S_3$

The in-out table for the above series is under the following

<table>
<thead>
<tr>
<th>Machines</th>
<th>A cutting (C)</th>
<th>Stitching (S)</th>
<th>Fixing School Emblem (F)</th>
<th>Ironing (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In</td>
<td>Out</td>
<td>In</td>
<td>In</td>
</tr>
<tr>
<td>5</td>
<td>15.450</td>
<td>30.138</td>
<td>30.138</td>
<td>32.244</td>
</tr>
<tr>
<td>2</td>
<td>30.138</td>
<td>36.900</td>
<td>36.900</td>
<td>40.588</td>
</tr>
<tr>
<td>3</td>
<td>36.900</td>
<td>46.275</td>
<td>46.275</td>
<td>48.381</td>
</tr>
</tbody>
</table>

- Minimum total elapsed time = 56.531 hrs
- Idle time of Cutting (C), $C = 56.531 - 46.275 = 10.256$ hrs
- Idle time of Stitching (S), $S = 10.762 + 4.088 + 12.582 + 4.656 + 5.687 + 8.150 = 45.925$ hrs
- Idle time of Fixing School Emblem (F), $F = 11.362 + 3.194 + 14.088 + 4.781 + 5.511 + 6.044 = 44.980$ hrs
- Idle time of Ironing (I), $I = 14.362 + 8.987 + 2.375 + 0.595 = 26.319$ hrs
- Rental Cost for Cutting (C), $C = 46.275 \times 10 = Rs. 462.75$
- Rental Cost for Stitching (S), $S = 48.381 - 45.925 = 2.456 \times 20 = Rs. 49.12$
- Rental Cost for Fixing School Emblem (F), $F = 50.487 - 44.980 = 5.507 \times 15 = Rs. 82.605$
- Rental Cost for Ironing (I), $I = 56.531 - 26.319 = 30.212 \times 5 = Rs. 151.06$
- Total Cost given for all the work = 462.75 + 49.12 + 82.605 + 151.06 = Rs. 745.53

**Comparison with Existing Methods**

We tabulate the comparison of the proposed method with the existing process, which clearly shows that the proposed method provides the same results.
7. Conclusion

Here flow shop scheduling problem is formulated under an intuitionistic fuzzy number and discussed a new approach is to find the optimal sequence and rental cost calculated and compared. The application of the above procedure is useful in many uncertain conditions.

References


Department of Mathematics,
Vels Institute of Science, Technology and Advanced Studies,
Chennai, Tamilnadu, India.
Email address: selvafeb6@gmail.com

Department of Mathematics,
Vels Institute of Science, Technology and Advanced Studies,
Chennai, Tamilnadu, India.
Email address: Santhosh.mitha@gmail.com