ON SOFT FUZZY SOFT TOPOLOGICAL VECTOR SPACES AND DIFFERENTIATIONS

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ABSTRACT. Basic properties of soft fuzzy soft topological vector spaces are introduced and discussed. Few results of soft fuzzy soft tangent to 0 and SFS differentiations have been established.

1. INTRODUCTION


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2. Preliminaries

Definition 2.1. [6] Let $P(U)$ be the power set of $U$ and $A$ be the subset of parameter $E$. Then $(F, A)$ is known as soft set over $U$ where $F : A \to P(U)$.

Definition 2.2. [4] A fuzzy soft set over the universe $U$ is a pair $(F, A)$ where $F : A \to F(U)$ and $A$ is the subset of a set of parameter $E$.

Definition 2.3. [3] $\delta \subset I^E$ is a fuzzy topology on $E$ iff

(i) For all $\alpha$ constant, $\alpha \in \delta$,

(ii) For all $\mu, \eta \in \delta \Rightarrow \mu \land \eta \in \delta$.

(iii) For all $(\mu_j)_{j \in J} \subset \delta \Rightarrow \sup_{j \in J} \mu_j \in \delta$.

The member in $\delta$ is known as open fuzzy set. $\mu \in I^E$ is said to be closed iff $\mu^c$ is open.

Definition 2.4. [10] Let $X$ be a set, $\eta \in I^X$ and $M$ is any subset of $X$. Then $(\eta, M)$ is said to be a soft fuzzy set in $X$.

3. On Soft Fuzzy Soft Sets

Throughout the paper $X \neq \phi$, $E$ denote the collection of parameters and $I = [0, 1]$. Also soft fuzzy soft is denoted by $SFS$.

Definition 3.1. A $SFS$ $\delta_E : X \to I \times P(E)$ with membership $\delta_E(p) = (\delta(p), A)$ where $\delta : X \to I$, $A$ is the member of the collection $P(E)$ of all subsets of $E$. Moreover its family is represented by $SFS(X, E)$.

Definition 3.2. $SFS$ characteristic function $\chi_A : X \to \{(1, E), (0, \phi)\}$ is defined as

$$\chi_A(p) = \begin{cases} 
(1, E), & \text{if } p \in A \subset X; \\
(0, \phi), & \text{otherwise}. 
\end{cases}$$

Definition 3.3. Let $\delta_E$ be a $SFS$ set. Define

$$p_{\delta_E}(q) = \begin{cases} 
(\delta(p), A)(\delta(p) \in [0, 1]), & \text{if } p = q; \\
(0, \phi), & \text{otherwise}. 
\end{cases}$$

$p_{\delta_E}$ is a sfs point (in short, SFSP) in $SFS(X, E)$.
Definition 3.4. Let $\lambda_E \in SFS(X, E)$ such that the universal SFS set is $\lambda_E(p) = (1, E)$, $\forall p \in X$ and it is represented by $(1, E)^\sim$. The null SFS set is defined as follows $\lambda_E(p) = (0, \phi)$, $\forall p \in X$ and it is represented as $(0, \phi)^\sim$.

Definition 3.5. Let $\delta_E \in SFS(X, E)$ such that $\delta_E(p) = (\delta(p), \mathcal{A})$, then the complement of $\delta_E$ is denoted by $\delta_E^c$ where $\delta_E^c(p) = (1, E)^\sim - \delta_E(p) = (1 - \delta(p), E\setminus \mathcal{A})$, $\forall p \in X$.

Definition 3.6. Let $\delta_E$ and $\mu_E$ be any two SFS sets such that $\delta_E(p) = (\delta(p), \mathcal{A})$ and $\mu_E(p) = (\mu(p), \mathcal{B})$. Then

(i) $\delta_E(p) \subseteq \mu_E(p) \Leftrightarrow \delta(p) \leq \mu(p)$, $\forall p \in X$, $\mathcal{A} \subseteq \mathcal{B}$.

(ii) $\delta_E(p) \supseteq \mu_E(p) \Leftrightarrow \delta(p) \geq \mu(p)$, $\forall p \in X$, $\mathcal{A} \supseteq \mathcal{B}$.

(iii) $\delta_E(p) \cap \mu_E(p) = \{\min\{\delta(p), \mu(p)\}, \mathcal{A} \cap \mathcal{B}\}$, $\forall p \in X$.

(iv) $\delta_E(p) \cup \mu_E(p) = \{\max\{\delta(p), \mu(p)\}, \mathcal{A} \cup \mathcal{B}\}$, $\forall p \in X$.

Definition 3.7. Let $\delta_E$ and $\mu_E$ be any two SFS sets. Then

(i) $\delta_E \subseteq \mu_E \Leftrightarrow \delta_E(p) \subseteq \mu_E(p)$, $\forall p \in X$.

(ii) $\delta_E \supseteq \mu_E \Leftrightarrow \delta_E(p) \supseteq \mu_E(p)$, $\forall p \in X$.

(iii) $\delta_E = \mu_E \Leftrightarrow \delta_E(p) = \mu_E(p)$, $\forall p \in X$.

Definition 3.8. Let $f : X \rightarrow Y$. If $\delta_E \in SFS(Y, E)$, then $f^{-1}(\delta_E)(p) = \delta_E \circ f(p) = \delta_E(f(p))$, $\forall p \in X$.

Definition 3.9. Let $f : X \rightarrow Y$. If $\mu_E \in SFS(X, E)$, then

$$f(\mu_E)(q) = \begin{cases} \cup_{p \in f^{-1}(q)}\mu_E(p), & \text{if } f^{-1}(q) \neq \phi; \\ (0, \phi), & \text{otherwise}. \end{cases}$$

Definition 3.10. A constant membership function is represented by $\mathcal{R}_E$ and if $\mathcal{R}_b(p) = b$ for all $p \in X$, where $0 < b \leq 1$.

Definition 3.11. Let $\delta_{j_E}$ be any SFS sets and $J$ be an indexed set. A SFS topology on $X$ is a collection $\mathcal{T}$ of SFS sets satisfying:

(i) $\mathcal{R}_b \in X$ and $\forall A \in P(E), \mathcal{R}_{b_A} \in \mathcal{T}$ where $\mathcal{R}_{b_A} = (\mathcal{R}_b(p), A)$ for all $p \in X$.

(ii) $\delta_{j_E} \in \mathcal{T}$, $j \in J \Rightarrow \psi_{j \in J} \delta_{j_E} \in \mathcal{T}$.

(iii) For any finite $J$, $\delta_{j_E} \in \mathcal{T}$, $\Rightarrow \bigcap_{j \in J} \delta_{j_E} \in \mathcal{T}$.

Then $(X, \mathcal{T})$ is said to be SFS topological space, SFSTS. Any member of $\mathcal{T}$ is SFS open set, SFSETS. SFS closed set is the complement of SFS open set and whose collection is denoted by SFSCS.
Remark 3.1. \( \mathcal{T} = \{ \chi_M : \forall M \in \tau \} \cup \{ \mathfrak{R}_{k_A} : \forall k_b \in X \text{ and } \forall A \in P(E) \} \) forms an usual SFS topology.

Proposition 3.1. Let \( f : X \to Y \). If \( \delta_1, \delta_2 \in \mathcal{X} \) be any two SFS sets in \( X \) and \( \mu_1, \mu_2 \in \mathcal{Y} \) be any two SFS sets in \( Y \). Then
(i) \( \delta_1 \subseteq \delta_2 \Rightarrow f(\delta_1) \subseteq f(\delta_2) \).
(ii) \( \mu_1 \subseteq \mu_2 \Rightarrow f^{-1}(\mu_1) \subseteq f^{-1}(\mu_2) \).

Definition 3.12. Let \( \delta_E \in SFS(X, E) \) is said to be a SFS neighbourhood(nghbd) of a SFS point \( p_{\delta_E} \) in \( X \) iff \( \exists a \mu_E \in \mathcal{T} \ni p_{\delta_E} \in \mu_E \in \delta_E \).

Definition 3.13. A system of SFS nghbds of a SFS point \( p_{\delta_E} \) is a set \( \mathcal{B}(p_{\delta_E}) \) of SFS nghbds of \( p_{\delta_E} \) such that for each SFS nghbd \( \delta_E \) of \( p_{\delta_E} \) there is a \( \mu_E \in \mathcal{B}(p_{\delta_E}) \) such that \( \mu_E \subseteq \delta_E \).

Proposition 3.2. Let \( f : (X, \mathcal{T}) \to (Y, \mathcal{S}) \), then we have the equivalence.
(i) \( f \) is SFS continuous.
(ii) \( \delta_E \in SFS(X, E) \) and each SFS nghbd \( \delta_E \) of \( f(\delta_E) \), \( \exists \) a SFS nghbd \( \mu_E \) of \( \delta_E \) such that \( f(\mu_E) \subseteq \delta_E \).

4. On SFS Topological Vector Spaces

Throughout the section \( \mathcal{V} \) is a vectorspace over the field \( K \).

Definition 4.1. Let \( \{\delta_{j}E \} \in SFS(\mathcal{V}, E) \), \( j = 1, 2, 3, \ldots, n \). The sum \( \delta_E = \delta_{1E} + \delta_{2E} + \delta_{3E} + \ldots + \delta_{nE} \) of \( \{\delta_{j}E \} \), is the SFS set having membership, \( \delta_E(p) = \bigcup_{p_1 \ldots p_n = p}(\delta_{1E}(p_1) \cap \delta_{2E}(p_2) \ldots \cap \delta_{nE}(p_n)) \), \( p \in \mathcal{V} \). The scalar product \( \alpha \delta_E \), of \( \alpha \in K \) and \( \delta_E \) is a SFS set in \( \mathcal{V} \) that has \( \alpha \delta_E(p) = \alpha \delta_E(p) \) \( \forall \alpha \neq 0 \) and \( \delta_E \) is a SFS set in \( \mathcal{V} \) that has \( \alpha \delta_E(p) = \alpha \delta_E(p) \) \( \forall \alpha \neq 0 \) or \( \bigcup_{q \in \mathcal{V}} \delta_E(q), p = 0 \).

Proposition 4.1. If \( f : \mathcal{V}_1 \to \mathcal{V}_2 \). Then for all SFS sets \( \delta_E, \mu_E \) in \( \mathcal{V}_1 \) and all scalars \( \alpha, f(\delta_E + \mu_E) = f(\delta_E) + f(\mu_E) \) and \( f(\alpha \delta_E) = \alpha f(\delta_E) \).

Proposition 4.2. If \( \delta_E, \mu_E \in SFS(\mathcal{V}, E) \) and \( \alpha \in K \), \( \alpha \neq 0 \), then \( \alpha \delta_E \in \mu_E \Rightarrow \delta_E \subseteq \mu_E \).

Proposition 4.3. Let \( \delta_{1E}, \delta_{2E}, \ldots, \delta_{nE} \in SFS(\mathcal{V}, E) \) and \( \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n \) scalars. Then the following are equivalent.
Proposition 4.4. Let $0$ point $B$ nghbds $\sigma$ SFS nghbd $\mu \delta$ there exists a SFS nghbd $\sigma$ of $A$ $\delta$. Let $\mu \delta$. Proposition 5.1. If the function $f$ is SFS tangent to $0\delta E$ such that $0\delta E = (\delta(0), \mathcal{A}), (0, \phi) \subset (\delta(0), \mathcal{A}) \subset (1, E)$, $\exists$ a system of SFS nghbds $B(0\delta E) \in \mathcal{V}$ holds.

(i) $\forall \delta E \in B(0\delta E) \exists a \mu E \in B(0\delta E)$ with $\mu E \subset \mu E \in \delta E$.

(ii) $\forall \delta E \in B(0\delta E) \exists a \mu E \in B(0\delta E)$ for which $k\mu E \subset \mu E \forall k \in K, |k| \leq 1$.

(iii) Every $\delta E \in B(0\delta E)$ is SFS balanced.

5. On SFS Differentiations

Definition 5.1. $\sigma : (\mathcal{V}_1, \mathcal{T}_1) \to (\mathcal{V}_2, \mathcal{T}_2)$ is called SFS tangent to $0$ if for each SFS nghbd $\mu E$ of $0\delta E$ where $0\delta E(0) = (\delta(0), \mathcal{A}), (0, \phi) \subset (\delta(0), \mathcal{A}) \subset (1, E)$, in $\mathcal{V}_2$ there exists a SFS nghbd $\delta E$ of $0\eta E$ where $0\eta E(0) = (\eta(0), F), (0, \phi) \subset (\eta(0), F) \subset (\delta(0), A)$ in $\mathcal{V}_1$ such that $\sigma(\delta E) \subset \rho(t)\mu E$ for some function $\rho(t)$.

Proposition 5.1. If the function $\sigma$ is SFS tangent to $0$, then $\sigma$ is SFS continuous at $0 \in \mathcal{V}_1$.

Proposition 5.2. If $\sigma$ and $\eta$ are two functions SFS tangent to $0$ then $\sigma + \eta$ is a function SFS tangent to $0$.

Proposition 5.3. Let $(\mathcal{V}_1, \mathcal{T}_1), (\mathcal{V}_2, \mathcal{T}_2), (\mathcal{V}_3, \mathcal{T}_3)$ be any three SFS topological vector spaces over $K$ with $E$ as the set of all parameters. Composition of SFS continuous linear map and SFS tangent to zero is SFS tangent to zero.

Definition 5.2. Let $(\mathcal{V}_1, \mathcal{T}_1)$ and $(\mathcal{V}_2, \mathcal{T}_2)$ be any two SFS topological vector spaces, each of them is a SFS $\mathcal{T}_1$ space. A SFS continuous function $f : \mathcal{V}_1 \to \mathcal{V}_2$ is called a SFS differentiable at $p \in \mathcal{V}_1$ if there is a linear SFS continuous function $u$ on $\mathcal{V}_1$ satisfies $f(p + q) = f(p) + u(q) + \sigma(q), q \in \mathcal{V}_1$ where $\sigma$ is SFS tangent to $0$ and $u$ is SFS derivative of $f$ at $p$.

Proposition 5.4. Let $(\mathcal{V}_1, \mathcal{T}_1), (\mathcal{V}_2, \mathcal{T}_2), (\mathcal{V}_3, \mathcal{T}_3)$ be any three SFS topological vector spaces and also SFS $\mathcal{T}_1$ space. Let $f$ and $g$ be a SFS continuous function on $\mathcal{V}_1$ and $\mathcal{V}_2$ respectively. Composition of two SFS differentiable function is SFS differentiable.
Proof. Assume that \( f \) and \( g \) are SFS differentiable. Hence \( f(p + r) - f(p) = f'(p)(r) + \sigma(r), \) \( r \in V_1, \) \( g(q + s) - g(q) = g'(q)(s) + \eta(s), \) \( s \in \mathcal{Y}_2, \) where \( \sigma \) and \( \eta \) are each SFS tangent to \( 0. \) Defining \( h = f \circ g, \) after substitution we get,

\[
h(p + r) - h(p) = g'(q)(f'(p)(r)) + g'(q)(\sigma(r)) + \eta(f'(p)(r) + \sigma(r)), \quad r \in \mathcal{Y}_1.
\]

By Proposition 5.3, \( g'(q) \circ \sigma \) is SFS tangent to \( 0. \) Consider the function \( \eta \circ (f'(p) + \sigma). \) For every SFS nghbd \( \mu_E \) of \( 0_{\mathcal{Y}_E} \) where \( 0_{\mathcal{Y}_E}(0) = (\nu(0), F), \) \( (0, \phi) \subset (\nu(0), F) \subseteq (1, E) \) in \( \mathcal{Y}_3 \) there is a SFS nghbd \( \delta_E \) of \( 0_{\mathcal{Y}_E} \) where \( 0_{\mathcal{Y}_E}(0) = (\delta(0), A), \) \( (0, \phi) \subset (\delta(0), A) \subseteq (\nu(0), F) \) in \( \mathcal{Y}_2 \) such that \( \eta(\delta_E(z)) \subseteq \rho(t)\mu_E(z), \) \( z \in \mathcal{Y}_3. \)

Given \( \delta_E \) there epists a SFS nghbd \( \delta'_E \) of \( 0_{\mathcal{Y}_E} \) such that \( \delta'_E + \delta_E \subseteq \delta_E. \) Suppose that both \( \delta_E \) and \( \delta'_E \) belongs to a system of SFS nghbds \( \mathbb{B}(0_{\mathcal{Y}_E}). \) By the SFS continuity of \( f'(p) \) there is a SFS nghbd \( \gamma_E \) of \( 0_{\mathcal{Y}_E} \) where \( 0_{\mathcal{Y}_E}(0) = (\beta(0), G), \) \( (0, \phi) \subset (\beta(0), G) \subseteq (\delta(0), A) \) in \( \mathcal{Y}_1 \) such that \( f'(p)(\gamma_E)(q) \subseteq \delta'_E(q), \) which implies that \( t^f'(p)(\gamma_E)(q) \subseteq t\delta'_E(q), \) that is \( f'(p)(t(\gamma_E))(q) \subseteq t\delta'_E(q), \) \( q \in \mathcal{Y}_2. \) For everq \( \delta'_E \) there exists a SFS nghbd \( \gamma'_E \) of \( 0_{\mathcal{Y}_E} \) in \( \mathcal{Y}_1 \) for which \( \sigma(t\gamma'_E)(q) \subseteq \rho(t)\delta'_E(q) \) and, for \( |\rho(t)|/t| \leq 1, \rho(t)\delta'_E(q) \subseteq t\delta'_E(q), \) \( q \in \mathcal{Y}_2. \) Let \( \gamma_{1_E} = \gamma_E \cap \gamma'_E \) and using Proposition 3.1, we obtain \( [\sigma(t\gamma_{1_E}) + f'(p)(t\gamma_{1_E})](q) \subseteq t\delta_E(q), \) which implies that \( \eta(\sigma(t\gamma_{1_E}) + f'(p)(t\gamma_{1_E})) \subseteq \eta(t\delta_E) \subseteq \rho(t)\mu_E, \) that is the function \( \eta \circ (f'(p) + \sigma) \) from \( \mathcal{Y}_1 \) to \( \mathcal{Y}_3 \) is SFS tangent to \( 0. \) Thus \( h(p + r) - h(p) = g'(q) \circ f'(p)(r) + \zeta(r), \) \( r \in V_1, \) where \( g'(q) \circ f'(p) \) is linear and SFS continuous, and \( \zeta, \) is SFS tangent to \( 0. \)

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REFERENCES


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