RAYLEIGH WAVE PROPAGATION WITH THE EFFECT OF INITIAL STRESS, MAGNETIC FIELD AND TWO TEMPERATURE IN THE DUAL PHASE LAG THERMOELASTICITY

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ABSTRACT. In the present study the governing equation of generalized thermoelasticity is formulated by considering Lord and Shulman theory under the influence of two temperature, initial stress, magnetic field and diffusion. The equations thus formed are considered for isotropic medium in xy-plane. Surface wave solution method is used to find the solution of these equations. The secular equation for Rayleigh wave thus obtained also satisfies radiation conditions. Effect of two-temperature, magnetic field, diffusion, initial stress, frequency has been shown graphically for a partial material.

1. INTRODUCTION


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2010 Mathematics Subject Classification. 74F05, 53C35.

Key words and phrases. Rayleigh waves, initial stress, magnetic field, dual phase lag, two-temperature.

2. Basic Equations

Following Sherief et al. (2004), the governing equations for a linear, isotropic and homogeneous elastic solid with generalized thermodiffusion at constant temperature $T$ in the absence of body force are:
(i) The displacement-strain relation
\[ e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \]

(ii) The energy equation
\[ -\dot{q}_{i,i} = \rho T_0 \dot{S}. \]

(iii) The modified Fourier’s law
\[ -K_{ij} \Phi_{,j} = q_i + \tau_0 \dot{q}_i. \]

(iv) The equation of motion
\[ \mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta_1 \Theta_{,i} - \beta_2 C_{,i} = \rho \ddot{u}_i. \]

(v) The equation of heat conduction:
\[ \rho c_E (\dot{\Theta} + \tau_0 \ddot{\Theta}) + \beta_1 T_0 (\dot{\Theta} + \tau_0 \ddot{\Theta}) + \alpha T_0 \dot{C} + \tau_0 \dot{C} = K \Phi_{,ii}. \]

(vi) The equation of mass diffusion:
\[ D^* \beta_2 \dot{C}_{,ii} + D^* a \Theta_{,ii} + \dot{C} + \tau \ddot{C} - D^* b C_{,ii} = 0. \]

(vii) Maxwell stresses
\[ \tilde{\sigma}_{ij} \mu_e [H_i h_j + H_j h_i - (H \cdot h) \delta_{ij}]. \]

(viii) The constitutive equations
\[
\begin{align*}
\sigma_{ij} &= 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta_1 \Theta - \beta_2 C), \\
\rho T_0 S &= \rho c_E \Theta + \beta_1 T_0 e_{kk} + a T_0 C, \\
P &= -\beta_2 e_{kk} + b C - a \Theta,
\end{align*}
\]

where \( \rho \) is the density, \( \lambda, \mu \) are constant of Lame, \( \sigma_{ij} \) is the stress tensor, \( e_{ij} \) is the strain tensor, \( u_i \) is the displacement vector, \( w_{ij} \) is the rotation tensor, \( S \) is the entropy per unit mass, \( C \) is the mass-concentration, \( c_E \) is the specific-heat at constant strain, \( K \) is the thermal-conductivity, \( D^* \) is the thermal diffusion, \( \tau_0 \) is thermal relaxation, \( \tau \) is diffusion relaxation time, \( a^* > 0 \) is the two-temperature parameter, \( a, b \) are the thermal diffusion effects and diffusive effects, \( p_0 \) is the initial stress parameter, \( h \) is the perturbed magnetic field over, \( j \) is the electric current density, \( \mu_e \) is the magnetic permeability, \( h = \nabla \times (\mu \times H_0) \) and \( H = H_0 + h \), \( \beta_1 = (3\lambda + 2\mu)\alpha_t \) and \( \beta_2 = (3\lambda + 2\mu)\alpha_c \), \( \alpha_t \) is the Linear thermal-expansion, \( \alpha_c \) is the diffusion expansion.
The temperature relation:

\begin{equation}
\Phi - \Theta = a^{*}\Phi_{(ii)},
\end{equation}

and \( \Theta = T - T_0 \) is the small temperature increment, \( T \) is the absolute temperature, \( T_0 \) is the reference uniform temperature such that \( \frac{T}{T_0} < < 1 \). \( \Phi \) is the conductive temperature and \( a^{*} \) is the parameter of two temperature.

3. FORMULATION OF PROBLEM AND SOLUTION

We consider a isotropic and homogenous medium of Rayleigh wave with initial stress, diffusion and magneto-thermo-elastic half-space of an infinite extent with Cartesian coordinate \((x,y,z)\) which is at uniform temperature previously. The origin is being taken on plane surface and consider \((z > 0)\) normal into the medium. We consider the plane stress parallel to the \(x-z\) plane in the present study and the displacement vector is \((u_1, 0, u_3)\)

\begin{equation}
u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x},\end{equation}

By using of equation (2.4) and (3.1), after solving the equations from (2.1)-(2.3) then we have

\begin{equation}(\mu - \frac{p_0}{2})(\psi_{,11} + \psi_{,33}) = \rho \ddot{\psi},\end{equation}

\begin{equation}(\lambda + 2\mu + \mu_e H_0^2)(\phi_{,11} + \phi_{,33}) - \beta_1 \Theta - \beta_2 C = \rho \ddot{\phi},\end{equation}

\begin{equation}K(\Phi_{,11} + \Phi_{,33}) = \rho c_T \tau_m \dot{\Phi} + \beta_1 T_0 \tau_m \frac{\partial}{\partial t} \nabla^2 \phi + a T_0 \tau_m \dot{C},\end{equation}

\begin{equation}D*\beta_2 \nabla^2 (\phi_{,11} + \phi_{,33}) + D*a(\Theta_{,11} + \Theta_{,33}) - D*b(C_{,11} + C_{,33}) + \tau_n \dot{C} = 0,\end{equation}

where \( \tau_m = 1 + \tau_n \frac{\beta}{M} \) and \( \tau_n = 1 + \tau_n \frac{\beta}{M} \). Surface waves moves along the \(x\)-axis in magneto-thermo-elastic with isotropic medium, so potential functions \( \psi, \phi, \Phi, C \) are considered in the form of which are given as:

\begin{equation}[\psi, \phi, \Phi, C] = [\hat{\psi}(z), \hat{\phi}(z), \hat{\Phi}(z), \hat{C}(z)]e^{i(nz - \chi t)}.
\end{equation}

Now put the value of \( \psi \) in equation (3.2) from equation(3.6), we have

\begin{equation}\frac{r_4^2}{\eta^2} = 1 - \frac{\rho c^2}{\mu - p_2/2}.
\end{equation}
Again, after put the value of $\phi, \Phi, C$ in equations (3.3)-(3.5) from (3.6), then we get the following non trivial solution

$$D^6 - LD^4 + MD^2 - N = 0,$$

where $L = 3\eta^2 - [(K - a^*)(\chi^2 \gamma_n \cdot \frac{\lambda + 2\mu + \mu_H^2}{\rho} - D^*a\chi^2 - \beta_2 D^* \beta_2)$$

$- D^*a(\frac{\lambda + 2\mu + \mu_H^2}{\rho} + \beta_1 \epsilon_1) + D^*(\chi^2 \epsilon_2 a^* - \epsilon_2 \frac{\lambda + 2\mu + \mu_H^2}{\rho})$

$+ \bar{\beta}_2 D^* \beta_2 a^*)/[D^*a(a^* \epsilon_1 \bar{\beta}_1 - \frac{\lambda + 2\mu + \mu_H^2}{\rho} (K - a^*))$

$+ a^* D^*b(\frac{\lambda + 2\mu + \mu_H^2}{\rho} \epsilon_2 + \bar{\beta}_2 \epsilon_1)],$

$M = 3\eta^2 - 2\eta^2[(K - a^*)(\chi^2 \gamma_n \cdot \frac{\lambda + 2\mu + \mu_H^2}{\rho} - D^*a\chi^2 - \beta_2 D^* \beta_2)$

$- D^*a(\frac{\lambda + 2\mu + \mu_H^2}{\rho} + \beta_1 \epsilon_1) + D^*(\chi^2 \epsilon_2 a^* - \epsilon_2 \frac{\lambda + 2\mu + \mu_H^2}{\rho})$

$+ \bar{\beta}_2 D^* \beta_2 a^*) + \tau_n^*((K - a^*)\chi^4 + \beta_1 \epsilon_1 + \frac{\lambda + 2\mu + \mu_H^2}{\rho})$

$- \chi^2(D^*a + \epsilon_2 D^*b) - D^*(\beta_2 + \beta_1 \epsilon_1)/[D^*a(a^* \epsilon_1 \bar{\beta}_1)$

$- \frac{\lambda + 2\mu + \mu_H^2}{\rho} (K - a^*)) + a^* D^*b(\frac{\lambda + 2\mu + \mu_H^2}{\rho} \epsilon_2 + \bar{\beta}_2 \epsilon_1)],$

$N = \eta^6 - 2\eta^2[(K - a^*)(\chi^2 \gamma_n \cdot \frac{\lambda + 2\mu + \mu_H^2}{\rho} - D^*a\chi^2 - \beta_2 D^* \beta_2)$

$- D^*a(\frac{\lambda + 2\mu + \mu_H^2}{\rho} + \beta_1 \epsilon_1) + D^*(\chi^2 \epsilon_2 a^* - \epsilon_2 \frac{\lambda + 2\mu + \mu_H^2}{\rho})$

$+ \bar{\beta}_2 D^* \beta_2 a^*) + \tau_n^*((K - a^*)\chi^4 + \beta_1 \epsilon_1 + \frac{\lambda + 2\mu + \mu_H^2}{\rho})$

$- \chi^2(D^*a + \epsilon_2 D^*b) - D^*(\beta_2 + \beta_1 \epsilon_1)/[D^*a(a^* \epsilon_1 \bar{\beta}_1)$

$- \frac{\lambda + 2\mu + \mu_H^2}{\rho} (K - a^*)) + a^* D^*b(\frac{\lambda + 2\mu + \mu_H^2}{\rho} \epsilon_2 + \bar{\beta}_2 \epsilon_1)],$

$\epsilon_1 = \frac{\beta_1 \gamma_0}{\rho \epsilon_2}, \epsilon_2 = \frac{\gamma_0}{\epsilon_2}, K = \frac{K}{\chi \epsilon_2}, \tau_m' = \frac{\tau_m}{\chi}, \tau_n' = \frac{\tau_n}{\chi}$

Some general solutions of equations (3.2) to (3.5) are given as

$$\psi(z) = [E \exp(-r_4 z) + E' \exp(r_4 z)] e^{(\eta x - \chi^2 t)},$$

$$\phi(z) = \sum_{i=1}^{3} [F_i \exp(-r_i z) + F_i' \exp(r_i z)] e^{(\eta x - \chi^2 t)},$$

$$\Phi(z) = \sum_{i=1}^{3} [G_i \exp(-r_i z) + G_i' \exp(r_i z)] e^{(\eta x - \chi^2 t)},$$

$$C(z) = \sum_{i=1}^{3} [H_i \exp(-r_i z) + H_i' \exp(r_i z)] e^{(\eta x - \chi^2 t)},$$

where $E, F_i, G_i, H_i, E', F_i', G_i', H_i'$ are the arbitrary constants. In general the roots are complex, therefore W.L.O.G we assume Real($r_1)$ $> 0$. With the help
of the $\psi(z), \phi(z), \Phi(z), C(z) \to 0$ as $z \to \infty$ as. Then the solution (3.7)-(3.10) reduces to the particular solution in the half space if $z > 0$

\begin{align}
\psi(z) &= E \exp(-r_4 z) e^{i(\eta x - \chi t)}, \\
\phi(z) &= \sum_{i=1}^{3} F_i \exp(-r_i z) e^{i(\eta x - \chi t)}, \\
\Phi(z) &= \sum_{i=1}^{3} P_i F_i \exp(-r_i z) e^{i(\eta x - \chi t)}, \\
C(z) &= \sum_{i=1}^{3} P_i^* F_i \exp(-r_i z) e^{i(\eta x - \chi t)},
\end{align}

here $G_i = P_i F_i$ and $H_i = P_i^* F_i$ and

\begin{align}
P_i &= \eta^2 \frac{\epsilon_2 [\mu - p_0] (-1 + \frac{r_i^2}{\eta^2}) + \beta_2 \epsilon_1 (-1 + \frac{r_i^2}{\eta^2})}{\epsilon_2 \beta_1 [1 - a^* \eta^2 (-1 + \frac{r_i^2}{\eta^2})] - \beta_1 [K \eta^2 (-1 + \frac{r_i^2}{\eta^2}) + 1 - a^* \eta^2 (-1 + \frac{r_i^2}{\eta^2})]}, \\
P_i^* &= \eta^2 \frac{\beta_1 D^* \beta_2 (-1 + \frac{r_i^2}{\eta^2})^2 + D^* a \frac{\mu - p_0}{\rho} (-1 + \frac{r_i^2}{\eta^2}) + \chi^2}{\beta_2 D^* b (-1 + \frac{r_i^2}{\eta^2}) + \frac{\chi^2}{\eta^2]} + \beta_2 D^* a (-1 + \frac{r_i^2}{\eta^2})} (i = 1, 2, 3).
\end{align}

4. Boundary Condition

The appropriate boundary conditions at stress free surface $z = 0$ are

\begin{align}
\sigma_{zz} + \bar{\sigma}_{zz} &= 0, \quad \sigma_{xx} + \bar{\sigma}_{xx} = 0, \quad \frac{\partial P}{\partial z} = 0, \quad \frac{\partial \Theta}{\partial z} + h \Theta = 0.
\end{align}

Thus, applying the boundary condition (4.1) in equation equations (3.11) to (3.14) then find the homogeneous system of equation $E, F_1, F_2$ and $F_3$

\begin{align}
4 \mu \frac{r_4}{\eta} - \frac{h}{\eta} \left[ r_1 S_2 r_3 Z_3 - S_3 r_2 Z_2 - r_2 S_1 r_3 Z_3 - S_3 r_1 Z_1 + r_3 S_1 r_2 Z_1 - S_2 r_1 Z_2 \right] \\
- \left[ (\mu + \frac{p_0}{2}) + \frac{r_4}{\eta} (\mu - \frac{p_0}{2}) - \frac{h}{\eta} \right] \left[ R_1 S_2 r_3 Z_3 - S_3 r_2 Z_2 - R_2 S_1 r_3 Z_3 - S_3 r_1 Z_1 + R_3 S_1 r_2 Z_1 - S_2 r_1 Z_2 \right]
\end{align}
where

\[
R_i = -(\lambda + \mu e H_0^2) + (\lambda + 2\mu e H_0^2) \frac{r_i^2}{\eta^2} - \beta_1 [1 - a^* (-1 + \frac{r_i^2}{\eta^2})] \frac{P_i^2}{\eta^2} - \beta_2 \frac{P_i^*}{\eta^2},
\]

\[(i = 1, 2, 3)\]

\[
S_i = \beta_2 (-1 + \frac{r_i^2}{\eta^2}) + b \frac{P_i^*}{\eta^2} - a [1 - a^* (-1 + \frac{r_i^2}{\eta^2})] \frac{P_i}{\eta^2}.
\]

\[
Z_i = 1 - a^* (-1 + \frac{r_i^2}{\eta^2}).
\]

Then the expression (4.2) is known as frequency or dispersive equation of propagating Rayleigh-Surface waves with magnetic, diffusion and initial stress in two-temperature-thermo-elastic medium along with dual-phase lag.

5. Special Case

For thermally insulated case \((h \to 0)\) and in the absence of initial stress, magnetic field, two temperature, Diffusion, the frequency equation (4.2) reduces to

\[
(2 - \frac{c_1^2}{c_2^2})^2 = \sqrt{4(1 - \frac{c_1^2}{c_2^2})(1 - \frac{c_2^2}{c_2^2})},
\]

where \(X_i, Y_i, P_i, P_i^*, R_i, S_i, Z_i\) calculated accordingly. Expression (5.1) is the Rayleigh wave speed equation for elastic medium.

6. Numerical results and Discussion

For the calculation of propagation of non dimensional speed of Rayleigh-waves, following Kumari and Singh (2016) the constant are taken for two-temperature thermo-elasticity solid half-space with diffusion, such as

\[
T_0 = 300 K, K = 0.494 \times 10^3 W.m^{-1}.s^{-1}.deg^{-1} \lambda = 5.775 \times 10^{10} N.m^2, c_E = 2.361 \times 10^2 J.Kg^{-1}.deg^{-1}, \tau_0 = 0.05s, a = 0.005, b = 0.05, \rho = 2.7 \times 10^3 Kg.m^{-3}.
\]

We obtain the dimensionless wave speed of Propagation of Rayleigh wave from equation (4.2) for different range of diffusion relaxation time \((\tau)\), thermos-diffusion \((D^*)\), two-temperature \(a^*\), frequency \((\chi)\), which is solved by the method of Fortran iteration method, using with the above value of physical different parameter.
Figure 1 shows that the speed is plotted against frequency for the different values of the $H_0=-10,0,10$. It shows that for the value of magnetic field -10,10 speed sharply decreases and then remain constant for the increase of the frequency. For the magnetic field 0 the value slowly decreases and then remains constant for the increase of the frequency.

Figure 2 Shows that the speed plotted against the Magnetic field for the different values of the two temperature parameter $a^*=0.1,0.5,0.9$. It shows that the speed remains constant for the values of two temperature parameter 0.1 and 0.5. For the value of two temp. parameter 0.9 it slowly decrease and then slowly increase with increase of the Magnetic field.
Figure 3. Speed Plotted against Diffusion

Figure 3 Shows that the speed is plotted against the Diffusion parameter for the different values of the magnetic field. Speed remains constant for all the values of magnetic field against the diffusion parameter. It shows the comparison for the different values of magnetic field parameter \((-10, 0, 10)\).

7. Conclusion

The general surface wave solutions of the governing equations of isotropic initial stress, magnetic field, two-temperature and diffusion thermo-elasticity are obtained. With the help of suitable radiation conditions, the general solutions are reduced to particular solutions in the half-space. The particular solutions satisfy the required boundary conditions at stress free thermally insulated or isothermal surface and we obtain the frequency equation of Rayleigh wave. Some particular cases of the frequency equation are derived. In absence of initial stress, Magnetic field, diffusion and two temperature parameters, the frequency equation reduces to the classical isotropic elastic case. The frequency equation is approximated for numerical purpose and then solved numerically for a particular model of the material. The non-dimensional speed of propagation is plotted against the frequency, Initial stress, Magnetic field, two-temperature and diffusion parameters. The numerical results describe the effects of frequency, initial stress, magnetic field, two-temperature and diffusion on the non-dimensional speed of propagation.
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