BIPOLAR VALUED FUZZY $d$-ALGEBRA

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ABSTRACT. In this paper, we introduce and study the concept of bipolar fuzzy subalgebra of $d$-algebra and we characterize bipolar fuzzy subalgebra to the crisp $d$-algebra. Further, we discuss the relation between bipolar fuzzy subalgebra and their level cuts. Also, we prove that the homomorphic image and inverse image of a bipolar fuzzy subalgebra is a bipolar fuzzy subalgebra.

1. INTRODUCTION

The concept of fuzzy subsets of a set was introduced by Zadeh, L.A. [9] in 1965. After that, there are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, etc. In fuzzy sets the membership degree of elements range over the interval $[0,1]$. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set and membership degree 0 indicates that an element does not belong to fuzzy set. The membership degrees on the interval $(0,1)$ indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. The concept of bipolar-valued fuzzy sets, first introduced by Zhang, W.R. [10] in 1994, is an

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extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree [-1,0) indicates that elements somewhat satisfy the implicit counter-property.

Neggers, J. and Kim, H.S. [8] introduced and studied the concept of $d$-algebra, which is another generalization of BCK-algebras and investigated relations between $d$-algebras and BCK-algebras. After that, Jun, Y.B., Neggers, J. and Kim, H.S. [7] introduced the concepts of fuzzy $d$-subalgebra, fuzzy $d$-ideal and fuzzy $d^*$-ideal, and investigated relations among them. Further, they discussed $d$-ideals in $d$-algebras.

In this paper, we introduce and study the concept of bipolar fuzzy subalgebra of $d$-algebra and we characterize bipolar fuzzy subalgebra to the crisp $d$-algebra. Further, we discuss the relation between bipolar fuzzy $d$-algebra and their level cuts. Also, we prove that the homomorphic image and inverse image of a bipolar fuzzy subalgebra is a bipolar fuzzy subalgebra.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

**Definition 2.1.** ([8]) A nonempty set $X$ with a constant 0 and a binary operation $*$ is called a $d$-algebra, if for all $x, y \in X$ it satisfies the following axioms:

$(dA1)$ $x * x = 0$

$(dA2)$ $0 * x = 0$

$(dA3)$ $x * y = 0$ and $y * x = 0 \Rightarrow x = y.$

**Definition 2.2.** ([8]) Let $Y$ be a non-empty subset of a $d$-algebra $X$, then $Y$ is called subalgebra of $X$ if $x * y \in Y$, for all $x, y \in Y$.

**Definition 2.3.** ([8]) Let $X$ and $Y$ be two $d$-algebras. A mapping $f : X \to Y$ is called a homomorphism if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$.

**Definition 2.4.** ([9]) Let $X$ be a non-empty set. A fuzzy subset $\mu$ of the set $X$ is a mapping $\mu : X \to [0, 1]$. 
Definition 2.5. ([10]) Let $X$ be the universe of discourse. A bipolar-valued fuzzy set $\mu$ in $X$ is an object having the form $\mu = \{x, \mu^-(x), \mu^+(x)/x \in X\}$, where $\mu^- : X \rightarrow [-1, 0]$ and $\mu^+ : X \rightarrow [0, 1]$ are mappings.

For the sake of simplicity, we shall use the symbol $\mu = (X; \mu^-, \mu^+)$ for the bipolar-valued fuzzy set $\mu = \{x, \mu^-(x), \mu^+(x)/x \in X\}$ and use the notion of bipolar fuzzy sets instead of the notion of bipolar-valued fuzzy sets.

Definition 2.6. ([10]) Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set and $s \times t \in [-1, 0] \times [0, 1]$, the sets $\mu_s^N = \{x \in X/\mu^-(x) \leq s\}$ and $\mu_t^P = \{x \in X/\mu^+(x) \geq t\}$ are called negative $s$-cut and positive $t$-cut respectively. For $s \times t \in [-1, 0] \times [0, 1]$, the set $\mu_{(s,t)} = \mu_s^N \cap \mu_t^P$ is called $(s,t)$-set of $\mu = (X; \mu^-, \mu^+)$. 

Definition 2.7. Let $\mu = (X; \mu^-, \mu^+)$ and $\sigma = (X; \sigma^-, \sigma^+)$ be two bipolar fuzzy sets of a universe of discourse $X$.

The intersection of $\mu$ and $\sigma$ is defined as

$$(\mu^- \cap \sigma^-)(x) = \min\{\mu^-(x), \sigma^-(x)\} \text{ and } (\mu^+ \cap \sigma^+)(x) = \min\{\mu^+(x), \sigma^+(x)\}.$$ 

The union of $\mu$ and $\sigma$ is defined as

$$(\mu^- \cup \sigma^-)(x) = \max\{\mu^-(x), \sigma^-(x)\} \text{ and } (\mu^+ \cup \sigma^+)(x) = \max\{\mu^+(x), \sigma^+(x)\}.$$ 

A bipolar set $\mu$ is contained in another bipolar set $\sigma$, $\mu \subseteq \sigma$ if and only if $\mu^-(x) \geq \sigma^-(x)$ and $\mu^+(x) \leq \sigma^+(x)$, for all $x \in X$.

Definition 2.8. Let $f : X \rightarrow Y$ be a homomorphism from a set $X$ onto a set $Y$ and let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set of $X$ and $\sigma = (Y; \sigma^-, \sigma^+)$ be two bipolar fuzzy set of $Y$, then the homomorphic image $f(\mu)$ of $\mu$ is $f(\mu) = ((f(\mu))^-, (f(\mu))^+)$ defined as for all $y \in Y$.

$$(f(\mu))^-(x) = \left\{ \begin{array}{ll}
\max\{\mu^-(x)/x \in f^{-1}(y) & \text{if } f^{-1}(y) \neq \emptyset \\
0 & \text{otherwise}
\end{array} \right.$$ 

and

$$(f(\mu))^+(x) = \left\{ \begin{array}{ll}
\max\{\mu^-(x)/x \in f^{-1}(y) & \text{if } f^{-1}(y) \neq \emptyset \\
0 & \text{otherwise}
\end{array} \right.$$ 

The pre-image $f^{-1}(\sigma)$ of $\sigma$ under $f$ is a bipolar set defined as $(f^{-1}(\sigma))^-(x) = \sigma^-(f(x))$ and $(f^{-1}(\sigma))^+(x) = \sigma^+(f(x))$, for all $x \in X$. 
3. Bipolar Fuzzy $d$-algebra

In this section, we introduce and study the concept of bipolar fuzzy subalgebra of $d$-algebra and we characterize bipolar fuzzy subalgebra to the crisp $d$-algebra. Further, we prove that the homomorphic image and inverse image of a bipolar fuzzy subalgebra is a bipolar fuzzy subalgebra.

Throughout this section $X$ stands for a $d$-algebra unless otherwise mentioned.

Now, we introduce the following.

**Definition 3.1.** A Bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ in $X$ is called a bipolar fuzzy subalgebra if it satisfies the following properties: for any $x, y \in X$,

(i). $\mu^+(x \ast y) \geq \min\{\mu^+(x), \mu^+(y)\}$

(ii). $\mu^-(x \ast y) \leq \max\{\mu^-(x), \mu^-(y)\}$.

**Example 1.** Consider a $d$-algebra $X = \{0, 1, 2, 3\}$ with the following Cayley table

<table>
<thead>
<tr>
<th>*</th>
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</table>

Define a bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$, where $\mu^- : X \to [-1, 0]$ and $\mu^+ : X \to [0, 1]$ as

$$\mu^-(x) = \begin{cases} -0.7 & \text{when } x = 0 \\ -0.2 & \text{when } x \neq 0 \end{cases}$$

and

$$\mu^+(x) = \begin{cases} 0.8 & \text{when } x = 0 \\ 0.1 & \text{when } x \neq 0 \end{cases}.$$  

Then $\mu$ is a bipolar fuzzy subalgebra.

**Proposition 3.1.** If $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra of $X$, then $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$, for all $x \in X$.

**Proof.** Let $x \in X$. Now, $\mu^-(0) = \mu^-(x \ast x) \leq \max\{\mu^-(x), \mu^-(x)\} = \mu^-(x)$ and $\mu^+(0) = \mu^+(x \ast x) \geq \min\{\mu^+(x), \mu^+(x)\} = \mu^+(x)$. Thus $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$.

□
Theorem 3.1. Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set. Then the two level cuts $\mu^N_{s_1}$, $\mu^P_{t_1}$, and $\mu^N_{s_2}$, $\mu^P_{t_2}$ are equal i.e., $\mu^N_{s_1} = \mu^N_{s_2}$ and $\mu^P_{t_1} = \mu^P_{t_2}$ if and only if there is no $x \in X$ such that $s_1 \geq \mu^-(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$.

Proof. Suppose $\mu^N_{s_1} = \mu^N_{s_2}$ and $\mu^P_{t_1} = \mu^P_{t_2}$. Suppose if possible there exist $x \in X$ such that

$$s_1 \geq \mu^-(x) \geq s_2 \quad \text{and} \quad t_1 \leq \mu^+(x) \leq t_2.$$ 

Now, $\mu^-(x) \leq s_1 \Rightarrow x \in \mu^N_{s_1} \Rightarrow \mu^-(x) \leq s_2$ and $\mu^+(x) \leq t_2 \Rightarrow x \in \mu^P_{t_2} \Rightarrow \mu^+(x) \leq t_1$, which is a contradiction. Thus there is no $x \in X$ such that $s_1 \geq \mu^-(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$.

Conversely, suppose that there is no $x \in X$ such that $s_1 \geq \mu^-(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$. Suppose $\mu^N_{s_1} \neq \mu^N_{s_2}$ and $\mu^P_{t_1} \neq \mu^P_{t_2}$. That implies there exist $x \in \mu^N_{s_1} \& x \notin \mu^N_{s_2}$ and there exist $y \in \mu^P_{t_1} \& y \notin \mu^P_{t_2}$. This implies $\mu^-(x) \leq s_1 \& \mu^-(x) \geq s_2$ and $\mu^+(x) \geq t_1 \& \mu^+(x) \leq t_2$, i.e., $s_1 \geq \mu^+(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$. Which is a contradiction. Thus $\mu^N_{s_1} = \mu^N_{s_2}$ and $\mu^P_{t_1} = \mu^P_{t_2}$. \qed

Theorem 3.2. A bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ of $X$ is a bipolar fuzzy subalgebra of $X$ if and only if the level cuts are subalgebras i.e., for all $s \times t \in [-1, 0] \times [0, 1], \emptyset \neq \mu^N_{s}$ and $\emptyset \neq \mu^P_{t}$ are subalgebras of $X$.

Proof. Suppose $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra. Let $s \times t \in [-1, 0] \times [0, 1]$ such that $\mu^N_{s} \neq \emptyset$ and $\mu^P_{t} \neq \emptyset$. Let $x, y \in \mu^P_{t}$ and $g, h \in \mu^N_{s}$. Therefore $\mu^+(x) \geq t, \mu^+(y) \geq t, \mu^-(g) \leq s$ and $\mu^-(h) \leq s$. Since $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra, we have

\begin{itemize}
  \item [(i)] $\mu^+(x \ast y) \geq \min\{\mu^+(x), \mu^+(y)\} \geq t$
  \item [(ii)] $\mu^-(x \ast y) \leq \max\{\mu^-(x), \mu^-(y)\} \geq s \Rightarrow x \ast y \in \mu^P_{t}$ and $g \ast h \in \mu^N_{s}$.
\end{itemize}

Thus $\mu^N_{s}$ and $\mu^P_{t}$ are subalgebras of $X$.

Conversely suppose that the level cuts $\mu^N_{s}$ and $\mu^P_{t}$ are subalgebras of $X$. Let $x, y \in X$. Then $\mu^+(x), \mu^+(y) \in [0, 1]$ and $\mu^-(x), \mu^-(y) \in [-1, 0]$. Choose $t = \min\{\mu^+(x), \mu^+(y)\}$ and $s = \max\{\mu^-(x), \mu^-(y)\}$. That implies $\mu^+(x) \geq t, \mu^-(y) \geq t$ and $\mu^-(x) \leq s$, i.e., $x, y \in \mu^P_{t}$ and $x, y \in \mu^N_{s}$, and further $x \ast y \in \mu^P_{t}$ and $x \ast y \in \mu^N_{s}$, $\mu^+(x \ast y) \geq \min\{\mu^+(x), \mu^+(y)\}$ and $\mu^-(x \ast y) \leq \max\{\mu^-(x), \mu^-(y)\}$.

Thus $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra. \qed

Theorem 3.3. Let $Y$ be a subalgebra of $X$, then for any $s \times t \in [-1, 0] \times [0, 1]$ there exist a bipolar fuzzy subalgebra $\mu$ of $X$ such that $\mu^N_{s} = Y$ and $\mu^P_{t} = Y$. 

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Clearly \( \mu_s = Y \) and \( \mu_t = Y \) Let \( x, y \in X \). If \( x, y \in Y \), then \( x \ast y \in Y \). So, \( \mu^-(x) = \mu^-(y) = \mu^-(x \ast y) = -1 \) and \( \mu^+(x) = \mu^+(y) = \mu^+(x \ast y) = t \). Therefore, (i) \( \mu^-(x \ast y) \leq \max\{\mu^-(x), \mu^-(y)\} \) and (ii) \( \mu^+(x \ast y) \geq \min\{\mu^+(x), \mu^+(y)\} \).

If \( x, y \notin Y \), then \( \mu^-(x) = \mu^-(y) = s \) and \( \mu^+(x) = \mu^+(y) = 0 \). Therefore, (i) \( \mu^-(x \ast y) \leq \max\{\mu^-(x), \mu^-(y)\} \) and (ii) \( \mu^+(x \ast y) \geq \min\{\mu^+(x), \mu^+(y)\} \).

If at most one of \( x, y \in Y \), then at least one of \( \mu^-(x) \& \mu^-(y) \) is equal to \( s \) and \( \mu^+(x) \& \mu^+(y) \) is equal to \( 0 \). Therefore, (i) \( \mu^-(x \ast y) \leq \max\{\mu^-(x), \mu^-(y)\} \) and (ii) \( \mu^+(x \ast y) \geq \min\{\mu^+(x), \mu^+(y)\} \).

Thus \( \mu \) bipolar fuzzy subalgebra of \( X \) such that \( \mu^N = Y \) and \( \mu^P = Y \). □

**Theorem 3.4.** If \( \mu = (X; \mu^-, \mu^+) \) and \( \sigma = (X; \sigma^-, \sigma^+) \) are two bipolar fuzzy subalgebras, then \( \mu \cap \sigma \) is a bipolar fuzzy subalgebra.

**Proof.** Let \( x, y \in X \). Now,

\[
(\mu^- \cap \sigma^-)(x \ast y) = \min\{\mu^-(x \ast y), \sigma^-(x \ast y)\}
\leq \min\{\max\{\mu^-(x), \mu^-(y)\}, \max\{\sigma^-(x), \sigma^-(y)\}\}
\leq \min\{\max\{\mu^-(x), \sigma^-(x)\}, \max\{\mu^-(y), \sigma^-(y)\}\}
= \min\{(\mu^- \cap \sigma^-)(x), (\mu^- \cap \sigma^-)(y)\}.
\]

Also,

\[
(\mu^+ \cap \sigma^+)(x \ast y) = \min\{\mu^+(x \ast y), \sigma^+(x \ast y)\}
\geq \min\{\min\{\mu^+(x), \mu^+(y)\}, \min\{\sigma^+(x), \sigma^+(y)\}\}
\geq \min\{\max\{\mu^+(x), \sigma^+(x)\}, \max\{\mu^+(y), \sigma^+(y)\}\}
= \min\{(\mu^+ \cap \sigma^+)(x), (\mu^+ \cap \sigma^+)(y)\}.
\]

Thus \( \mu \cap \sigma \) is bipolar fuzzy subalgebra. □
Corollary 3.1. The intersection of arbitrary family of bipolar fuzzy subalgebras is a bipolar fuzzy subalgebra.

In general union of two fuzzy $\Gamma$-semirings may not be a fuzzy $\Gamma$-semiring.

Example 2. Consider a $d$-algebra $X = \{0, 1, 2\}$ with the following Cayley table

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<tr>
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<tbody>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>2</td>
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<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Define a bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$, where $\mu^- : X \to [-1, 0]$ and $\mu^+ : X \to [0, 1]$ as

$$
\mu^-(x) = \begin{cases} 
-0.7 & \text{if } x = 0 \\
-0.5 & \text{if } x = 1 \\
-0.4 & \text{if } x = 2 
\end{cases} \quad \text{and} \quad
\mu^+(x) = \begin{cases} 
0.9 & \text{if } x = 0 \\
0.8 & \text{if } x = 1 \\
0.6 & \text{if } x = 2
\end{cases}.
$$

Define a bipolar fuzzy set $\sigma = (X; \sigma^-, \sigma^+)$, where $\sigma^- : X \to [-1, 0]$ and $\sigma^+ : X \to [0, 1]$ as

$$
\sigma^-(x) = \begin{cases} 
-0.7 & \text{if } x = 0 \\
-0.5 & \text{if } x = 1 \\
-0.4 & \text{if } x = 2 
\end{cases} \quad \text{and} \quad
\sigma^+(x) = \begin{cases} 
0.8 & \text{if } x = 0 \\
0.7 & \text{if } x = 1 \\
0.5 & \text{if } x = 2
\end{cases}.
$$

Clearly, $\mu$ and $\sigma$ are bipolar fuzzy subalgebras. Here $(\mu^+ \cup \sigma^+)(1 \ast 0) = 0.6$ is not greater than or equal to $0.8 = \min \{\mu^+ \cup \sigma^+)(1), (\mu^+ \cup \sigma^+)(0)\}$. Therefore $\mu^+ \cup \sigma^+$ is not a bipolar fuzzy subalgebra. Thus union of bipolar fuzzy subalgebras is not a bipolar fuzzy subalgebra.

In particular, we have the following theorem.

Theorem 3.5. Let $\mu = (X; \mu^-, \mu^+)$ and $\sigma = (X; \sigma^-, \sigma^+)$ be two bipolar fuzzy subalgebras, then $\mu \cup \sigma$ is a bipolar fuzzy sub algebra only if $\mu \subseteq \sigma$ or $\sigma \subseteq \mu$. 
Proof. Suppose $\mu \subseteq \sigma$. Let $x, y \in X$. Now,

\[(\mu^- \cap \sigma^-)(x \ast y) = \max\{\mu^-(x \ast y), \sigma^-(x \ast y)\}\]
\[= \sigma^-(x \ast y)\]
\[\leq \max\{\sigma^-(x), \sigma^-(y)\}\]
\[\leq \max\{\max\{\mu^-(x), \sigma^-(x)\}, \max\{\mu^+(y), \sigma^-(y)\}\}\]
\[= \max\{(\mu^- \cup \sigma^-)(x), (\mu^- \cup \sigma^-)(y)\}.\]

Also,

\[(\mu^+ \cap \sigma^+)(x \ast y) = \max\{\mu^+(x \ast y), \sigma^+(x \ast y)\}\]
\[= \sigma^+(x \ast y)\]
\[\geq \min\{\sigma^+(x), \sigma^+(y)\}\]
\[\geq \min\{\max\{\mu^+(x), \sigma^+(x)\}, \max\{\mu^+(y), \sigma^+(y)\}\}\]
\[= \max\{(\mu^+ \cap \sigma^+)(x), (\mu^+ \cap \sigma^+)(y)\}.\]

Similarly, we can prove if $\sigma \subseteq \mu$. Thus $\mu \cup \sigma$ is bipolar fuzzy subalgebra. 

Theorem 3.6. Let $f$ be a homomorphism from a $d$-algebra $X$ onto a $d$-algebra $Y$. Let $\sigma$ be a bipolar fuzzy subalgebra of $Y$, then the pre-image $f^{-1}(\sigma)$ of $\sigma$ is a bipolar fuzzy subalgebra of $X$.

Proof. Let $x, y \in X$. Now,

\[(f^{-1}(\sigma))^- (x \ast y) = \sigma^-(f(x \ast y))\]
\[\leq \sigma^-(f(x) \ast f(y))\]
\[\leq \max\{\sigma^-(f(x)), \sigma^-(f(y))\}\]
\[= \max\{(f^{-1}(\sigma))^-(x), (f^{-1}(\sigma))^-(y)\}.\]

Also,

\[(f^{-1}(\sigma))^+(x \ast y) = \sigma^+(f(x \ast y))\]
\[\geq \sigma^+(f(x) \ast f(y))\]
\[\geq \min\{\sigma^+(f(x)), \sigma^+(f(y))\}\]
\[= \min\{(f^{-1}(\sigma))^+(x), (f^{-1}(\sigma))^+(y)\}.\]

Thus $f^{-1}(\sigma)$ is a bipolar fuzzy subalgebra of $X$. 

Theorem 3.7. Let $f$ be a homomorphism from a $d$-algebra $X$ onto a $d$-algebra $Y$. Let $\mu$ be a bipolar fuzzy subalgebra of $X$, then the homomorphic image $f(\mu)$ of $\mu$ is a bipolar fuzzy subalgebra of $Y$.

Proof. Let $x, y \in Y$. Suppose neither $f^{-1}(x)$ nor $f^{-1}(y)$ is non-empty. Since $f$ is homomorphism and so there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = y$ it follows that $a * b \in f^{-1}(x * y)$. Now,

$$(f(\mu))^{-}(x * y) = \max\{\mu^{-}(z)/z \in f^{-1}(x * y)\}$$

$$\leq \max\{\mu^{-}(a * b)/a \in f^{-1}(x), b \in f^{-1}(y)\}$$

$$\leq \max\{\max\{\mu^{-}(a), \mu^{-}(b)\}/a \in f^{-1}(x), b \in f^{-1}(y)\}$$

$$= \max\{\max\{\mu^{-}(a)/a \in f^{-1}(x)\}, \max\{\mu^{-}(b)/b \in f^{-1}(y)\}\}$$

$$= \max\{(f(\mu))^{-}(x), (f(\mu))^{-}(y)\}.$$

Also,

$$(f(\mu))^{+}(x * y) = \max\{\mu^{+}(z)/z \in f^{-1}(x * y)\}$$

$$\geq \max\{\mu^{+}(a * b)/a \in f^{-1}(x), b \in f^{-1}(y)\}$$

$$\geq \max\{\min\{\mu^{+}(a), \mu^{+}(b)\}/a \in f^{-1}(x), b \in f^{-1}(y)\}$$

$$= \min\{\max\{\mu^{+}(a)/a \in f^{-1}(x)\}, \max\{\mu^{+}(b)/b \in f^{-1}(y)\}\}$$

$$= \min\{(f(\mu))^{+}(x), (f(\mu))^{+}(y)\}.$$

Thus $f(\mu)$ is a bipolar fuzzy subalgebra of $Y$. \qed

References


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