THE PACIFYING AND SHRINKING EDGES OF SOME GRAPHS

RAM KUMAR R.¹ AND KANNAN BALAKRISHNAN

ABSTRACT. Given a vertex $v$ of a graph $G$, an edge $e$ of $G^c$ is called a pacifying edge of $v$ if the addition of the edge $e$ to $G$ decreases the eccentricity of $v$ the most. We identify the pacifying edges of the vertices of cycles and a more general class of graphs called Symmetric Even Graphs. An $e$ in $G^c$ is called a shrinking edge of $G$ if the addition of $e$ to $G$ decreases the radius of $G$ the most. Shrinking edges of the above mentioned classes are identified.

1. INTRODUCTION

Centrality is one of the most important concepts in graph theory as it measures the relative importance of a vertex in the graph. Eccentricity measures how far a vertex is from the furthest in the graph. In some cases it is desirable to reduce the eccentricity of a vertex by introducing additional edges to the graph. This is in fact an optimization problem as the reduction in the eccentricity caused by different set of edges are different. Hence the objective becomes to determine the minimum number of edges that may be added so that the eccentricity is reduced the most. One special case of this problem is when addition of only a single edge is permissible. That is we determine a pair of non-adjacent vertices such that adding an edge between them reduces the eccentricity to a minimum. This has applications in various types of networks such as computer networks where the problem is to connect an optimal pair of nodes such that

¹corresponding author

2010 Mathematics Subject Classification. 05C12, 05C35.

Key words and phrases. Pacifying Edges, Shrinking Edges, Symmetric Even Graphs.

7027
the maximum distance of a node to any part of the network is reduced, social networks where a link between two persons reduces the maximum distance of a person in the network.

We consider only finite simple connected graphs. For the graph $G$, $V(G)$ denotes the vertex set and $E(G)$ denotes the edge set. For a vertex $w$ of $G$ the eccentricity, $e_G(w)$ is $\max_{u \in V} d(u, w)$. When the underlying graph is obvious we shall use $V$, $E$ and $e(w)$ for $V(G)$, $E(G)$ and $e_G(w)$ respectively. The radius of $G$ is the minimum eccentricity among the vertices of $G$ and the diameter of $G$ is the maximum eccentricity among the vertices of $G$. A vertex $v$ is an eccentric vertex of $u$ if $e(u) = d(u, v)$. A graph $G$ is called even if for each vertex $u$ of $G$ there is a unique eccentric vertex, $\bar{u}$, such that $d(u, \bar{u}) = \text{diam}(G)$. Even graphs are referred to as diametrical graphs in [5] and as self-centered unique eccentric point graphs in [7].

Graphs having extremal properties with respect to distance parameters like radius and diameter have been studied extensively. Ore in [6] defined a graph to be diameter maximal if the addition of any edge to the graph decreases the diameter of the graph and gave a characterisation of such graphs. Caccetta and Smyth [1] gave a general form of diameter maximal graphs with edge connectivity $k$, diameter $d$, number of vertices $n$ and having the maximum number of edges.

A graph $G$ is diameter minimal if the deletion of any edge increases the diameter of $G$. This class of graphs were studied by many authors [1–3, 8].

A graph $G$ is called radius minimal if radius of $G - e$ is greater than radius of $G$ for every edge of $G$. Gliviak [2] proved that a graph is radius minimal if and only if it is a tree.

Any graph $G$ such that radius of $G + e \leq$ radius of $G$ for every $e \in G^c$ is called a radially maximal graph. Vizing in [9] found an upper bound on the number of edges in radially maximal graphs and a lower bound was found by Nishanov. Nishanov studied some properties of radially maximal graphs with radius $r \geq 3$ and diameter $2r - 2$.

Knor characterized unicyclic, non-selfcentric, radially-maximal graphs on the minimum number of vertices. He further proved that the number of such graphs is $\frac{1}{18}r^3 + O(r^2)$. It was conjectured that if $G$ is a non-selfcentric radially-maximal graph with radius $r \geq 3$ on the minimum number of vertices then $G$ is planar,
has exactly $3r - 1$ vertices, the maximum degree of $G$ is 3 and the minimum
degree of $G$ is 1. Knor with the help of exhaustive computer search proved this
result for $r = 4$ and 5.

In this paper we focus on the extremal properties of the individual vertices of
the graph.

2. Pacifying edges

For a vertex $w \in G$ an edge $uv$ is called a pacifying edge of $w$ if $e_{G+uv}(w) \leq$
e_{G+xy}(w)$ for all $xy \in E(G)$. It is not necessary that every vertex of a graph
has pacifying edges. One trivial example is the complete graph where every
vertex has eccentricity one. There are other non trivial examples. Take the
complete bipartite graph $K_{m,n}$ where $m, n > 2$. Each vertex of this graph has
eccentricity two. Since $m, n > 2$ by adding an edge between any single pair of
non-adjacent vertices the eccentricity of none of the vertices reduces. In other
words no vertex of $K_{m,n}$ has a pacifying edge. $C_5$ is another example of a graph
in which no vertex has a pacifying edge.

The following is an example of a graph in which some vertices have pacifying
edges while some others do not have any pacifying edge. Here, $xw$, $uy$ and $zv$

![Figure 1](image_url)

are the pacifying edges of $x,y$ and $z$ respectively as they reduce the eccentricity
of these vertices from 2 to 1. But, the vertices $u,v$ and $w$ do not have any
pacifying edge.

An even graph $G$ is symmetric if for every $u \in V(G)$ there exists a $v \in V(G)$
such that $I(u,v) = G$, see Gobel et.al [4]. Hypercubes, even cycles etc are well
known examples of symmetric even graphs. From the definition of symmetric
even graphs it is clear that they are unique eccentric vertex graphs and every
vertex of such a graph is an eccentric vertex. For more examples of symmetric
even graphs and one way of constructing such graphs, see, Gobel et.al [4].
Before we state the next theorem we introduce the following notations.

For integers $i$ and $j$,

\[
    i + n j = \begin{cases} 
        i + j & \text{if } i + j \leq n \\
        i + j - n & \text{if } i + j > n
    \end{cases}
\]

\[
    i - n j = \begin{cases} 
        i - j & \text{if } i - j > 0 \\
        n + i - j & \text{if } i - j \leq 0
    \end{cases}
\]

\textbf{Theorem 2.1.} Consider the odd cycle $C_{2n+1}(n > 2)$ with vertex set $\{v_1, \ldots, v_{2n+1}\}$.

(1) If $n$ is even the pacifying edges of a vertex $v_i$ are

(a) $v_i v_{i+2n+1}$

(b) $v_i v_{i+2n+1(n+1)}$

(c) $v_i v_{i+2n+1(n+2)}$

(d) $v_i v_{i+2n+1(n-1)}$

(e) $v_i v_{i+2n+1(n+1)}$

(f) $v_i v_{i+2n+1}$

(g) $v_i v_{i+2n+1}$

(h) $v_i v_{i+2n+1}$

(2) If $n$ is odd the pacifying edges $v_i$ are $v_i v_{i+2n+1(n+1)}$ and $v_i v_{i+2n+1(n+1)}$.

\textbf{Proof.}

(1) Suppose $n$ is even. Add the edge $v_i v_{i+2n+1}$.

Then we get two cycles, say $C'_1$ and $C'_2$, both containing $v_i$ and having $n + 2$ and $n + 1$ edges. $n + 2$ is even and $v_i$ has eccentricity $\frac{n}{2} + 1$ in $C'_1$ and hence in the graph
$G + v_i v_{i+2n+1}$. Similarly by adding the edge $v_i v_{i+2n+1(n+1)}$ the eccentricity of $v_i$ reduces to $\frac{n}{2} + 1$.

Adding the edge $v_i v_{i+2n+1(n+2)}$ we get cycles $C'_1$ and $C'_2$ where $C'_1$ has $n + 3$ edges and $C'_2$ has $n$ edges and both contain the vertex $v_i$. $C'_1$ has radius $\frac{n}{2} + 1$ and $C'_2$ has radius $\frac{n}{2}$. Therefore $v_i$ has eccentricity $\frac{n}{2} + 1$ in the new graph. Similarly adding the edge $v_{i+2n+1(n-1)}$ reduces the eccentricity of $v_i$ to $\frac{n}{2} + 1$.

Adding an edge between $v_i$ and a vertex other than $v_{i+2n+1(n)}$, $v_{i+2n+1(n+1)}$, $v_{i+2n+1(n+2)}$, $v_{i+2n+1(n-1)}$ we get two cycles $C''_1$ and $C''_2$, both containing $v_i$, and one of them having radius greater than $\frac{n}{2} + 1$. Therefore eccentricity of $v_i$ in such a graph is greater than $\frac{n}{2} + 1$. Now we add an edge between $v_j$ and $v_k$ such that $j, k \neq i$. Let $C'_1$ and $C'_2$ be the resulting two cycles. Take two cases.

**Case-I:** $v_i \in C'_1$ where $|E(C'_1)| < |E(C'_2)|$. That is, $C'_2$ has atleast $n + 2$ edges or radius of $C'_1$ is atleast $\frac{n}{2} + 1$. Assume $d(v_i, v_j) \leq d(v_i, v_k)$. Let $\bar{v}_j$ be the eccentric vertex of $v_j$ in $C'_2$. That is $d(v_j, \bar{v}_j) \geq \frac{n}{2} + 1$. Therefore $d(v_k, \bar{v}_j) \geq \frac{n}{2}$. Since $n > 2$, $\frac{n}{2} > 1$.

$$d(v_i, \bar{v}_j) = \min\{d(v_i, v_j) + d(v_j, \bar{v}_j), d(v_i, v_k) + d(v_k, \bar{v}_j)\}$$

$$\geq \min\{d(v_i, v_j) + \frac{n}{2} + 1, d(v_i, v_k) + \frac{n}{2}\},$$

$d(v_i, \bar{v}_j) = \frac{n}{2} + 1$ only when $d(v_i, v_k) = d(v_i, v_j) = 1$ and this implies $n = 2$. Since $n > 2$ we have $d(v_i, \bar{v}_j) > \frac{n}{2} + 1$. Hence $v_j v_k$ is not a pacifying edge of $v_i$.

**Case-II:** $v_i \in V(C'_2)$ where $E(C'_2) > E(C'_1)$. Here we shall consider two sub cases.

**SubCase-I:** $|E(C'_2)| = n + 2$ and $|E(C'_1)| = n + 1$. Assume $d(v_i, v_j) \leq d(v_i, v_k)$ Let $\bar{v}_j$ be the vertex that is eccentric to both $v_j$ and $v_k$ in $C'_1$. Then $d(v_i, \bar{v}_j) = d(v_i, v_j) + \frac{n}{2}$. But $d(v_i, \bar{v}_j) = \frac{n}{2} + 1$ when $v_i$ is adjacent to $v_j$. In this case we have that the eccentricity of $v_i$ is $\frac{n}{2} + 1$. In other words, for the vertex $v_i$, the edge $v_j v_k$ such that $v_j$ is adjacent to $v_i$ and $d_{C_{2n+1}}(v_j, v_k) = d_{C_{2n+1}}(v_i, v_k) = n$ is a pacifying edge of $v_i$. Thus the edges $v_{i+2n+1(n+1)} v_{i+2n+1(n+1)}$ and $v_{i-2n+1(n+1)} v_{i+2n+1(n+1)}$ are pacifying edges of the vertex $v_i$. 


Subcase-II: \(|E(C'_2)| = n + 3\) and \(|E(C'_1)| = n\). Let \(\bar{v}_j\) be the vertex eccentric to \(v_j\) in \(C'_1\). Then

\[
d(v_i, \bar{v}_j) = \begin{cases}
    d(v_i, v_j) + \frac{n}{2} & \text{if } d(v_i, v_j) < d(v_i, v_k) \\
    d(v_i, v_k) + d(v_j, \bar{v}_j) - 1 & \text{if } d(v_i, v_j) = d(v_i, v_k) \\
    = d(v_i, v_j) + \frac{n}{2} - 1 & \text{if } d(v_i, v_j) = d(v_i, v_k)
\end{cases}
\]

and

\[
d(v_i, v_j) = d(v_i, v_k) = 2 \implies n = 2.
\]

But we have \(n > 2\). Therefore \(d(v_i, v_j) = d(v_i, v_k) \implies \) both are greater than 2. Hence \(d(v_i, \bar{v}_j) \geq \frac{n}{2} + 2\). Therefore \(v_jv_k\) is not a pacifying edge. Hence we assume that \(d(v_i, v_j) < d(v_i, v_k)\). Then \(d(v_i, \bar{v}_j) = d(v_i, v_j) + \frac{n}{2}\).

Thus \(d(v_i, \bar{v}_j) = \frac{n}{2} + 1\) if and if only if \(v_i\) is adjacent to \(v_j\). In other words, for the vertex \(v_i\), the edge \(v_jv_k\) such that \(v_j\) is adjacent to \(v_i\), \(d_{C_2n+1}(v_j, v_k) = n - 1\) and \(d(v_i, v_k) = n\) is a pacifying edge of \(v_i\). Thus the edges \(v_{i+2n+1}v_{i+2n+1n}\) and \(v_{i-2n+1}v_{i+2n+1(n+1)}\) are pacifying edges of the vertex \(v_i\).

Subcase-III: \(|E(C'_2)| > n + 3\). In this case \(e_{C'_2}(v_i) \geq \frac{n}{2} + 2\) or \(e_{C_{2n+1}}(v_i) \geq \frac{n}{2} + 2\).

Thus we get that the pacifying edges of \(v_i\) are precisely

(a) \(v_i v_{i+2n+1n}\)
(b) \(v_i v_{i+2n+1(n+1)}\)
(c) \(v_i v_{i+2n+1(n+2)}\)
(d) \(v_i v_{i+2n+1(n-1)}\)
(e) \(v_{i+2n+1}v_{i+2n+1(n+1)}\)
(f) \(v_{i-2n+1}v_{i+2n+1n}\)
(g) \(v_{i+2n+1}v_{i+2n+1n}\)
(h) \(v_{i-2n+1}v_{i+2n+1(n+1)}\)

(2) Assume \(n\) is odd. Joining \(v_i\) to \(v_{i+2n+1n}\) we get two cycles \(C'_1\) and \(C'_2\) having \(n + 1\) and \(n + 2\) edges respectively. Then \(C'_1\) and \(C'_2\) have radii \(\frac{n+1}{2}\). Therefore the eccentricity of \(v_i\) in both \(C'_1\) and \(C'_2\) is \(\frac{n+1}{2}\) or eccentricity of \(v_i\) in the \(G + v_i v_{i+2n+1n}\) is \(\frac{n+1}{2}\). Similarly by adding the edge \(v_i v_{i+2n+1(n+1)}\) the eccentricity of \(v_i\) reduces to \(\frac{n+1}{2}\).
Now, let \( v_i \) be joined to any vertex other than \( v_{i+2n+1} \) and \( v_{i+2n+1(n+1)} \). Then one of the cycles formed contains atleast \( n + 3 \) edges. That is the radius of that cycle is \( \frac{n+3}{2} \) or eccentricity of \( v_i \) in the new graph is atleast \( \frac{n+3}{2} \). Hence any such edge cannot be a pacifying edge of \( v_i \). Suppose we join \( v_j \) and \( v_k \) where \( j, k \neq i \). Let \( C'_1 \) and \( C'_2 \) be the two cycles formed where \( |E(C'_1)| \leq |E(C'_2)| \). Here we shall consider two cases.

**Case-I:** Suppose \( v_i \in V(C'_1) \). Take the following subcases.

**Subcase-I:** \( |E(C'_1)| = n + 1 \) and \( |E(C'_2)| = n + 2 \). Then \( C'_1 \) is an even cycle. Let \( d(v_i, v_j) < d(v_i, v_k) \). Let \( v_\bar{j} \) be the eccentric vertex of \( v_j \) in \( C'_2 \). \( d(v_i, v_\bar{j}) = d(v_i, v_j) + d(v_j, v_\bar{j}) = d(v_i, v_j) + \frac{n+1}{2} > \frac{n+1}{2} \). Hence \( e_{G+v_jv_k} < \frac{n+1}{2} \). That is, \( v_jv_k \) is not a pacifying edge.

**Subcase-II:** If \( |E(C'_2)| \geq n + 3 \) then the radius of \( C'_2 \geq \frac{n+1}{2} + 1 \). Then \( d(v_i, v_\bar{j}) \geq \frac{n+1}{2} + 1 \) where \( v_\bar{j} \) is the eccentric vertex of \( v_j \) in \( C'_2 \). That is, the eccentricity of \( v_i \) in \( G + v_jv_k \) is atleast \( \frac{n+1}{2} + 1 \). That is \( v_jv_k \) is not a pacifying edge.

**Case-II:** Suppose \( v_i \in V(C'_2) \). We have that \( |E(C'_2)| \geq n + 2 \). Again we consider two subcases cases.

**Subcase-I:** \( |E(C'_2)| = n + 2 \). Let \( v_\bar{j} \) be the eccentric vertex of \( v_j \) in \( C'_1 \).

\[
d(v_i, v_\bar{j}) = \begin{cases} 
  d(v_i, v_j) + d(v_j, v_\bar{j}) & \text{if } d(v_i, v_k) > d(v_i, v_j) \\
  d(v_i, v_j) + d(v_j, v_\bar{j}) - 1 & \text{if } d(v_i, v_k) = d(v_i, v_j)
\end{cases}
\]

\[
d(v_i, v_k) = d(v_i, v_j) \implies d(v_i, v_\bar{j}) = d(v_i, v_j) + \frac{n+1}{2} - 1.
\]

\[
d(v_i, v_k) = d(v_i, v_j) = 1 \implies \text{our cycle is } C_3 \text{ which is not the case.}
\]

Hence \( d(v_i, v_\bar{j}) > 1 \) or \( d(v_i, v_\bar{j}) < \frac{n+1}{2} \).

So \( v_jv_k \) cannot be a pacifying edge of \( v_i \).

If \( d(v_i, v_k) > d(v_i, v_j) \) then

\[
d(v_i, v_\bar{j}) = d(v_i, v_j) + d(v_j, v_\bar{j}) = d(v_i, v_j) + \frac{n+1}{2} > \frac{n+1}{2}.
\]

Hence \( e_{G+v_jv_k}(v_i) > \frac{n+1}{2} \). That is, \( v_jv_k \) is not a pacifying edge.
Theorem 2.2. Let $G$ be a Symmetric Even graph having diameter $d$. Then

1. If $d$ is even then the only pacifying edge of a vertex $v$ is $v\bar{v}$. 
2. If $d$ is odd the pacifying edges of $v$ are
   (a) All edges $vy$ such that $y$ is either $\bar{v}$ or a vertex adjacent to $\bar{v}$.
   (b) All edges $x\bar{v}$ such that $x$ is either $v$ or a vertex adjacent to $v$.

Proof. Let $v_1$ and $v_2$ be vertices such that $d(v_1, v) = r_1, d(v_2, v) = r_2$ and $d(v_1, v) \leq d(v_2, v)$. Now consider the graph $G + v_1v_2$. If $d(v_1, v) = d(v_2, v)$, then $d_{G+v_1v_2}(v, \bar{v}) = d$ and hence the eccentricity of $v$ does not decrease. So we can assume that $d(v_1, v) < d(v_2, v)$. Let $u$ be a vertex belonging to a shortest $v - v_2$ path. If $d(u, v) = m$, then, since $G$ is symmetric even, $d(u, \bar{v}) = m$. Therefore $d(v_2, u) \leq r_2 + m$. $d(v_2, \bar{u}) = r_2 + m - \ell$ implies $d(u, \bar{u}) = d - r_2 - m + r_2 + m - \ell = d - \ell$, a contradiction to fact that $G$ is self centered. Therefore $d(v_2, u) = r_2 + m$. That is the length of the shortest path from $v$ to $\bar{u}$ in $G + v_1v_2$ passing through the edge $v_1v_2$ is $r_1 + 1 + r_2 + m$. Hence $d_{G+v_1v_2}(v, \bar{u}) = \min\{d - m, r_1 + 1 + r_2 + m\}$. Let $w$ be a vertex in the shortest $v - v_2$ path such that $d(w, v) = k$ (ie $d(\bar{w}, \bar{v}) = k$) and\r
\r
$1 + 1 + r_2 + k = d - k$ or $d - k - 1$ according to the parity of $r_1 + r_2 + 1 + d$. For any vertex $x$ such that $d(\bar{v}, x) < k$ we have that $d_{G+v_1v_2}(v, x) < r_1 + r_2 + 1 + k$ and for any vertex $x$ such that $d(\bar{v}, x) > k$ we have $d_{G+v_1v_2}(v, x) < d - k$. That is $\bar{w}$ is an eccentric vertex of $v$ in $G + v_1v_2$. Hence the eccentricity of $v$ is $d_{G+v_1v_2}(\bar{w}, v)$.

Now we shall consider two cases.

1. Assume $d$ is even. When $r_1 + r_2$ is odd $r_1 + r_2 + 1$ is even and hence $r_1 + r_2 + 1 + k = d - k$ or $k = \frac{d}{2} - \frac{r_1 + r_2 + 1}{2}$ and therefore $e_{G+v_1v_2}(v) = r_1 + r_2 + 1 + k = \frac{d}{2} + \frac{r_1 + r_2 + 1}{2}$.

When $r_1 + r_2$ is even $r_1 + r_2 + 1$ is odd and hence $r_1 + r_2 + 1 + k = d - k - 1$ or $k = \frac{d}{2} - \frac{r_1 + r_2 + 2}{2}$ and therefore $e_{G+v_1v_2}(v) = r_1 + r_2 + 1 + d = \frac{d}{2} + \frac{r_1 + r_2}{2}$. □
Thus \( e_{G+v_{1}v_{2}}(v) = \frac{d}{2} + \lceil \frac{r_1 + r_2}{2} \rceil \). This is a minimum when \( r_1 = r_2 = 0 \). That is, the only pacifying edge is \( \bar{v}v \).

(2) Assume \( d \) is odd. When \( r_1 + r_2 \) is odd, \( r_1 + r_2 + 1 \) is even and hence \( r_1 + r_2 + 1 + k = n - k - 1 \) or \( x = \frac{d - 1}{2} - \frac{r_1 + r_2 + 1}{2} \) and therefore \( e_{G+v_{1}v_{2}}(v) = r_1 + r_2 + 1 + d = \frac{d - 1}{2} + \frac{r_1 + r_2 + 1}{2} \). When \( r_1 + r_2 \) is even \( r_1 + r_2 + 1 \) is odd and hence \( r_1 + r_2 + 1 + k = n - k \) or \( x = \frac{d - 1}{2} - \frac{r_1 + r_2}{2} \) and therefore \( e_{G+v_{1}v_{2}}(v) = r_1 + r_2 + 1 + k = \frac{d - 1}{2} + \frac{r_1 + r_2 + 1}{2} \).

Thus \( e_{G+v_{1}v_{2}}(v) = \frac{d}{2} + \lceil \frac{r_1 + r_2 + 1}{2} \rceil \). This is a minimum when \( r_1 = r_2 = 0 \) or \( r_1 = 1, r_2 = 0 \) or \( r_1 = 0, r_2 = 1 \). That is, the pacifying edges are

(a) All edges \( vy \) such that \( y \) is either \( \bar{v} \) or a vertex adjacent to \( \bar{v} \).

(b) All edges \( x\bar{v} \) such that \( x \) is either \( v \) or a vertex adjacent to \( v \).

\[ \square \]

3. Shrinking Edges

Definition 3.1. For a graph \( G \), an edge \( uv \in E(G^c) \) is called a Shrinking Edge if \( \text{rad}(G+uv) \leq \text{rad}(G+xy) \) for every \( xy \in E(G^c) \).

The following corollary identify the shrinking edges of an odd cycle.

Corollary 3.1. (to theorem 2.1) Consider the cycle \( C_{2n+1} \) having vertex set \( \{v_1, \ldots, v_{2n+1}\} \). An edge \( v_{i}v_{j} \) in \( C_{2n+1}^{c} \) is a shrinking edge if and only if it is the pacifying edge of some vertex \( v_{i} \).

Proof. Let \( n \) be even. If \( v_{i}v_{j} \), an edge of \( C_{2n+1}^{c} \), is a pacifying edge of a vertex \( v_{k} \) then \( e_{G+v_{i}v_{j}}(v_{k}) = \frac{n}{2} + 1 \) and also for all \( v_{r} \neq v_{k} \), we have \( e_{G+v_{i}v_{j}}(v_{r}) \geq \frac{n}{2} + 1 \). Therefore \( \text{rad}(G + v_{i}v_{j}) = \frac{n}{2} + 1 \). By adding a single edge(any of the pacifying edges) the eccentricity of every vertex can be reduced exactly to \( \frac{n}{2} + 1 \). Therefore an edge is a shrinking edge if and only if it is a pacifying edge of some vertex. Similarly the case when \( n \) is odd. Here instead of \( \frac{n}{2} + 1 \) we have \( \frac{n+1}{2} \). \( \square \)

We have a similar result for symmetric even graphs and the proof is also the same.

Corollary 3.2. (to theorem 2.2) Consider the symmetric even graph \( G \). An edge \( uv \) in \( G^c \) is a shrinking edge if and only if it is the pacifying edge of some vertex \( v \).
4. CONCLUSION

Here we introduced the concept of pacifying edges and shrinking edges of the vertices of a graph and the same has been identified for odd cycles and symmetric even graphs. For these classes of graphs, the pacifying edges of any vertex depends on the parity of the radius of the graph. For odd cycles and symmetric even graphs, any edge that is a pacifying edge of some vertex is shown to be a shrinking edge of the graph.

5. ACKNOWLEDGEMENT

The work is a part of the thesis submitted by first author to Cochin University of Science and Technology (CUSAT) for the award of the degree of Doctor of Philosophy (Ph.D) and has not been submitted elsewhere.

REFERENCES


DEPARTMENT OF MATHEMATICS, MAHATMA GANDHI COLLEGE, THIRUVANANTHAPURAM
Email address: ram.k.mail@gmail.com

DEPARTMENT OF COMPUTER APPLICATIONS, COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY, KOCHI
Email address: mullayilkannan@gmail.com