Advances in Mathematics: Scientific Journal **9** (2020), no.9, 7375–7384
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.9.84

THE EFFECT OF PEER TUTORING ON THE PROCESS OF LEARNING MATHEMATICAL PROOFS

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ABSTRACT. Writing mathematical proof and understanding mathematical knowledge are two preeminent skills ideally to be mastered by every student in the Department of Mathematical Sciences (JSM). Despite the abundance of proving techniques, students still face difficulties in the drafting of proof. Therefore, this study seeks to discover the effectiveness of peer tutors in improving students' abilities to devise mathematical proof and to understand mathematical concepts. A peer tutor network is used to determine whether or not students that excel in proving and mathematical comprehension are taken as peer tutors. The aim of this study is to identify peer tutors by measure of in-degree centrality on the network and later analyse characteristics of the peer tutors. On one hand, these tutors should be groomed and given comprehensive understanding since they have a strong ripple effect on the networks of the whole class. On the other hand, students with good scores and understanding of proofs need to be encouraged and incentivized to be peer tutors. The resulting analysis on the network will be able to help educators identify gaps and improve overall students' performance in learning mathematical proof and comprehension.

1. INTRODUCTION

Mathematical proving is an activity that involves skills, understanding and creativity to solve and prove mathematical statements. Although numerous

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2020 *Mathematics Subject Classification.* 97c60, 68m10.
Key words and phrases. in-degree, mathematical comprehension, networks, peer tutor.

studies have been developed such as two-column proof by [1], the Reasoning Control Matrix for the Proving Process by [2] and the Rangka Enam Baris Tiga Lajur introduced by [3], many students view mathematics as a complex subject.

On the other hand, this study seeks to examine the impact of peer tutors in helping their peers acquire mathematical proving capabilities and mathematical knowledge. Peer tutoring is an academic activity where the referred student (tutors) and the referring student (tutees) collaborate in a learning process that usually occurs naturally outside of the formal teaching and learning sessions. [4] stated that peer tutor is an activity that is able to help both tutees and tutors gain knowledge and skills. It has been found that most students prefer to ask their peers before asking guidance from official tutors or teachers. It is usually beneficial for both parties. [5] found that the peer tutoring process is able to improve students' self-confidence, presentation skills, social skills and organizational skills.

However, peer tutoring can be damaging if the tutors themselves do not have a solid understanding of the content. To gauge the level of understanding of both tutors and tutees, we look at their performances in two particular instances which is firstly a quiz that focuses on mathematical proofs and secondly the final examination that has a general focus on mathematical knowledge and skills. Therefore the objectives of this paper are, (1) to compare students' ability in proving and general mathematical knowledge based on comparison of the quiz and final exam, (2) to analyse peer tutoring dynamics and selection in the learning of mathematical proofs, and (3) to identify peer tutoring dynamics and selection in the learning of general mathematical knowledge.

2. Mathematical Proofs and general Mathematical knowledge

We focus on a group of 138 students in the Department of Mathematical Science (JSM), UKM taught in a class requiring some knowledge of mathematical proofs. We assume that the marks of the given quiz ([6]) represents students' proving capability and that the final exam represents the students' general mathematical comprehension. In Figure 1, the percentage marks for quiz on y-axis represents the percentage marks for the proving based quiz. Meanwhile, the x-axis is the percentage marks for the final examination results representing the general mathematical comprehension of each student.

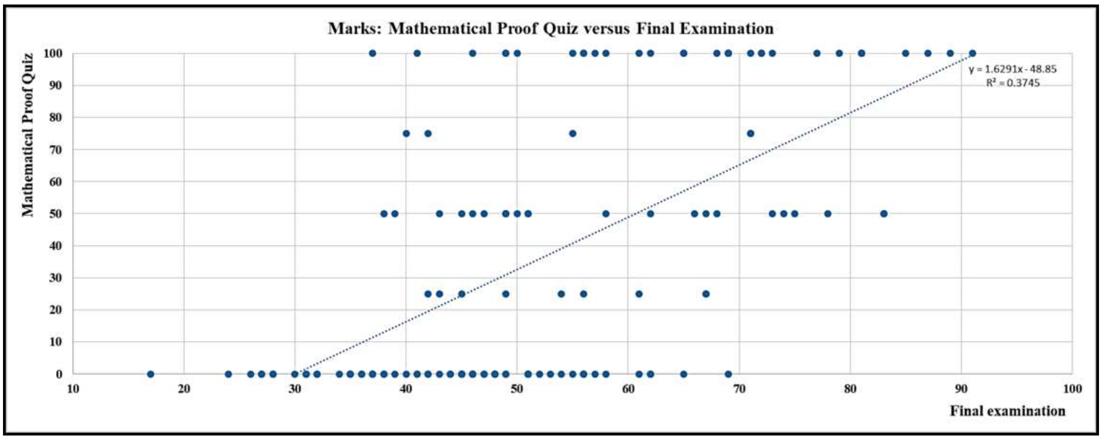

FIGURE 1. Correlation analysis is used to evaluate the strength of the relationship between the quiz and final examination

From Table 1, there is a weak but significant positive correlation between the scores in the quiz and the final examination, $r(138) = 0.612, p < 0.001$. However, Figure 1 highlights that the relationship between the two variables is not strong (correlation coefficient of 1.000 being the strongest) as some of the data points are quite far from the best-of-fit line.

TABLE 1. Regression statistics for the relationship between the quiz and final examination

Regression Statistics	
Multiple R	0.611965239
R Square	0.374501454
Observations	138

In Figure 1, there are five horizontal clusters of data points consisting of highest, average (middle-top, middle, middle-low) and lowest of the score in a mathematical proof. The clusters are formed since the marks for the mathematical proof quiz were given in five different levels. For the most top cluster (100% in proving), there are some number of data points away from the best-of-fit line. Among the data points, five data points scatter to the left of the graph obtaining the failing grade in final examination with the percentage marks in between 37%-49%. This is worrying since this indicates that some students that are good in proving did not perform so well overall or perhaps they lost momentum towards the end of the semester.

On the other side, the lowest cluster (0% in proving) scatters at the bottom of the graph in Figure 1. In this clustering, most of the data points are at the percentage marks between 30%-49% for final examination. This indicates that the students in this cluster did not make progress in their study strategies. The student with low performance in proving generally remained as a low achiever in the final examination. This finding agrees with [7] that the student who devalues mathematics will be less motivated to achieve the goals.

There are some caveats. The quiz covers only initial topics (with less questions) while the final examination covers all topics studied during the semester including some proving. To further analyze the dynamics of the learning process, this study analyzes the peer tutor network and the tutors' proving ability.

3. Peer Tutor Network Visualization

The peer tutor network was obtained from a previous study, [8] on the same set of students. The obtained data is re-analysed to focus on the ability to complete the mathematical proof as opposed to general mathematical comprehension investigated in [8].

A network can be viewed as a diagram that has a set of vertices, V which are connected by a set of edges, E. Such networks are directed when the edges are directed from one vertex to another, [9]. Figure 2 contains a directed network A with four vertices in $V = \{1, 2, 3, 4\}$ representing students and five edges in $E = \{(1, 2), (2, 4), (4, 3), (3, 1), (1, 4)\} = \{e_1, e_2, e_3, e_4, e_5\}$ representing their peer tutoring relationships.

In a directed network, there are two types of degrees; in-degree, k_i^{in}, and out-degree $k_i^{out}, i \in V$. [10] stated that the in-degree is the number of edges that point inwards to vertex. In Figure 2, there are no edges pointing towards vertex 1 thus $k_1^{in} = 0$. And the remaining in-degrees are $k_2^{in} = 1$,$k_3^{in} = 2$ and $k_4^{in} = 2$. Meanwhile, the out-degree, the number of edges pointing outwards from vertex 1 is $k_1^{out} = 3$. The remaining out-degrees are $k_2^{out} = 1$, $k_3^{out} = 0$ and $k_4^{out} = 1$. In short, Students 3 and 4 are the top tutors because they have the highest in-degree and while Student 1 is the least referred person by having the highest out-degree (has a lot of tutors).

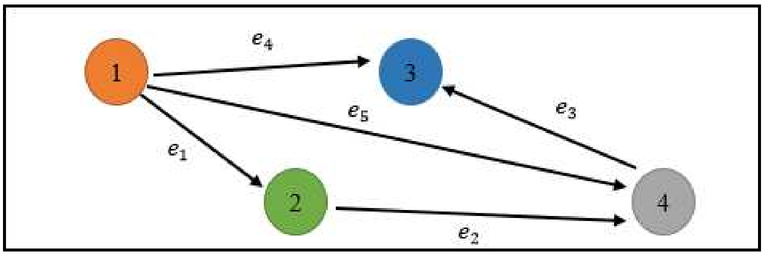

FIGURE 2. Simple directed graph that consists of four vertices and five edges

Therefore, the in-degree of a vertex (student) shall be taken as an indicator of a student's peer tutoring level (the more people refer to you the higher your level would be) since the value of in-degree counts the number of students considering you as their peer tutor. An open-source network analysis tool called Gephi is used for the following network analysis.

4. A NETWORK ANALYSIS OF PEER TUTORING BASED ON MATHEMATICAL PROVING CAPABILITIES MEASURED THROUGH THE QUIZ

Figure 3 is the directed network of peer tutors, where the colours of each vertex indicate the score for mathematical proving based quiz and the size of each vertex represents the value of its in-degree. The aim for the analysis in Figure 3 is to examine whether the students who obtained excellent scores in proving are taken as the peer tutors or otherwise. The darkest coloured vertices are students with the highest score in the quiz. The best case scenario would be that the students who excel in proving (dark blue vertices) also have high in-degree (big vertex size), in other words, students are taking peer tutor that have a strong grasp of proving.

However, our results are not as clear cut as the best case scenario, since the top five vertices with high in-degree number, or the most referred among 138 students are vertices 65, 95, 113, 59 and 131 (descending order). Vertex 65 has the biggest in-degree with 19 students referring to him/her. Vertex 65 has dark blue colour, which means that the tutees are referring to a suitable able student. Vertices 95 and 113 have in-degree 13 respectively. However, vertex 113 has moderate-light blue colour showing that he/she obtained a moderate score in proving. Next, vertices 59 and 131 are referred to by 10 students respectively.

Vertex 59 is dark blue colour, in contrast with vertex 131. Vertex 131 is a white vertex, meaning that the vertex obtained a fail grade in completing the proof. This means that the other 10 students may be taking the wrong person as a peer tutor and this could be detrimental to their learning process. Figure 3 highlights that peer tutors are not necessarily excellent students in mathematical proving.

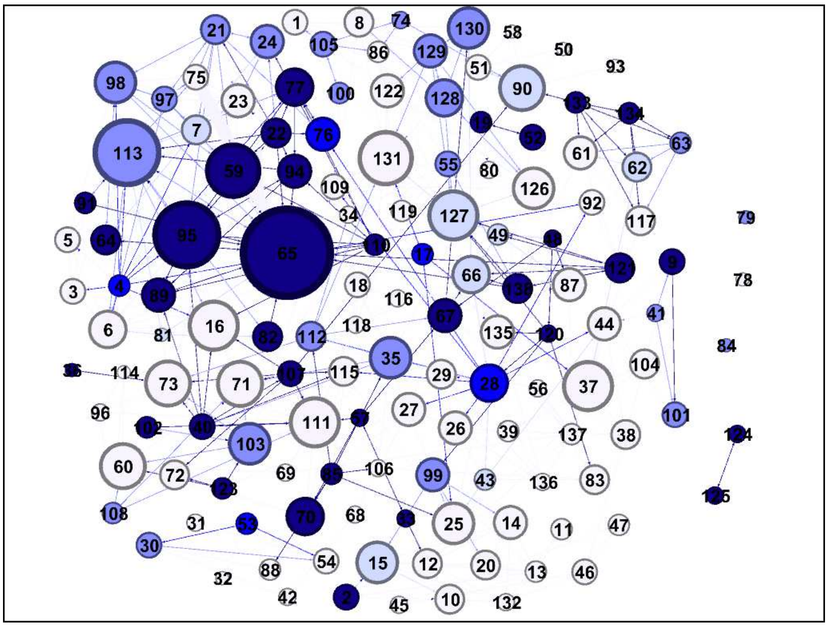

FIGURE 3. Peer tutor network where colours of vertices represent the score in the quiz (the darker the higher the marks) and the size of vertices represents the value of its in-degree (the bigger circle, the higher the in-degree value)

To strengthen the analysis, we plot the in-degree versus percentage marks of the quiz in Figure 4. In the Figure, one can observe that the relationship is not linear. The most referred to student with in-degree 19 did score full marks on the quiz which is a good sign. However, there are also a lot of peer tutors with a significant number of tutees (as much as 10 people referring to them) who scored zero in the quiz. This is a worrying trend as this may be damaging to the class as a whole (through the tutees) where incorrect information is disseminated. Moreover there are also students that scored full marks in the quiz who are not tutoring anybody else.

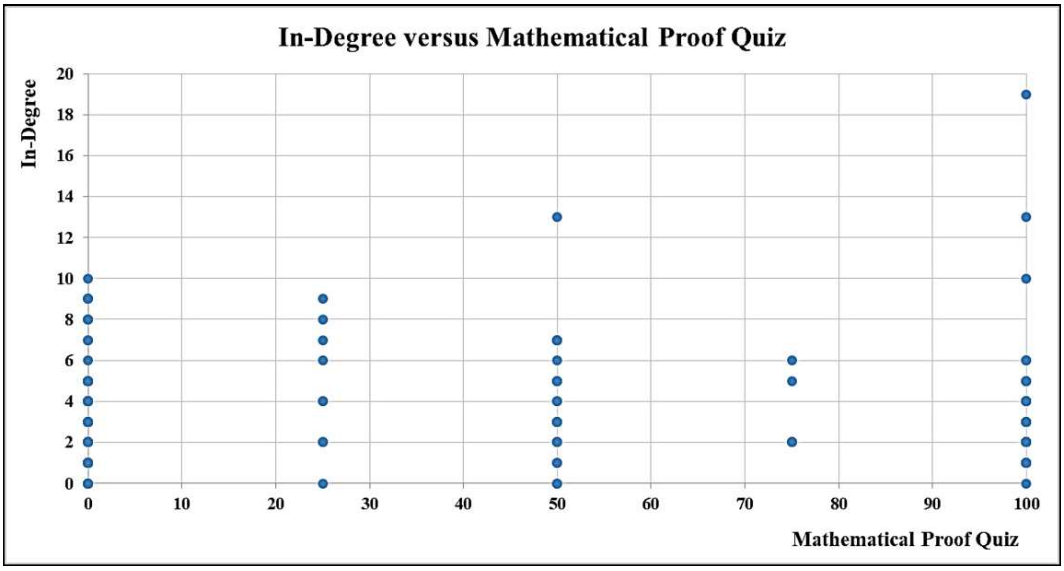

FIGURE 4. Relationship between the peer tutor selection (in-degree) and their mathematical proving abilities (marks of the quiz)

5. A NETWORK ANALYSIS OF PEER TUTORING BASED ON MATHEMATICAL COMPREHENSION MEASURED THROUGH THE FINAL EXAMINATIONS

Figure 5 is the peer tutor network, where the colours of the vertices represent the scores for the final examination and the sizes are for the value of its in-degree. The aim of this analysis is to examine whether students that achieved high marks in final examination are chosen as peer tutors or not.

From the analysis on the quiz (since the in-degree remains the same), the top vertices 65, 95, 113, 59 and 131. Vertex 65 has dark red colour, meaning that the student obtained high marks in the final examination. 19 students are referring to him/her. Similarly, vertices 95 and 113 with 13 tutee students also obtained high scores in the said examination, thus both vertices have red vertices. Next, vertices 59 and 131 have dark and light coloured vertices respectively. Vertex 131 with light red vertex is a student with moderate final examination results (not outright failing as before), perhaps he/she has improved or it may be that teaching others have made the student better in his/her studies.

Based on Figure 6, there is a somewhat positive relationship between peer tutoring and their mathematical knowledge based on the scores for the final

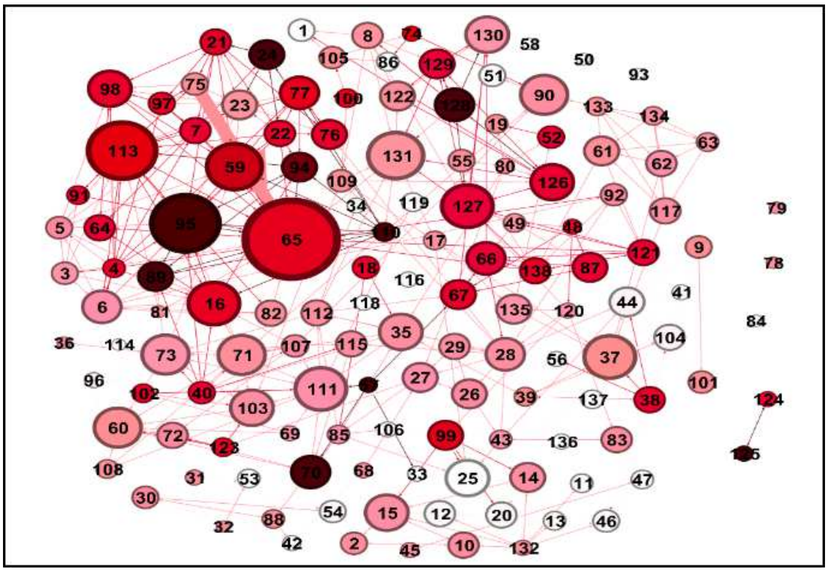

FIGURE 5. Peer tutor network where colours of vertices represent the score in the final exams (the redder the higher the marks) and the size of vertices represents the value of its in-degree (the bigger circle, the higher the in-degree value)

examination and the in-degree of the vertices. The student with the highest in degree of 19, obtained marks of above 70%. Similarly the second and third ranked peer tutor scored around 80% in the finals. This indicates that many students are referring to suitable peer tutors in studying. However, most of data scatter in the middle of the graph with percentage marks in between 40%-60%. These students are referring to the moderate achievers in the final examination.

6. DISCUSSION AND CONCLUSION

The final results in Figures 3 and 5 highlight that high achievers in mathematical proving based quiz and final examination are not necessarily the most referred to by other students. For example, vertex 131 that obtained low marks in both proof quiz and final examination is still taken as a peer tutor by the higher value of its in-degree. However, the remaining top four vertices (vertices 65, 95, 113 and 59) with high in-degree values are high achievers and generally suitable to be tutors. Furthermore, students in vertices 78, 79 and 84 did not

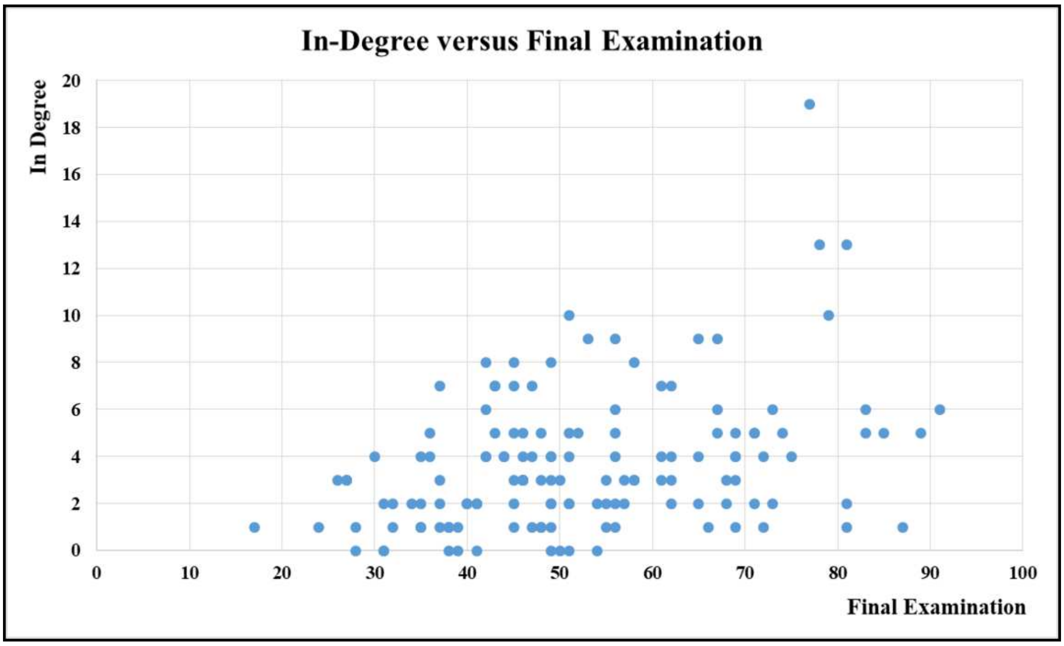

FIGURE 6. Relationship between the peer tutor selection (in-degree) and their general mathematical knowledge (marks of the final examination)

make any connection with other students. It is possible that these three student prefer to ask for help from more experienced individuals such as seniors and teachers. These three students are moderate achievers, and perhaps as an initiative, the teacher can encourage them to join high achievers in their study group sessions.

Figure 1 indicate that proving capabilities does not necessarily reflect overall mathematical capabilities. Therefore, our suggestions for teachers are twofold. Firstly, encourage the high achiever to be peer tutors using some incentives. On the other hand, identify top peer tutors that are not performing so well and perhaps give them a little bit more attention to make them perform better and hopefully they will spread the knowledge to their peers.

Moreover, this network view can help teachers to pay extra attention to low achievers in both proving and mathematical comprehension. An initiative can be made to mix low achievers with high achievers in separate groups assignments to encourage interaction and knowledge dissemination. Further network analysis can be done to scrutinize the results by examining other measures on networks such as degree, closeness centrality and betweenness centrality.

ACKNOWLEDGMENT

The authors acknowledge support from a grant GUP-2019-504 of Universiti Kebangsaan Malaysia.

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