SIMILARITY MEASURE USING SIGN DISTANCE

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ABSTRACT. In this paper, we proposed the sign distance between two trapezoidal intuitionistic fuzzy numbers. Using the sign distance we proposed similarity measure for trapezoidal intuitionistic fuzzy numbers. Some properties are discussed for the proposed method. Finally illustrations are given for that proposed method.

1. INTRODUCTION

Similarity measure are to see how two objects are related together. Many real world application make use of similarity measure. Wang and Xin [4] also analyzed the similarity measure and distance between intuitionistic fuzzy set by proposing some new axioms. Stephen Dinagar and Fany Helena [2,3] studied about the similarity measures of TrIFNs using centroids of horizontal and vertical axes and value $\&$ ambiguity indices and also the Similarity Measure of Generalized Interval-Valued trapezoidal intuitionistic fuzzy numbers. Using above references we proposed a new similarity measure with the help of sign distance.

2020 Mathematics Subject Classification. 03F55,94D05.

Key words and phrases. Trapezoidal Intuitionistic Fuzzy Number, Sign Distance, Similarity Measure.
This paper is arranged as follows: We gave the basic definitions in section 2. In section 3 we proposed the sign distance for trapezoidal intuitionistic fuzzy numbers. In section 4 we proposed similarity measure for TrIFNs using the sign distance and some properties are also discussed followed by illustration. Finally conclusion is given in section 5.

2. PRELIMINARIES

**Definition 2.1.** A trapezoidal intuitionistic fuzzy number $\tilde{A}^i$ with parameters $b_1 \leq a_1, b_2 \leq a_2, a_3 \leq b_3, a_4 \leq b_4$ is denoted as $\tilde{A}^i = [(a_1, a_2, a_3, a_4) (b_1, b_2, b_3, b_4)]$ is the set of real numbers $\mathbb{R}$ is an intuitionistic fuzzy number whose functions are given as

$$
\mu_{\tilde{A}^i}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \leq x \leq a_2 \\
1, & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3}, & \text{if } a_3 \leq x \leq a_4 \\
0, & \text{otherwise}
\end{cases}
$$

$$
\nu_{\tilde{A}^i}(x) = \begin{cases} 
\frac{b_2-x}{b_2-b_1}, & \text{if } b_1 \leq x \leq b_2 \\
0, & \text{if } b_2 \leq x \leq b_3 \\
\frac{x-b_3}{b_4-b_3}, & \text{if } b_3 \leq x \leq b_4 \\
1, & \text{otherwise}
\end{cases}
$$

**Definition 2.2.** The sign distance between $a, b$ are defined as $D(a, b) = a - b$. Since $D(a, 0) = a$ and $D(b, 0) = b$, we have $D(a, b) = D(a, 0) + D(b, 0)$. If $a > 0 D(a, 0) = a$, then $a$ with sign distance $a$. If $a < 0 D(a, 0) = a$, then $a$ with sign distance $-a$.

3. PROPOSED SIGN DISTANCE OF TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS [1]

A trapezoidal intuitionistic fuzzy number is denoted as $\tilde{A}^i = [(a_1, a_2, a_3, a_4) (b_1, b_2, b_3, b_4)]$. Sign distance of $\tilde{A}^i$ can be calculated using the $\alpha$ and $\beta$-cuts of intuitionistic fuzzy number, i.e.,
\[
D^S(\tilde{A}^i, \tilde{B}^i) = \frac{1}{4} \left( \int_0^1 (a_1 + \alpha (a_2 - a_1)) \, d\alpha + \int_0^1 (a_4 - \alpha (a_4 - a_3)) \, d\alpha \right.
+ \int_0^1 (b_1 - (1 - \beta) (b_2 - b_1)) \, d\beta + \int_0^1 (b_4 + (1 - \beta) (b_4 - b_3)) \, d\beta \biggr)
= \frac{a_1 + a_2 + a_3 + a_4 + 3b_1 - b_2 - b_3 + 3b_4}{8}
\]

4. PROPOSED SIMILARITY MEASURE USING SIGN DISTANCE

The similarity measure between two TrIFNs using the sign distance can be calculated as follows:

Algorithm:

Step 1: Calculate \(\tilde{A}^i\) and \(\tilde{B}^i\) i.e. \(\sum_{i=1}^{4} [a_i + b_i]\) and \(\sum_{i=1}^{4} [c_i + d_i]\).

Step 2: Calculate the Sign distance for TrIFNs as
\[
D^S(\tilde{A}^i, \tilde{0}^i) = \frac{a_1 + a_2 + a_3 + a_4 + 3b_1 - b_2 - b_3 + 3b_4}{8}.
\]
Similar for \(D^S(\tilde{B}^i, \tilde{0}^i)\).

Step 3: Finally calculate similarity measure as
\[
S(\tilde{A}^i, \tilde{B}^i) = \left(1 - \frac{|\tilde{A}^i - \tilde{B}^i|}{4}\right) \times \frac{\min \left[ D^S(\tilde{A}^i, \tilde{0}^i), D^S(\tilde{B}^i, \tilde{0}^i) \right]}{\max \left[ D^S(\tilde{A}^i, \tilde{0}^i), D^S(\tilde{B}^i, \tilde{0}^i) \right]}.
\]


Property 1. \(\tilde{A}^i\) and \(\tilde{B}^i\) are identical iff \(S(\tilde{A}^i, \tilde{B}^i) = 1\).
Proof. (i) \(\sum_{i=1}^{4} (a_i + b_i) = \sum_{i=1}^{4} (c_i + d_i)\) and \(DS(\hat{A}^i, \hat{0}^i) = DS(\hat{B}^i, \hat{0}^i) = 1\).

Therefore \(S(\hat{A}^i, \hat{B}^i) = 1\).

(ii) If \(S(\hat{A}^i, \hat{B}^i) = \left[1 - |\hat{A}^i - \hat{B}^i|\right] \times \frac{\min\{DS(\hat{A}^i, \hat{0}^i), DS(\hat{B}^i, \hat{0}^i)\}}{\max\{DS(\hat{A}^i, \hat{0}^i), DS(\hat{B}^i, \hat{0}^i)\}} = 1\).

Therefore \(\hat{A}^i\) and \(\hat{B}^i\) are identical. \(\square\)

Property 2. \(S(\hat{A}^i, \hat{B}^i) = S(\hat{B}^i, \hat{A}^i)\)

Proof. The proof is obvious. \(\square\)

Property 3. If \(\hat{A}^i\) and \(\hat{B}^i\) are two real numbers then \(S(\hat{A}^i, \hat{B}^i) = 1 - \frac{|\hat{A}^i - \hat{B}^i|}{4}\).

Proof. \(\hat{A}^i\) and \(\hat{B}^i\) are two real numbers. So, \(\hat{A}^i = [(a_1, a_2, a_3, a_4) (b_1, b_2, b_3, b_4)]\) and \(\hat{B}^i = [(c_1, c_2, c_3, c_4) (d_1, d_2, d_3, d_4)]\). Assume that \(DS(\hat{A}^i, \hat{0}^i) = DS(\hat{B}^i, \hat{0}^i) = 1\).

Therefore, \(S(\hat{A}^i, \hat{B}^i) = \left[1 - \frac{|\hat{A}^i - \hat{B}^i|}{4}\right]\). \(\square\)

4.2. Illustrations.

Example 1. Two Trapezoidal intuitionistic fuzzy numbers are

\(\hat{A}^i = [(0.57, 0.69, 0.7, 0.85) (0.45, 0.6, 0.8, 0.9)]\)

and

\(\hat{B}^i = [(0.56, 0.6, 0.74, 0.89) (0.39, 0.59, 0.83, 0.94)]\).

Using the above algorithm we get \(S(\hat{A}^i, \hat{B}^i) = 0.9768\).

Example 2. Two Trapezoidal intuitionistic fuzzy numbers are

\(\hat{A}^i = [(0.34, 0.59, 0.68, 0.73) (0.29, 0.4, 0.7, 0.82)]\)

and

\(\hat{B}^i = [(0.37, 0.48, 0.74, 0.86) (0.3, 0.4, 0.8, 0.9)]\).

Using the above algorithm we get \(S(\hat{A}^i, \hat{B}^i) = 0.8716\).
5. Conclusion

The similarity is a measure which is used to measure the strength of relationship between two objects, images, documents. According to different objects type similarity calculation method is also different. We may extend this similarity measure to some other fuzzy number in future.

References


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