BASE PARACOMPACTNESS IN BITOPOLOGICAL SPACES

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ABSTRACT. This paper defines base paracompactness in bitopological spaces and also studied the properties of pairwise base paracompact spaces and their relations with other bitopological spaces. In this study, many well known theorems are generalized concerning base paracompactness.

1. INTRODUCTION

The concept of bitopological spaces can be represented as a space of the form \((S, \mu_1, \mu_2)\) where \(\mu_1, \mu_2\) are two topologies on \(S\). This is related to previous study that has been done on bitopological spaces whereby each of the topologies are set of points that has nearby points related to satisfy a set of axioms. Pairwise Hausdorff (P-Hausdorff), Pairwise regular (P-regular), Pairwise normal (P-normal) spaces were explain by Kelly (1963) \[2\] with common several standard result referred to as Tietze extintion. Further work in the field of bitopological spaces was carried out by Kim (1968) \[3\] and Patty (1967) \[4\]. A pairwise open cover of \(S\) and a cover \(\tilde{U}\) of \((S, \mu_1, \mu_2)\) known a pairwise open cover of \((S, \mu_1, \mu_2)\) if \(\tilde{U} \subseteq \mu_1 \cup \mu_2, \tilde{U} \cap \mu_i \neq \emptyset, i = 1, 2\), was explained by Fletcher et al (1969) \[1\]. Bitopological space \((S, \mu_1, \mu_2)\) is pairwise paracompact if it

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has locally finite parallel subtle distinction for every pairwise open cover $\tilde{U}$ of $(S, \mu_1, \mu_2)$ and totally paracompact if it has a locally limited subcover for every open base of it (Ford, 1963, [7]). The relationship between minimum and maximum inductive magnitude in metric spaces was shown and conclusion was drawn that locally compact spaces can be totally paracompact. Base paracompact is a common to the concept of totally paracompactness, Porter (2003) [5] investigated many properties of base paracompact spaces. In this study, the concept of pairwise base paracompact spaces will be introduced. First we will define pairwise bases of the bitopologicl spaces and the concept of pairwise base paracompact, and also introduce some of their properties. In addition we will state a well known definition which will be used to state some theorems and results. In this study $(S, \mu)$ denotes a topological space and $(S, \mu_1, \mu_2)$ denotes the bitopological spaces. Let $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{N}$ and $\mathbb{Q}$ represent the set of all real, integer, natural and rational number respectively. Furthermore $\mu_{coc}$, $\mu_{dis}$, $\mu_{s}$, $\mu_{u}$, $\mu_{l}$, $\mu_{r}$, $\mu_{cof}$ represent cocountable, discrete, sorgenfrey, usual, left ray, right ray and cofinite topologies on a non empty set $S$. The smallest closed set which contains the given set $A$ is represented by $\text{cl}A$, $w(S)$ denotes the weight of the space $S$, therefore, $w(S) = |\tilde{B}|$ where $\tilde{B}$ has the smallest cardinal number among all other bases of $S$.

2. BASE PARACOMPACTNESS IN BITOPOLITICAL SPACES

The concept, properties and relationship among base paracompactness in bitopological spaces and other spaces will be introduced and discussed here. However let us recall two important definitions which will be need in sequance.

**Definition 2.1.** If every $\mu_i$-free variable set of $\tilde{V}$ is included in some $\mu_i$-free variable set of $\tilde{V}$, $i=1,2$. Then a pairwise free variable cover $\tilde{V}$ of $S$ is said to be parallel subtil distinction of a pairwise free variable $\tilde{V}$.

**Definition 2.2.** If for each $s \in S$ and $E \subseteq S$, then we would refer to $E$ as $\mu_{us}$ open and if $E$ is $\mu_1$open (resp $\mu_2$ open) set of $\tilde{U}$.

**Definition 2.3.** If for each $s \in S$, there exist a $\mu_{us}$ neighborhood of $x$ having at least one element in common with a limited number of members of $\tilde{V}$. A subtle distiction $\tilde{V}$ of a pairwise free variable cover $\tilde{U}$ of $S$ is said to be locally limited.
Definition 2.4. \[8\] Let's represent a bitopological space as \((S, \mu_1, \mu_2)\):

- A chosen set \(K\) is said to be pairwise open if \(K\) is equally \(\mu_1\)-open and \(\mu_2\)-open in \(S\).
- A chosen set \(K\) is said to be pairwise closed if \(K\) is equally \(\mu_1\)-closed and \(\mu_2\)-closed in \(S\).
- If the elements of a cover of a bitopological space \((S, \mu_1, \mu_2)\) are members of \(\mu_1\) and \(\mu_2\), the cover is called pairwise open and it includes at minimum one occupied member of each \(\mu_1\) and \(\mu_2\).

Definition 2.5. \[9\] A set \(\tilde{B} \subseteq P(S)\) is said to be pairwise basis of the bitopological space \((S, \mu_1, \mu_2)\) if and only if:

- If \(\tilde{B} \subseteq \mu_1 \cap \mu_2\), i.e., \(\tilde{B}\) is collection of pairwise open sets.
- For all \(s \in S\) and for each open set \(U\) containing \(s\), \(\beta \in \tilde{B}\) exists \(s \in \beta \subseteq \tilde{B}\).

Definition 2.6. A bitopological space \((S, \mu_1, \mu_2)\) is called pairwise base paracompact if there is a pairwise base for \((S, \mu_1, \mu_2)\) with \(w(S) = |\tilde{B}|\). Each pairwise open cover of \(S\) has pairwise locally finite subtle distinction by \(\tilde{B}\) members.

Remark 2.1. A bitopological space \((S, \mu_1, \mu_2)\) is called pairwise base paracompact if both \((S, \mu_1)\) and \((S, \mu_2)\) are base paracompact.

Example 1. Consider \((S, \mu_{cof}, \mu_u)\) is pairwise base paracompact, since \(\mu_{cof}\) and \(\mu_u\) are both base paracompact spaces.

Definition 2.7. A subset \(M\) of space \(S\) is pairwise base paracompact relative to \(S\), if \(\exists\) pairwise open base \(\tilde{B}\) of \(S\), \(w(S) = |\tilde{B}|\), every pairwise open cover (in \(S\)) of \(M\) has pairwise locally limited (in \(S\)) partial refinement \(B \subseteq \tilde{B}, M \subseteq \cup B\).

Lemma 2.1. Each pairwise closed subset of pairwise base paracompact space \(S\) is pairwise base paracompact relative to \(S\).

Proof. Assume \(S\) to be pairwise base paracompact space and \(\tilde{B}\) be a pairwise basis for \(S\), let \(M\) be a pairwise closed of \(S\), and assume \(\tilde{U}\) to be a pairwise open cover in \(S\) of \(M\). The open cover \(\tilde{U} \cup (S \setminus M)\) in \(S\) has a pairwise locally limited open subtle distinction \(B\) by \(\tilde{B}\) members. Thus, \(W = \{\beta \in B, \beta \cap M \neq \emptyset\}\) is pairwise locally limited (in \(S\)) partial subtle distinction of \(\tilde{U}\).

Theorem 2.1. Assume \(S\) to be a pairwise base paracompact. If \(M\) is pairwise closed of \(w(S) = w(M)\), in that case \(M\) is pairwise base paracompact.
Proof. By Lemma 2.1, M is pairwise base paracompact relative to S, because w(S)=w(M) and M is pairwise base paracompact. □

Definition 2.8. A pairwise cover $\tilde{A}$ of a set S is pairwise star finite subtle distinction of pairwise cover $\tilde{B}$ if for each $\beta \in \tilde{B}$, $\exists A \in \tilde{A}$, star ($\beta, \tilde{B}$) = $\cup\{c \in \tilde{B}, c \cap \beta \neq \emptyset\}$ $\subseteq \tilde{A}$.

Remark 2.2. Each open cover of a paracompact space has an open star subtle distinction.

Theorem 2.2. S is pairwise base paracompact, If S is pairwise paracompact, and it is denumerable union of closed pairwise base paracompact sets relative to S.

Proof. To show $(S, \mu_1, \mu_2)$ is pairwise base paracompact, it is sufficient to show that both topological spaces $(S, \mu_1)$ and $(S, \mu_2)$ are base paracompact. Let $S = \bigcup_{i=1}^{\infty} F_i$ where every $F_i$ is closed and base paracompact relative to S. Now $\forall i, \exists$ a basis $\tilde{B}$ for S certifies base paracompactness relative to S for $F_i$. Assume that $\tilde{B} = \bigcup_{i=1}^{\infty} \beta_i$, be a ware that $\tilde{B}$ is a base for S and w(S) = $|\tilde{B}|$. In addition, $\tilde{B}$ certifies base paracompactness relative to S for each $F_i$. Assume $\tilde{U}$ to be an open cover for S. Now, there will be a locally limited subcollection $A_o$ of $\tilde{B}$ that covers $F_o$ and seperate $\tilde{U}$. By Remark 2.2, $A_o$ has $(A_o)^*$ which is open star subtle distinction of the open cover $A_o \cup (S \setminus F_o)$ of S. By induction, $\exists A_n \subseteq \tilde{B}$ for $i \leq n$ which covers $F_n$ and seperates $\tilde{U}$. Therefore, assume $(A_n)^*$ to be an star subtle distinction of the cover $A_n \cup (S \setminus F_n)$ of S and also every $(A_i)^*$ for each $i \leq n$. For each j set $V_j = \{v \subseteq A_j : vu \text{ for any } u \in A_j \text{ and for every } i \leq j\}$. Let $s \in S$ being given clearly, let $\bigcup_{i=1}^{\infty} A_i$ which covers S, assume j to be the minimum positive integer such $s \in V$ for some $V \in A_j$. Assume $V \subseteq U \subseteq A_j$ for some $i \leq j$. Then $s \in U \in A_i$, which disconfirm the choice of j. Hence $v \in V_j$ and V covers S. Assume $s \in S$, then $s \in F_i$ for some i. Then $s \in W$ for some $W \in (A_i)^*$. Assume O be neighborhood of s with O $\subseteq W$, which meets limitedly many member of $\bigcup_{i=1}^{\infty} A_i$. Let $V \cap W \neq \emptyset$, for each $v \in V_j$ with $i \leq j$, because $A_j$ seperates $A_{j-1}$, $V \subseteq \text{star}(W, A_i)$ $\subseteq \text{star}(W, (A_i)^*) \subseteq U \subseteq (A_i)$. This contradicts the choice of $V_j$. Hence V is locally finite refinement of U. Similarly, we show that $(S, \mu_2)$ is base paracompact, and hence $(S, \mu_1, \mu_2)$ is pairwise base paracompact. □

Corollary 2.1. Let S represents a pairwise base paracompact. M is base pairwise paracompact, if $M \subseteq S$ is $F_\sigma$ with w(S)=w(M).
Proof. Because M is an $F_{\sigma}$ set, then $M = \bigcup_{n=1}^{\omega} M_n$, where $M_n$ is pairwise closed, by Lemma 2.1 every $M_n$ is pairwise base paracompact relative to M because $w(S) = w(M)$. Now with Theorem 2.2, M is pairwise base paracompact. □

Definition 2.9. A pairwise open cover $\tilde{U} = \{u_\alpha, \alpha \in \Delta\}$ for a bitopological spaces $(S, \mu_1, \mu_2)$ is called pairwise shrinkable $\iff$ a pairwise open cover $\tilde{V} = \{v_\alpha, \alpha \in \Delta\}$ such that $cl_1 v_\alpha, cl_2 v_\alpha \subseteq u_\alpha$.

Definition 2.10. A bitopological space $(S, \mu_1, \mu_2)$ is called pairwise locally base paracompact, if for each $s \in S$, a pairwise open neighborhood $O_s$ of $s$ exists then $cl_1 O_s, cl_2 O_s$ are base paracompact spaces.

Theorem 2.3. A pairwise paracompact, pairwise locally base paracompact are base paracompact spaces.

Proof. Let $s \in S$; let $O_s$ be a pairwise open neighborhood of $s$; $cl_1 O_s, cl_2 O_s$ are pairwise base paracompact; $O = \{O_s, s \in S\}$ be a pairwise open cover of $(S, \mu_1, \mu_2)$. Since $(S, \mu_1, \mu_2)$ is pairwise base paracompact, O has a pairwise locally limited open subtile distinction $V$, $|V| \leq w(S)$. Assume W shrinks O, that for each $\omega \in W$ there will be $O_\omega \in O$ such that $cl_1 W \subseteq O_\omega$ and $cl_2 W \subseteq O_\omega$. Hence, $clW$ will be pairwise base paracompact for $\omega \in W$. Assume $B_x$ to be a pairwise abasis for $O_\omega$ which certifies base paracompactness relative to $clW$. Suppose $B = \{B_\omega, \omega \in W\}$ and assume $\tilde{U}$ to be a pairwise open cover of $(S, \mu_1, \mu_2)$. Consider $\tilde{U}_\omega = \{U, U \cap clW \neq \emptyset\}$ there is a pairwise locally limited (in S) subtile distinction $B'_\omega$ of $\tilde{U}_\omega$, $\cup B'_\omega \subseteq O_\omega$. It should be noted that $B'_\omega$ is pairwise locally limited in S. Therefore, $B' = \cup\{B'_\omega, \omega \in W\}$ is pairwise locally limited subtile distinction of $\tilde{U}$. □

Definition 2.11. A pairwise bases $\tilde{B}$ for a topological space $(S, \mu_1, \mu_2)$ is pairwise steady if for each point $s \in S$ and any pairwise neighborhood $U$ of $s$, a pairwise neighborhood $V \subseteq U$ of the point $s$ exist, the set of all elements of $\tilde{B}$ meets finitly many of both $V$ and $S-U$.

Theorem 2.4. The sets of greatest members from $\tilde{B}$, $(\tilde{B})^m = \{\beta \in \tilde{B}, if \beta \subseteq \beta' \in \tilde{B}, then \beta = \beta'\}$ is a pairwise locally finite cover of $(S, \mu_1, \mu_2)$, if $\tilde{B}$ is pairwise steady bases for a space $(S, \mu_1, \mu_2)$.

Proof. We shall show $\cup(\tilde{B})^m = S$, for each $s \in S$ a pairwise $U_0$. Let $s$ not be contained in any member of $(\tilde{B})^m$. Thus, we can define an infinite sequence
of $\tilde{B}$ elements, $U_i \neq U_{i+1}$ for $i=1,2,\ldots$, i.e., an infinite subfamily of pairwise open sets $\{U_i, 1 \leq i \leq \infty\}$ of $\tilde{B}$ whose members contain $s$ and meet $S-U_0$, which is impossible, hence $s \in (\tilde{B})^m$ and $\cup (\tilde{B})^m = S$. Now it can be seen if the pairwise base $\tilde{B}$ is pairwise regular, then the cover $(\tilde{B})^m$ is $(\tilde{B})^m$ which contains $s$, the set of all $\tilde{B}$ elements that meets both $S-U$ and $s$ and a pairwise neighborhood $V$ of $s$. However, every $U' \in (\tilde{B})^m - V$ that contains $s$ (meets $V$) also meets $S-U$, so that only finitly many members of $(\tilde{B})^m$ contains $s$ (meets $V$) which shows that $(\tilde{B})^m$ is pairwise locally finite cover of $S$. $\square$

**Definition 2.12.** A pairwise basis $\tilde{B}$ for a topological space $(S, \mu_1, \mu_2)$ is pairwise steady if for each point $s \in S$ and any pairwise neighborhood $U$ of $s$, exist a pairwise neighborhood $V \subseteq U$ of the point $s$, the set of all $\tilde{B}$ elements that meets both $S-U$ and $s$ and a pairwise neighborhood $V$ of $s$. However, every $U' \in (\tilde{B})^m - V$ that contains $s$ (meets $V$) also meets $S-U$, so that only finitly many members of $(\tilde{B})^m$ contains $s$ (meets $V$) which shows that $(\tilde{B})^m$ is pairwise locally finite cover of $S$. $\square$

**Theorem 2.5.** The sets of greatest members from $\tilde{B}$, $(\tilde{B})^m = \{ \beta \in \tilde{B}, \text{if } \beta \subseteq \beta' \in \tilde{B}, \text{then } \beta = \beta' \}$ is a pairwise locally finite cover of $(S, \mu_1, \mu_2)$, if $\tilde{B}$ is pairwise steady bases for a space $(S, \mu_1, \mu_2)$.

**Proof.** We shall show $\cup (\tilde{B})^m = S$, for each $s \in S$ a pairwise $U_0$. Let $s$ not be contained in any member of $(\tilde{B})^m$. Thus, we can define an infinite sequence $U_0 \subseteq U_1 \subseteq U_2 \subseteq \ldots$ of $\tilde{B}$ elements, $U_i \neq U_{i+1}$ for $i = 1, 2, \ldots$, i.e., an infinite subfamily of pairwise open sets $\{U_i, 1 \leq i \leq \infty\}$ of $\tilde{B}$ whose members contain $s$ and meet $S-U_0$, which is impossible, hence $s \in (\tilde{B})^m$ and $\cup (\tilde{B})^m = S$.

Now it can be seen if the pairwise base $\tilde{B}$ is pairwise regular, then the cover $\tilde{B}^m$ is pairwise locally finite, take a point $s \in S$, with a pairwise open set $U \in (\tilde{B})^m$ which contains $s$, the set of all $\tilde{B}$ elements that meets both $S-U$ and $s$ and a pairwise neighborhood $V$ of $s$. However, every $U' \in \tilde{B}^m-V$ that contains $s$ (meets $V$) also meets $S-U$, so that only finitly many members of $(\tilde{B})^m$ contains $s$ (meets $V$) which shows that $(\tilde{B})^m$ is pairwise locally finite cover of $S$. $\square$

**Lemma 2.2.** A bitological spaces $(S, \mu_1, \mu_2)$ is pairwise measurable if, and only if, it is a pairwise $T_1$-space and poses a pairwise steady base.

**Proof.** First we will check that every pairwise measurable space has a pairwise regular base $(S, \rho, \tau)$ be metric space for $(S, \mu_1, \mu_2)$ and $B_i$ a pairwise locally limited open refinement of the pairwise open cover $\{B(s, \frac{1}{2^i}), s \in S\}$. Clearly $\tilde{B} = \{B_i, 1 \leq i \leq \infty\}$ is pairwise base for $S$. For each point $s \in S$ and for any pairwise neighborhood $U$ of $s$, exist $B(s, \frac{1}{2^i}) \subseteq U$. Let $V_o = B(s, \frac{1}{2^j})$ and for $j =$
1, 2, . . . , let $V_j$ be a pairwise neighborhood of $s$ that meets only finitly many elements $B_i$, which is the set of all elements of $\tilde{B}$ that meets both $V = \cap \{V_j, 0 \leq j \leq i \}$ and $S-U$ is limited.

**Theorem 2.6.** Metric bitopological spaces are pairwise base paracompact spaces.

**Proof.** Assume $(S, \mu_1, \mu_2)$ be a pairwise measurable and suppose $\tilde{B}$ is pairwise steady basis for $S$ and $|\tilde{B}| = w(S)$. Suppose $\tilde{\mathcal{U}}$ be a pairwise open cover of $S$, and suppose $\mathcal{C} = \{\beta \in \tilde{B}, \beta \subseteq u, u \in \tilde{\mathcal{U}}\}$. Note that $\mathcal{C}$ is pairwise steady bases for $S$, and hence $C^m$ is pairwise locally limited subtile distinction of $\tilde{\mathcal{U}}$ by $\tilde{B}$ elements.

**Theorem 2.7.** Let us make $\tilde{B}$ a pairwise basis for a bitopological spaces $S$ with $|\tilde{B}| = w(S)$. Thus, a pairwise bases $\tilde{B}'$ exists for $(S, \mu_1, \mu_2)$ with $|\tilde{B}'| = w(S)$ and $\tilde{B} \subseteq \tilde{B}'$, which is closed under finite unions and finite intersections, and complements of closure.

**Proof.** Suppose $\tilde{B}_0 = \tilde{B}$ and suppose $\tilde{B}_1$ be all finite unions, finite intersections, and complements of clousers by elements of $\tilde{B}_0$. Be aware that $|\tilde{B}_1| = w(S)$. Because $|\tilde{B}| = w(S)$, continue by induction, let $|\tilde{B}_{n-1}| = w(S)$, suppose $\tilde{B}_2$ be all finite unions, finite intersections and complements of clousers by members of $\tilde{B}_{n-1}$. Be aware that $|\tilde{B}_n| = w(S)$. Because $|\tilde{B}_{n-1}| = w(S)$, then the bases $\tilde{B}' = \cup \{B_n, n \leq \omega\}$ is the needed bases.

**Theorem 2.8.** A pairwise regular lindelof spaces are pairwise base paracompact.

**Proof.** Suppose $(S, \mu_1, \mu_2)$ be apairwise steady lindelof space, and suppose $\tilde{B}'$ be a pairwise bases for $S$ with $|\tilde{B}'| = w(S)$. By Theorem 2.7 there is a pairwise base $\tilde{B}$ with $\tilde{B}' \subseteq \tilde{B}$ which is closed under finite unions and finite intersections and complements of clousers. Let $\tilde{U}$ be a pairwise open cover of $S$. Thus, for every $s \in S$, there exist a pairwise open set $V_s, U_s \in \tilde{B}, s \in V_s \subseteq clV_s \subseteq U_s \subseteq U$ for some $U \subseteq \tilde{U}$.

**Definition 2.13.** [6] A map $f : (S, \mu_1, \mu_2) \rightarrow (T, \nu_1, \nu_2)$ is referred to as pairwise-continuous (pairwise open, pairwise closed, pairwise homemorphism, respectively) if the map $f_1 : (S, \mu_1) \rightarrow (T, \nu_1)$ is continuous and $f_2 : (S, \mu_2) \rightarrow (T, \nu_2)$ is continuous (open, closed, homemorphism, respectively).
Definition 2.14. A map \( f : (S, \mu_1, \mu_2) \rightarrow (T, \nu_1, \nu_2) \) is referred to as pairwise-perfect if the function \( f \) is pairwise continuous and pairwise closed and for all \( t \in T \), the set \( f^{-1} \) is pairwise compact.

Theorem 2.9. Let \( f : (S, \mu_1, \mu_2) \rightarrow (T, \nu_1, \nu_2) \) be a pairwise-perfect mapping, then \( S \) is pairwise base paracompact if \( T \) is so.

Proof. Suppose \( \tilde{B}_t \) be a pairwise basis for \( T \) which certifies base paracompactness. Note \( w(S) \geq w(T) \). Let \( \tilde{B}_s \) be a pairwise basis for \( S \) with \( | \tilde{B}_s | = w(S) \) and let \( \tilde{B}'_s = \{ \tilde{B}_s \cup f^{-1}(\tilde{B}), \tilde{B} \in \tilde{B}_t \} \cup \{ \tilde{B} \cap f^{-1}(\tilde{B}'), \tilde{B} \in \tilde{B}_s, \tilde{B}' \in \tilde{B}_t \} \). Claim \( \tilde{B}'_s \) is the pairwise base which witness base paracompactness of \( S \). Now \( | \tilde{B}'_s | = w(S) \). Suppose \( \tilde{U} = \{ U_p, p \in P \} \) be a pairwise open cover for \( S \). For each \( t \in T \), choose a finite subset \( I(y) \subseteq P \), \( f^{-1}(\tilde{B}) \subseteq \{ U_p, p \in I \} \), Since \( f \) is pairwise closed, \( \exists \) a pairwise set \( V_t \) of \( t \); \( f^{-1}(V_t) \subseteq \cup \{ U_p, p \in P \} \). The pairwise cover \( \{ V_t, t \in T \} \) has a pairwise locally limited subtle distinction \( \tilde{B}'_y \subseteq \tilde{B}_t \). Then \( \{ f^{-1}(\tilde{B}), \tilde{B} \in \tilde{B}'_y \} \), is pairwise locally limited and for each \( \tilde{B} \in \tilde{B}'_y \), \( f^{-1}(\tilde{B}) \subseteq f^{-1}(V_t) \subseteq \cup \{ U_p, p \in I(\tilde{B}) \} \), for some \( t \in T \). Now \( \tilde{B}'_x = \{ U_p \cap f^{-1}(\tilde{B}): \tilde{B} \in \tilde{B}'_y \text{and} p \in I(\tilde{B}) \} \) is pairwise locally finite refinement of \( \tilde{U} \) by members of \( \tilde{B}'_x \). Hence \( S \) is pairwise base paracompact.

Corollary 2.2. Let \( S \) be a pairwise base paracompact, and suppose \( T \) be a pairwise compact space. Hence, \( S \times T \) is pairwise base paracompact.

Proof. The projection map \( f: S \times T \rightarrow S \) is pairwise perfect mapping, and hence \( S \times T \) is pairwise base paracompact.

Theorem 2.10. Assuming \( S \) is a pairwise base paracompact space while \( T \) is a pairwise \( \sigma \)-compact. Then \( S \times T \) is pairwise base paracompact.

Proof. Suppose \( T = \cup \{ C_i, i \leq \omega \} \), where every \( C_i \) is a pairwise compact subset of \( T \). Let \( \tilde{B}_s \) be a pairwise base of \( S \) which certify base paracompactness. Also assuming \( \tilde{B}_t \) is a pairwise base for \( T \) with \( | \tilde{B}_t | = w(T) \). Be aware that \( \tilde{B}_s \times \tilde{B}_t \) is a pairwise basis for \( S \times T \) with \( | \tilde{B}_s \times \tilde{B}_t | = w(S \times T) \), Theorem 2.3 satisfies to prove \( S \times C_i \) is pairwise base paracompact relative to \( S \times T \). Hence we will prove that \( \tilde{U} \) poses a pairwise locally limited subcover \( S \times C_i \) by elements of \( \tilde{B}_s \times \tilde{B}_t \). Since \( s \times C_i \) is pairwise paracompact, finitly many elements of \( \tilde{U} \) exist, say \( \tilde{U}_1 \times V_1 \cdots \tilde{U}_n \times V_n \) that cover \( s \times C_i \) and define \( V_s = \{ V_1, \ldots, V_n \} \) and \( W_s = \tilde{U}_1 \cap \cdots \cap \tilde{U}_n \). Be aware that \( \tilde{W} = \{ W_s, s \in S \} \) covers \( S \). suppose \( \tilde{W} \) be a
pairwise locally limited refinement of $W$ by $\tilde{B}_s$ elements. For every $O \in W'$, $O \subseteq W_{so}$, for some $W_{so} \subseteq W$. Then $\cup \{O \times V, V \in V_{so}, O \in W'\}$ is a pairwise locally limited distinction of $\tilde{U}$ covering $S \times C_i$. 

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References


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