THE CONTINUOUS WAVELET TRANSFORM FOR A FOURIER-JACOBI TYPE OPERATOR

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ABSTRACT. The Jacobi operator is generalized by considering a singular differential difference operator \(\Lambda\) on \((0, \infty)\) and harmonic analysis corresponding to generalized Fourier transform is also investigated. To construct and investigate Fourier-Jacobi wavelet transform on half line, tools of harmonic analysis related to \(\Lambda\) is used.

1. INTRODUCTION

The wavelet transform of a function \(f \in L^2(\mathbb{R})\) of the wavelet \(\phi \in L^2(\mathbb{R})\) is defined by

\[
(W_{f\phi})(k, h) = \int_{-\infty}^{\infty} f(p)\tilde{\phi}_{k,h}(p)dp, \quad k \in \mathbb{R}, \ h > 0.
\]

where

\[
\phi_{h,k}(p) = h^{-1/2}\phi\left(\frac{p - k}{h}\right).
\]

In terms of translation \(\tau_b\) defined by

\[
\tau_k\phi(p) = \phi(p - k), \quad k \in \mathbb{R}
\]

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and dilation $D_h$ defined by
\[ D_h \phi(p) = h^{-1/2} \phi\left(\frac{p}{h}\right), \quad h > 0, \]
we can write
\[ \phi_{h,k}(p) = \tau_k D_k \phi(p). \]

It is known from (1.1), (1.2) and (1.3) that wavelet transform for a function is an integral transform and its kernel is dilated translate of wavelet $\phi$.

The wavelet transform (1.1) can also express in convolution:
\[ (W_{a,\phi} f)(k, h) = (f \ast g_{0,h})(k), \]
where
\[ g(p) = \bar{\phi}(-p). \]

2. Preliminaries

The generalized Legendre function $P_{\gamma}^{(\sigma_1, \sigma_2)}(y)$ defined by
\[ P_{\gamma}^{(\sigma_1, \sigma_2)}(y) = \frac{(1 + |y|)^{\sigma_2/2}}{\Gamma(1 - \sigma_1)(|y| - 1)^{\sigma_1/2}} \cdot F[\gamma + \frac{\sigma_2 - \sigma_1}{2} + 1, -\gamma + \frac{\sigma_2 - \sigma_1}{2}; 1 - \sigma_1; \frac{1 - |y|}{2}], \quad y \in \mathbb{R}^n, \]
where $F[u,v;w;z]$ denotes the Gauss hypergeometric function is a generalization of the Jacobi polynomial [7,p.343]. It reduces to the Jacobi polynomial $P_{\gamma}^{(\sigma_1, \sigma_2)}(y)$ for $\gamma = n$, a non-negative integer. Integral transforms along with generalized Legendre functions as kernels have been investigated by Braaksma and Meulenbeld [1]. Theory and application of these transforms can also be found in [2-8]. The convolution theory developed by Flensted-Jensen and Koornwinder [5] is basis for the present work. The following normalized form will be used in the sequel
\[ R_{\gamma}^{(\sigma_1, \sigma_2)}(y) = P_{\gamma}^{(\sigma_1, \sigma_2)}(y) / P_{\gamma}^{(\sigma_1, \sigma_2)}(1), y \in \mathbb{R}^n. \]

Let $\text{ch}(x)$ denote $\cosh(x)$ and $\text{sh}(x)$ denote $\sinh(x)$. Then set
\[ \phi_{\chi}(x) = R_{\gamma}^{(\sigma_1, \sigma_2)}(\sigma_1, \sigma_2)(\text{ch}2x). \]
Also, from [8] we know that $\phi_\lambda(t)$ is a solution of the IVP

$$\frac{1}{\Lambda(x)} \frac{d}{dx} \left( \Lambda(x) \frac{d}{dx} u(x) \right) = \Lambda u(x) = -(\chi^2 + \rho^2) u(x) \quad \text{where} \quad u(0) = 1, \ u'(0) = 0,$$

where

$$\Delta(x) = (e^x + e^{-x})^{2\sigma_2 + 1} (e^x + e^{-x})^{2\sigma_1 + 1} = 2^{2\rho} (shx)^{2\sigma_1 + 1} (chx)^{2\sigma_2 + 1},$$

$\rho = \sigma_1 + \sigma_2 + 1 > 0$. Let $\phi_\chi(x)$ be the second kind Jacobi function is a solution of (2.1) such that

$$\Phi_\chi(x) = 2^{\rho_i} [1 + o(1)]$$

as $x \to \infty$.

Thus

$$\Phi_\chi(x) = (e^x + e^{-x})^{i(x-\rho)} F\left( \frac{\sigma_2 - \sigma_1 + 1 - i\chi}{2}, \frac{\rho - i\chi}{2}; 1 - i\chi; - \frac{1}{(shx)^2} \right).$$

We know that

$$\phi_\chi(x) = c(\chi) \Phi_\chi(x) + c(-\chi) \Phi_\chi(x).$$

Let us define $L^q_{\sigma_1}$, $1 \leq q \leq \infty$, as the class of measurable functions $f$ on the half line for which $\|f\|_{q,\sigma_1} < \infty$, where

$$\|f\|_{q,\sigma_1} = (\int_0^\infty |f(x)|^q d\mu(y))^{1/q}, \text{ if } q < \infty,$$

and

$$\|f\|_{\infty,\sigma_1} = \|f\|_{\infty} = \text{esssup}_{x \geq 0} |f(y)|.$$

The Fourier-Jacobi transform defined for a function $f \in L^1_{\sigma_1}$ is given by

$$F_j(f)(\chi) = \hat{f}(\chi) = \int_0^\infty f(y) \phi_\chi(y) d\mu(y) = (2\pi)^{-1/2} \Lambda(y) dy,$$

and the inverse mapping is given by

$$g(y) = (2\pi)^{-1/2} \int_0^\infty \hat{g}(\chi) \phi_\chi(y) |c(\chi)|^2 d\chi = \int_0^\infty \hat{g}(\chi) \phi_\chi(y) dv(\chi),$$

where

$$dv(\chi) = (2\pi)^{-1/2} |c(\chi)|^2 d\chi$$

and

$$c(\chi) = \frac{2^{\rho - i\chi} \Gamma(i\chi) \Gamma(\sigma_1 + 1)}{\Gamma((\rho + i\chi)/2) \Gamma((\sigma_1 + \sigma_2 + 1 + i\chi)/2)}.$$
As in [5] he convolution is defined by

\[(f_1 * f_2)(y) = \int_0^\infty \int_0^\infty f_1(x)f_2(s)k(y, s, x)d\mu(x)d\mu(s),\]

where

\[K(x_1, x_2, x_3) = \frac{2^{(1/2)-\rho} \Gamma(\sigma_1 + 1)(\text{ch}x_1 \text{ch}x_2 \text{ch}x_3)^{\sigma_1-\sigma_2-1}}{\Gamma(\sigma_1 + (1/2)) (\text{sh}x_1 \text{sh}x_2 \text{sh}x_3)^{2\sigma_1}} \times F(\sigma_1 + \sigma_2, \sigma_1 - \sigma_2; \sigma_1 + 1/2; 1 - B/2),\]

with

\[B = \begin{cases} \frac{(\text{ch}x_1)^2 + (\text{ch}x_2)^2 + (\text{ch}x_3)^2 - 1}{2}, & |x_1 - x_2| < x_3 < x_1 + x_2 \\ 0, & \text{otherwise}. \end{cases} \]

Then \(K(x_1, x_2, x_3)\) satisfies the following properties:

(i) In all the variables \(K(x_1, x_2, x_3)\) is symmetric;
(ii) \(K(x_1, x_2, x_3) \geq 0;\)
(iii) \(\int_0^\infty K(x_1, x_2, x_3)d\mu(x_3) = 1.\)

Also it has been shown that in [5] that

\[(\varphi_1 \varphi_2)(x_3) = \int_0^\infty \varphi_1(x_3)K(x_1, x_2, x_3)d\mu(x_3).\]

Applying (1.2) and (1.3), we have

\[(\widehat{f_1} * \widehat{f_2})(\chi) = \int_0^\infty f_1(x)f_2(x)d\mu(x).\]

An inner product on \(L^2(\mu)\) is defined by

\[\langle f_1, f_2 \rangle = \int_0^\infty f_1(x)f_2(x)d\mu(x).\]

Similar definition is given to \(L^q(\mu)\). From [5] we have the following

**Lemma 2.1.** Let \(1 \leq q < 2, \frac{1}{q} + \frac{1}{s} = 1\) and \(f \in L^q(\mu)\). Then

\[|\widehat{f}(\chi)| \leq \|f\|_q \|\varphi_\chi\|_s.\]

If \(f \in L^1(\mu), \widehat{f}(\mu),\) is continuous in \(\mathcal{D}_1\) and for all \(\chi \in \mathcal{D}_1\)

\[|\widehat{f}(\chi)| \leq \|f\|_1.\]
Theorem 2.1. Let \( q, s, r \) satisfy \( \frac{1}{q} + \frac{1}{s} = 1 + \frac{1}{r}; 1 \leq q, s, r \leq \infty \) for \( f_1 \in L^q(\mu) \) and \( f_2 \in L^s(\mu) \), \( f_1 * f_2 \in L^r(\mu) \) and \( \|f_1 * f_2\|_r \leq \|f_1\|_q \|f_2\|_s \). Moreover, for \( f_1, f_2 \in L^1(\mu) \) we have

\[
(f_1 * f_2 \hat{\chi}(\chi)) = \hat{f}_1(\chi) \hat{f}_2(\chi).
\]

For any \( f_1 \in L^2(\mu) \), the below Parseval identity holds for the Fourier-Jacobi transform:

\[
\int_0^\infty |f_1(x)|^2 d\mu(x) = \int_0^\infty |\hat{f}_1(x)|^2 d\nu(x).
\]

The Fourier-Jacobi translation \( \tau_b \) of \( \varphi \in L^2(\mu) \) defined by

\[
\tau_b \varphi(y) = \varphi(y, b) = \int_0^\infty \varphi(z) K(y, b, z) d\mu(z), 0 < y, b < \infty,
\]

maps \( \tau_b(y) \) defined on the positive half of the real axis into the function \( \varphi(y, b) \) defined on the upper half of the positive half plane. \( \tau_b \) is also called generalized translation. Using Hölder’s inequality it can be shown that

\[
\|\tau_b f_1\|_{L^q(\mu)} \leq \|f_1\|_{L^q(\mu)}
\]

and the map \( y \to \tau_b f_1 \) is continuous for all \( f_1 \in L^q(\mu) \), \( q \in [1, \infty) \).

Definition 2.1. A function \( \omega \in L^2(\mu) \) is a Fourier-Jacobi wavelet, satisfies the condition of admissibility

\[
0 < C^\chi_\omega = \int_0^\infty |\mathcal{F}_j(\omega)(\chi)|^2 \frac{d\chi}{\chi} < \infty.
\]

Definition 2.2. Let \( \omega \in L^2(\mu) \) be a Jacobi wavelet, then for a suitable function \( f \) on \( L^2(\mu) \) the continuous Fourier-Jacobi wavelet transform is defined by

\[
J^\chi_\omega(f)(\sigma_1, \sigma_2) = \int_0^\infty f(y) \omega^\chi_{\sigma_1, \sigma_2}(y) d\mu(y),
\]

where \( \sigma_1 > 0, \sigma_2 \geq 0 \),

\[
\omega^\chi_{\sigma_1, \sigma_2}(y) = \int_0^\infty K(\sigma_2, y, z) \omega(\sigma_1, z) d\mu(z),
\]

and \( \omega_{\sigma_1}(y) = \omega(\sigma_1, y) \).

Theorem 2.2. Let a Fourier-Jacobi wavelet is \( \omega \in L^2(\mu) \). Then

(i) For all \( f \in L^2(\mu) \) then Plancherel formula we have

\[
\int_0^\infty |f(y)|^2 d\mu(y) = \frac{1}{C_\omega} \int_0^\infty \int_0^\infty |J^\chi_\omega(f)(\sigma_1, \sigma_2)|^2 d\mu(\sigma_2) d\mu(\sigma_1).
\]
(ii) Assume that $\|F_j(\omega)\|_\infty < \infty$. For $f \in L^2(\mu)$ and $0 < \varepsilon_1 < \varepsilon_2 < \infty$, the function
\[
f^{\varepsilon_1, \varepsilon_2}(y) = \frac{1}{C_\omega} \int_0^\infty \int_0^\infty J^\mu_\omega(f) \omega^{\mu}_{\varepsilon_1, \varepsilon_2}(y) d\mu(\sigma_2) d\mu(\sigma_1),
\]
belongs to $L^2(\mu)$ and satisfies $\lim_{\varepsilon_1 \to 0, \varepsilon_2 \to \infty} \|f^{\varepsilon_1, \varepsilon_2} - f\|_{2, \mu} = 0$.

(iii) For $f \in L^1(\mu)$ such that $F_\chi(f) \in L^1(\mu)$, we have
\[
f(y) = \frac{1}{C_\omega^\chi} \int_0^\infty (\int_0^\infty J^\mu_\omega(f) \omega^{\mu}_{\varepsilon_1, \varepsilon_2}(y) d\mu(\sigma_2)) d\mu(\sigma_1),
\]
for almost all $y \geq 0$.

3. HARMONIC ANALYSIS RELATED TO FOURIER-JACOBI OPERATOR $\Lambda$

Let the map $N$ be defined by $Nf(y) = \Lambda(y)f(y)$. Let $L^q(\mu)$, $1 \leq q \leq \infty$, be the class of measurable function $f$ on the half line for which $\|f\|_{q, \mu} = \|M^{-1}f\|_{q, \mu} < \infty$.

Generalized Fourier transform
For $\chi \in \mathbb{C}$ and $y \in \mathbb{R}$,
\[
(3.1) \quad \phi_\chi(y) = \Lambda(y)\phi_\chi(y).
\]
The generalized Fourier transform defined for a function $f_1 \in L^1(\mu)$ is given by
\[
(3.2) \quad F_\Lambda(f_1)(\chi) = \int_0^\infty f_1(y)\phi_\chi(y) d\mu(y).
\]

**Theorem 3.1.** Let $f_1 \in L^1(\mu)$ such that $F_\Lambda(f_1) \in L^1(\mu)$. Then for almost all $y > 0$,
\[
f_1(y) = \int_0^\infty F_\Lambda(f_1)(\chi)\phi_\chi(y) d\nu(\chi).
\]

**Proof.** By (3.1), (3.2) and Proposition 2.1(ii) we have
\[
\int_0^\infty (f_1)(\chi)\phi_\chi(y) d\nu(\chi) = \Lambda(y) \int_0^\infty F_{\sigma_1+2n}(M^{-1}f_1)(\chi)\phi_\chi(y) d\nu(\chi)
\]
\[
= \Lambda(y) M^{-1}f_1(y) = f_1(y).
\]
\[\square\]
Theorem 3.2.

(i) For every \( f_1 \in L^1(\mu) \cap L^1(\mu) \) the Plancherel formula we have
\[
\int_0^\infty |f_1(y)|^2d\mu(y) = \int_0^\infty |F_\Lambda(f_1)(\chi)|^2d\nu(\chi).
\]

(ii) Unique isometric isomorphism from \( L^2(\mu) \) onto \( L^2(\mu) \) is extend by generalized Fourier transform \( F_\Lambda \). And its inverse transform is given by
\[
F_\Lambda^{-1}(f_2)(y) = \int_0^\infty f_2(\chi)\phi_\chi(y)d\nu(\chi),
\]
where the integral is converges in \( L^2(\mu) \).

Proof. Let \( f_1 \in L^1(\mu) \cap L^1(\mu) \). By (3.1) we have
\[
\int_0^\infty |F_\Lambda(f_1)(\chi)|^2d\nu(\chi) = \int_0^\infty |F_\Lambda(M^{-1}f_1)(\chi)|^2d\nu(\chi)
\]
\[
= \int_0^\infty |M^{-1}f_1(y)|^2d\mu(y) = \int_0^\infty |f_1(y)|^2d\mu(y)
\]
which concludes that (i) and (ii) can be proved in standard manner. \( \square \)

4. Generalized convolution product

Definition 4.1. Define the generalized translation operator \( T^y \), \( 0 \leq y \), by the relation
\[
T^y f_1(b) = \tau^y_{\chi}(M^{-1}f_1)(b), 0 \leq b,
\]
where \( \tau^y_{\chi} \) is the Jacobi translation operator.

Definition 4.2. The generalized convolution product of two functions \( f_1 \) and \( f_2 \) on half line is defined by
\[
f_1 \ast f_2(y) = \int_0^\infty T^y f_1(b)f_2(b)d\mu(b), 0 \leq y.
\]

Proposition 4.1.

(i) Let \( f \) be in \( L^q(\mu), 1 \leq q \leq \infty \). Then \( \forall 0 \leq y \), the function \( T^y f \) \( f_1 \in L^q(\mu) \), and \( \|T^y f_1\|_{q,\mu} \leq \Lambda\|f_1\|_{q,\mu} \).

(ii) For \( f_1 \in L^q(\mu), q = 1 \) or \( 2 \), we have
\[
F_\Lambda(T^y f_1)(\chi) = \phi_\chi(y)F_\Lambda(f_1)(\chi).
\]
(iii) Let \( q, s \in [1, \infty] \) such that \( \frac{1}{q} + \frac{1}{s} = 1 \). If \( f_1 \in L^q(\mu) \) and \( f_2 \in L^s(\mu) \) then
\[
\int_0^\infty T^y f_1(b) f_2(b) d(b) = int_0^\infty f_1(b) T^y f_2(b) d(b).
\]

(iv) Let \( q, s, r \in [1, \infty] \) such that \( \frac{1}{q} + \frac{1}{s} - 1 = \frac{1}{r} \). If \( f_1 \in L^q(\mu) \) and \( f_2 \in L^s(\mu) \) then \( f_1 \circ f_2 \in L^r(\mu) \) and \( \| f_1 \circ f_2 \|_{r, \mu} \leq \| f_1 \|_{q, \mu} \| f_2 \|_{s, \mu} \).

(v) For \( f_1 \in L^1(\mu) \) and \( f_2 \in L^q(\mu) \), \( q = 1 \) or \( 2 \), we have
\[
F_\lambda (f_1 \circ f_2) = F_\lambda (f_1) F_\lambda (f_2).
\]

5. Generalized Wavelets

Definition 5.1. A generalized wavelet is a function \( \phi \in L^q(\mu) \) satisfying the condition of admissibility
\[
0 < C_\phi = \int_0^\infty |F_\lambda (f_2)(\chi)|^2 \frac{d\nu(\chi)}{\chi} < \infty.
\]

For \( f_2 \in L^2(\mu) \) and \((h, k) \in (0, \infty) \times (0, \infty) \) put
\[
\phi_{h,k}(y) = \int_0^\infty \phi(hz) K(k, y, z) d\mu(z).
\]

Proposition 5.1. For all \( h > 0 \) and \( 0 \leq k \) we have
\[
\phi_{h,k}(y) = k \Lambda(y) (M^{-1} \phi)^{\mu}_{h,k}(y).
\]

Proof. Using (2.13), (2.14) and (3.1) we can easily prove that
\[
\phi_{h,k}(y) = k \Lambda(y) (M^{-1} \phi)^{\mu}_{h,k}(y).
\]

Definition 5.2. Let a generalized wavelet be \( f_2 \in L^2(\mu) \). We define for regular functions \( f \) on the half line, the generalized continuous Fourier-Jacobi wavelet transform is given by
\[
L_\phi (f_1)(h, k) = \int_0^\infty f_1(y) \phi_{h,k}(y) d\mu(y),
\]
or
\[
L_\phi (f_1)(h, k) = f_1 \ast \phi_h(k),
\]
where the generalized convolution product \( \ast \) is given (4.2).
Proposition 5.2. We have

\[ L_\phi(f_1)(h, k) = S_{M^{-1} f_2}^\mu(M^{-1} f_1)(h, k). \]

Proof. From (2.6), (3.1) and (5.4) we deduce that

\[
L_\phi(f_1)(h, k) = \int_0^\infty f_1(y)\phi_{h,k}(y)\,d\mu(y) \\
= \int_0^\infty (M^{-1} f_1)(y)(M^{-1}\phi)_{h,k}(y)\,d\mu(y) = S_{M^{-1} f_2}^\mu(N^{-1} f_1)(h, k),
\]

which concludes the proof. □

Theorem 5.1. (Plancherel formula) Let \( \phi \in L^2(\mu) \) be a generalized wavelet. For every \( f_1 \in L^2(\mu) \), we have Plancherel formula

\[
\int_0^\infty |f_1(y)|^2\,d\mu(y) = 1 \cdot C_\phi \int_0^\infty \int_0^\infty |L_\phi(f_1)(h, k)|^2\,d\mu(k)\,\frac{d\mu(h)}{h}.
\]

Proof. By (2.19) and (5.4) we have

\[
\int_0^\infty \int_0^\infty |L_\phi(f_1)(h, k)|^2\,d\mu(k)\,\frac{d\mu(h)}{h} \\
= \int_0^\infty \int_0^\infty S_{M^{-1} f_2}^\mu(M^{-1} f_1)(h, k)\,d\mu(k)\,\frac{d\mu(h)}{h} \\
= C_{M^{-1} f_2}^\mu \int_0^\infty |M^{-1} f_1(y)|^2\,d\mu(y) = C_\phi \int_0^\infty |f_1(y)|^2\,d\mu(y).
\]

Which concludes the proof. □

Theorem 5.2. (Calderon’s formula) Let a generalized wavelet be \( \phi \in L^2(\mu) \) such that \( \|F_\Lambda(\phi)\|_\infty < \infty \). Then for \( f_1 \in L^2(\mu) \) and \( 0 < \varepsilon_1 < \varepsilon_1 < \infty \), then the function

\[
f_{\varepsilon_1,\varepsilon_2} = \frac{1}{C_\phi} \int_{\varepsilon_1}^{\infty} \int_{\varepsilon_1}^{\infty} L_\phi(f_1)(h, k)\phi_{\varepsilon_1,\varepsilon_2}(y)\,d\mu(k)\,\frac{d\mu(h)}{h}
\]

belongs to \( L^2_{\varepsilon_1,\varepsilon_2} \) and satisfies \( \lim_{\varepsilon_1 \to 0, \varepsilon_2 \to \infty} \|f_{\varepsilon_1,\varepsilon_2} - f_1\|_{2,\mu} = 0 \).

Proof. By (2.19), (3.2) and (5.4) we have

\[
f_{\varepsilon_1,\varepsilon_2} = \frac{\Lambda(y)}{C_{M^{-1} f_2}^\mu} \int_{\varepsilon_1}^{\infty} \int_{\varepsilon_1}^{\infty} S_{M^{-1} f_2}^\mu(M^{-1} f_1)(h, k)(M^{-1} f_2)_{h,k}(y)\,d\mu(k)\,\frac{d\mu(h)}{h}.
\]

□
Theorem 5.3. Let a generalized wavelet be \( \phi \in L^2(\mu) \). If \( f_1 \in L^1(\mu) \) and \( F_\Lambda(f_1) \in L^2(\mu) \) then we have
\[
f_1(y) = \frac{1}{C_\omega} \int_0^\infty \left( \int_0^\infty L_\omega(f_1)(h, k) \phi_{h,k} d\mu(k) \right) \frac{d\mu(h)}{h}
\]
for almost all \( 0 \leq y \).

Proof. By (2.16), (3.1) and (5.4) we have
\[
\frac{1}{C_\omega} \int_0^\infty \left( \int_0^\infty L_\omega(f_1)(h, k) \phi_{h,k} d\mu(k) \right) \frac{d\mu(h)}{h} = \frac{\Delta(y)}{C_{M^{-1}f_2}} \int_0^\infty \left( \int_0^\infty S_{N-1,\phi}^\mu(M^{-1}f_1)(h, k)(M^{-1}f\phi)_{h,k}(y) d\mu(k) \right) \frac{d\mu(h)}{h},
\]
and the result follows from Theorem 2.2. \( \square \)

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